On Pure PO-Ternary Γ-Ideals in Ordered Ternary Γ-Semirings

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Abstract: In this paper, we introduce the concepts of pure po-ternary Γ-ideal, weakly pure po-ternary Γ-ideal and purely prime po-ternary Γ-ideal in an ordered ternary Γ-semiring. We obtain some characterizations of pure po-ternary Γ-ideals and prove that the set of all purely prime po-ternary Γ-ideals is topologized. Note that the results on ternary Γ-semiring without order become then special cases.

Keywords: ternary Γ-semiring; ordered ternary Γ-semiring; weakly regular; pure po-ternary Γ-ideal; weakly pure po-ternary Γ-ideal; purely prime po-ternary Γ-ideal; topology.

I. Introduction

In [1], Ahsan and Takahashi introduced the notions of pure ideal and purely prime ideal in a semigroup. Recently, Bashir and Shabir [2] defined the concepts of pure ideal, weakly pure ideal and purely prime ideal in a ternary semigroup without order. The authors gave some characterizations of pure ideals and showed that the set of all purely prime ideals of a ternary semigroup is topologized. In this paper, we introduce the concepts of pure po-ternary Γ-ideal, weakly pure po-ternary Γ-ideal and purely prime po-ternary Γ-ideal in an ordered ternary Γ-semiring. We characterize pure po-ternary Γ-ideals and prove that the set of all purely prime po-ternary Γ-ideals of an ordered ternary Γ-semiring is topologized. Note that the results on ternary Γ-semiring without order become then special cases.

II. Preliminaries

Definition 2.1: Let T and Γ be two additive commutative semigroups. T is said to be a Ternary Γ-semiring if there exist a mapping from \( T \times T \times T \rightarrow T \) which maps \((x_1, \alpha, x_2, \beta, x_3) \rightarrow [x_1x_2\beta x_3] \) satisfying the conditions:

i) \([\alpha \beta \gamma x]d = [\alpha \beta \gamma y]d = [\alpha \beta \gamma x]d\]

ii) \((a + b)\alpha \beta d = [a\alpha \beta d] + [b\alpha \beta d]\)

iii) \([\alpha a + c]b = [\alpha a + c]b\]

iv) \([\alpha a\beta c + d] = [\alpha a\beta c] + [\alpha a\beta d]\) for all \(a, b, c, d \in T\) and \(\alpha, \beta, \gamma, \delta \in \Gamma\).

Obviously, every ternary semiring \(T\) is a ternary Γ-semiring. Let \(T\) be a ternary semiring and \(\Gamma\) be a commutative ternary semigroup. Define a mapping \(T \times \Gamma \times T \times \Gamma \rightarrow T\) by \(a\beta \gamma x = abc\) for all \(a, b, c \in T\) and \(\alpha, \beta, \gamma \in \Gamma\). Then \(T\) is a ternary Γ-semiring.

Note 2.2: (\(T, \Gamma, +, [\ ]\)) is a ternary Γ-semiring. For nonempty subsets \(A_1, A_2\) and \(A_3\) of \(T\), let \([\Gamma A_1 A_2]\) = \(\{x | a \in A_1, b \in A_2, c \in \Gamma, \alpha, \beta \in \Gamma\}\). For \(x \in T\), let \([x \Gamma A_1 A_2]\) = \(\{x | \Gamma A_1 A_2\}\). The other cases can be defined analogously.

Note 2.3: Let \(T\) be a ternary semiring. If \(A, B\) are two subsets of \(T\), we shall denote the set \(A + B = \{a + b : a \in A, b \in B\}\) and \(2A = \{a + a : a \in A\}\).

Definition 2.4: A ternary Γ-semiring \(T\) is called an ordered ternary Γ-semiring if there is a partial order \(\leq\) on \(T\) such that \(x \leq y\) implies that (i) \(a + c \leq b + c\) and \(c + a \leq c + b\)

(ii) \([a\alpha \beta d] \leq [a\alpha \beta d], [a\alpha \beta d] \leq [a\alpha \beta d]\) and \([a\alpha \beta d] \leq [a\alpha \beta d]\) for all \(a, b, c, d \in T\) and \(\alpha, \beta, \gamma \in \Gamma\).

Note 2.5: For the convenience we write \((x_1, \alpha, x_2, \beta, x_3)\) instead of \([x_1x_2\beta x_3]\).

III. PO-Ternary Γ-Ideals:

Definition 3.1: Let \(T\) be a PO-ternary Γ-semiring. A nonempty subset ‘\(S\)’ is said to be a PO-ternary Γ-subsemiring of \(T\) if

(i) \(S\) is an additive subsemigroup of \(T\),

(ii) \(a\alpha \beta c \in S\) for all \(a, b, c \in S, \alpha, \beta \in \Gamma\).
(iii) $\forall T, s \in S, t \leq s \Rightarrow t \in S.$

**Example 3.2:** Let $T = M_2(\mathbb{Z})$ and $\Gamma = M_2(\mathbb{Z})$ define the ordering as $a_{ij} \leq b_{ij}$. Then $T$ be the PO-ternary $\Gamma$-semiring of the set of all $2 \times 2$ square matrices over $\mathbb{Z}$, the set of all non-positive integers. Let $S = \{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in \mathbb{Z}\}$. Then $S$ is a PO-ternary $\Gamma$-subsemiring of $T$.

**Notation 3.3:** Let $T$ be PO-ternary $\Gamma$-semiring and $S$ be a nonempty subset of $T$. If $H$ is a nonempty subset of $S$, we denote $\{s \in S : s \leq h$ for some $h \in H\}$ by $(H)_S$.

**Notation 3.4:** Let $T$ be PO-ternary $\Gamma$-semiring and $S$ be a nonempty subset of $T$. If $H$ is a nonempty subset of $S$, we denote $\{s \in S : s \leq h$ for some $h \in H\}$ by $(H)_S$.

**Note 3.5:** $(H)_T$ and $(H)_T$ are simply denoted by $(H)$ and $[H]$ respectively.

**Note 3.6:** A nonempty subset $S$ of a PO-ternary $\Gamma$-semiring $T$ is apo-ternary $\Gamma$-subsemiring of $T$ if $(1)S + S \subseteq S$ $(2) STS \subseteq S, (3) S \subseteq S$.

**Theorem 3.7:** Let $S$ be po-ternary $\Gamma$-semiring and $A \subseteq S, B \subseteq S$. Then $(i)$ $A \subseteq (A), (ii) ((A)] = (A), (iii) (A][B][C] \subseteq (A\cap B\cap C)$ and $(iv) A \subseteq B \Rightarrow A \subseteq (B), (v) A \subseteq B \Rightarrow [A] \subseteq [B], (vi) (A \cap B) = (A] \cap (B]$, (vii) $(A \cup B) = (A] \cup (B]$.

**Definition 3.8:** A nonempty subset $A$ of a PO-ternary $\Gamma$-semiring $T$ is said to be left PO-ternary $\Gamma$-ideal of $T$ if $(1) a, b \in A$ implies $a + b \in A$. (2) $b, c \in T, a, \alpha, \beta \in \Gamma$ implies $bac \beta a \in A$. (3) $t \in T, a \in A, t \leq a \Rightarrow t \in A$.

**Note 3.9:** A nonempty subset $A$ of a PO-ternary $\Gamma$-semiring $T$ is a left PO-ternary $\Gamma$-ideal of $T$ if and only if $A$ is additive subsemigroup of $T$. $T^\Gamma TA \subseteq A$ and $(A) \subseteq A$.

**Note 3.10:** Let $T$ be a PO-ternary $\Gamma$-semiring.

Then the set $(T^\Gamma TA) = \{\sum_{i=1}^{n} x_i \alpha_i t_i \beta_i a_n : r, t, \alpha_i, \beta_i \in \Gamma$ and $n \in \mathbb{N}\}$.

**Example 3.11:** In the PO-ternary $\Gamma$-semiring $\mathbb{Z}^0, n\mathbb{Z}^0$ is a left PO-ternary $\Gamma$-ideal for any $n \in \mathbb{N}$.

**Theorem 3.12:** Let $T$ be a PO-ternary $\Gamma$-semiring. Then $(T^\Gamma TA)$ is a left PO-ternary $\Gamma$-ideal of $T$ for all $a \in T$.

**Definition 3.13:** A left PO-ternary $\Gamma$-ideal $A$ of a PO-ternary $\Gamma$-semiring $T$ is said to be the principal left PO-ternary $\Gamma$-ideal generated by $a$ if $A$ is a left PO-ternary $\Gamma$-ideal generated by $\{a\}$ for some $a \in T$. It is denoted by $L(a)$ or $<a>$.

**Theorem 3.14:** If $T$ is a PO-ternary $\Gamma$-semiring and $a \in T$ then

$L(a) = (A)$ where $A = \left\{\sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n : r, t, \alpha_i, \beta_i \in \Gamma$ and $n \in \mathbb{Z}^+$\right\}$, and $\sum$ denotes a finite sum and $\mathbb{Z}^+$ is the set of all positive integer with zero.

**Proof:** Given that $A = \left\{\sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n : r, t, \alpha_i, \beta_i \in \Gamma$ and $n \in \mathbb{Z}^+$\right\}$. Let $a, b \in A$.

$a, b \in A$. Then $a = \sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n$ and $b = \sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n$. For $r, t, \alpha_i, \beta_i \in \Gamma$ and $n \in \mathbb{Z}^+$.

Now $a + b = \sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n + \sum_{r=1}^{n} r \alpha_i t_i \beta_i a_n \Rightarrow a + b$ is a finite sum.

Therefore $a + b \in A$ and hence $A$ is an additive subsemigroup of $T$.

For $t_1, t_2 \in T$ and $a \in A$.

Then $t_1 \alpha t_2 \beta a = t_1 \alpha t_2 (\sum_{r=1}^{n} r \alpha_i t_i \beta_i t_1 t_2 a_n) = \sum_{r=1}^{n} r \alpha_i t_i \beta_i (t_1 \alpha t_2 t_1 t_2 a_n) + n(t_1 \alpha t_2 t_1 t_2 a_n) \in A$.
Therefore \( t_1a_1t_2\beta_2\alpha_2A \) and hence \( A \) is a left ternary \( \Gamma \)-ideal of \( T \). By theorem 3.18, we have \( \{ A \} \) is a left ordered ternary \( \Gamma \)-ideal of \( T \) containing \( a \). Thus \( L(a) \subseteq \{ A \} \). On the other hand, \( L(a) \) is also a left ordered \( \Gamma \)-ideal of \( T \) containing \( a \), so we have \( A \subseteq L(a) \). Thus \( \{ A \} \subseteq L(a) \) since \( \{ A \} \) is a left ordered ternary ideal of \( T \) generated by \( A \). Therefore \( L(a) = \{ A \} \), as required.

**Definition 3.15**: A nonempty subset of a PO-ternary \( \Gamma \)-semiring \( T \) is said to be a lateral PO-ternary ideal of \( T \) if

1. \( a, b \in A \) implies \( a + b \in A \).
2. \( b, c \in T, \alpha, \beta \in \Gamma, a \in A \) implies \( b\alpha a\beta c \in A \).
3. \( t_1T, a \in A, t \leq a \Rightarrow t \in A \).

**Note 3.16**: A nonempty subset \( A \) of a PO-ternary semiring \( T \) is a lateral PO-ternary \( \Gamma \)-ideal of \( T \) if and only if \( A \) is additive subsemigroup of \( T \), \( \Gamma \) \( \subseteq \) \( A \) and \( \{ A \} \subseteq \Gamma \).

**Theorem 3.18**: Let \( T \) be a PO-ternary \( \Gamma \)-semiring. Then \( \{ TT\Gamma\Gamma \} \) is a lateral PO-ternary \( \Gamma \)-ideal of \( T \) for all \( a \in T \).

**Theorem 3.18**: Let \( T \) be a PO-ternary \( \Gamma \)-semiring. Then \( \{ TT\Gamma\Gamma \} \) is a lateral PO-ternary \( \Gamma \)-ideal of \( T \) for all \( a \in T \).

**Definition 3.19**: A lateral PO-ternary \( \Gamma \)-ideal \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is said to be the principal lateral PO-ternary \( \Gamma \)-ideal generated by \( a \) if \( A \) is a lateral PO-ternary \( \Gamma \)-ideal generated by \( \{ a \} \) for some \( a \in T \). It is denoted by \( M(a) \) (or) \( \langle a \rangle \).

**Theorem 3.20**: If \( T \) is a PO-ternary \( \Gamma \)-semiring and \( a \in T \) then

\[
M(a) = \{ A \}, \text{ where } A = \left\{ \sum_{j=1}^{n} r \alpha, a_1 \beta_1, t_1 + \sum_{j=1}^{n} u \alpha, a_2 \beta_2, t_2 : r \alpha, t_1, u \alpha, t_2 \in T, \alpha, \beta_1, \alpha_2, \beta_2 \in \Gamma \right\}
\]

\( \Sigma \) denotes a finite sum and \( z_0^+ \) is the set of all positive integer with zero.

**Definition 3.21**: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is a right PO-ternary \( \Gamma \)-ideal of \( T \) if

1. \( a, b \in A \) implies \( a + b \in A \).
2. \( b, c \in T, \alpha, \beta \in \Gamma, a \in A \) implies \( a\alpha b\beta c \in A \).
3. \( t \in T, a \in A, t \leq a \Rightarrow t \in A \).

**Note 3.22**: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is a right PO-ternary \( \Gamma \)-ideal of \( T \) if and only if \( A \) is additive subsemigroup of \( T \), \( \Gamma \) \( \subseteq \) \( A \) and \( \{ A \} \subseteq \Gamma \).

**Definition 3.23**: A right PO-ternary \( \Gamma \)-ideal \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is said to be a principal right PO-ternary \( \Gamma \)-ideal generated by \( a \) if \( A \) is a right PO-ternary \( \Gamma \)-ideal generated by \( \{ a \} \) for some \( a \in T \). It is denoted by \( R(a) \) (or) \( \langle a \rangle \).

**Theorem 3.24**: If \( T \) is a po-ternary \( \Gamma \)-semiring and \( a \in T \) then

\[
R(a) = \{ A \}, \text{ where } A = \left\{ \sum_{j=1}^{n} a \alpha, r \beta_1, t_1 : r \alpha, t_1 \in T, \alpha, \beta_1 \in \Gamma \right\}
\]

\( \Sigma \) denotes a finite sum and \( z_0^+ \) is the set of all positive integer with zero.

**Definition 3.25**: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is a two sided PO-ternary \( \Gamma \)-ideal of \( T \) if

1. \( a, b \in A \) implies \( a + b \in A \).
2. \( b, c \in T, \alpha, \beta \in \Gamma, a \in A \) implies \( b\alpha a\beta c \in A \).
3. \( t \in T, a \in A, t \leq a \Rightarrow t \in A \).

**Note 3.26**: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is a two sided PO-ternary \( \Gamma \)-ideal of \( T \) if and only if it is both a left PO-ternary \( \Gamma \)-ideal and a right PO-ternary \( \Gamma \)-ideal of \( T \).
Definition 3.27: A two sided PO-ternary \( \Gamma \)-ideal \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is said to be the principal two sided \( \text{PO-ternary } \Gamma \text{-ideal} \) provided \( A \) is a two sided PO-ternary \( \Gamma \)-ideal generated by \( \{ a \} \) for some \( a \in T \). It is denoted by \( T(a) \) (or \( a^{\geq} \)).

Theorem 3.28: If \( T \) is a PO-ternary \( \Gamma \)-semiring and \( a \in T \) then \( T(a) = (A), \) where

\[
A = \left\{ \alpha \sum_{i=1}^{n} r_{i} \alpha_{i} s_{i} \beta_{i} A + \sum_{i=1}^{n} a \alpha_{i} t_{i} \beta_{i} A + \sum_{i=1}^{n} l_{i} \alpha_{i} m_{i} \beta_{i} A \gamma_{i} p_{i} \delta_{i} A + na : \right. \\
\left. \sum_{i=1}^{n} r_{i} s_{i} t_{i} u_{i} l_{i} m_{i} p_{i} q_{i} \in T, \alpha_{i}, \beta_{i}, \gamma_{i}, \delta_{i} \in \Gamma \text{ and } n \in \mathbb{Z}^{+} \right\}
\]

finite sum and \( \varepsilon \) is the set of all positive integer with zero.

Definition 3.29: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is said to be PO-ternary \( \Gamma \)-ideal of \( T \) if

1. \( a, b \in A \) if \( a \in A \) and \( b \in A \) implies \( a + b \in A \)
2. \( b, c \in T, \alpha, \beta, \varepsilon \in \Gamma, a \in A \) implies \( b \alpha c \beta \varepsilon A, b \alpha a \beta c \varepsilon A, a \alpha a \beta c \varepsilon A \).
3. \( T \in T, A, a \in A, t \leq a \Rightarrow t \in A \).

Note 3.30: A nonempty subset \( A \) of a PO-ternary \( \Gamma \)-semiring \( T \) is aPO-ternary \( \Gamma \)-ideal of \( T \) if and only if it is left PO-ternary \( \Gamma \)-ideal, lateral PO-ternary \( \Gamma \)-ideal and right PO-ternary \( \Gamma \)-ideal of \( T \).

Definition 3.31: An element \( a \) of a PO-ternary \( \Gamma \)-semiring. \( T \) is said to be regular if there exist \( x, y \in T \) such that \( a \leq ax \beta y \gamma \delta \) for all \( \alpha, \beta, \gamma, \delta \in \Gamma \).

IV. Pure po-ternary \( \Gamma \)-ideals in ordered ternary \( \Gamma \)-semiring

In this section we define pure po-ternary \( \Gamma \)-ideals in ordered ternary \( \Gamma \)-semiring.

Definition 4.1: Let \( T \) be an ordered ternary \( \Gamma \)-semiring. A two-sided po-ternary \( \Gamma \)-ideal \( A \) of \( T \) is called a left (respectively, right) pure two-sided po-ternary \( \Gamma \)-ideal if for each \( x \in A \) there exist \( y_{i}, z_{i} \in A, \alpha_{i}, \beta_{i} \in \Gamma \) where \( i \in \Delta \) such that \( x \leq \sum_{i=1}^{n} y_{i} \alpha_{i} z_{i} \beta_{i} x \) (respectively, \( x \leq \sum_{i=1}^{n} x \alpha_{i} y_{i} z_{i} \beta_{i} \)). Apo-ternary \( \Gamma \)-ideal \( A \) of \( T \) is called left (respectively, right) pure po-ternary \( \Gamma \)-ideal if for each \( x \in A \) there exist \( y_{i}, z_{i} \in A, \alpha_{i}, \beta_{i} \in \Gamma \) where \( i \in \Delta \) such that \( x \leq \sum_{i=1}^{n} y_{i} \alpha_{i} z_{i} \beta_{i} x \) (respectively, \( x \leq \sum_{i=1}^{n} x \alpha_{i} y_{i} z_{i} \beta_{i} \)). Similarly, we define one-sided left and right pure po-ternary \( \Gamma \)-ideals.

Theorem 4.2: Let \( T \) be an ordered ternary \( \Gamma \)-semiring and \( A \) a two-sided po-ternary \( \Gamma \)-ideal of \( T \). Then \( A \) is right pure po-ternary two-sided \( \Gamma \)-ideal if and only if \( B \alpha A = ([B \Gamma A \Gamma A]) \) for all right po-ternary \( \Gamma \)-ideals \( B \) of \( T \).

Proof: Assume that \( A \) is right pure two-sided po-ternary \( \Gamma \)-ideal. Let \( B \) be a right po-ternary \( \Gamma \)-ideal of \( T \). We have \([B \Gamma A \Gamma A] \subseteq [B \Gamma T \Gamma T] \subseteq B \). Then \([B \Gamma A \Gamma A] \subseteq [B \Gamma T \Gamma T] \subseteq A \), so \([B \Gamma A \Gamma A] \subseteq [A] \subseteq A \). Hence \([B \Gamma A \Gamma A] \subseteq B \). To prove the reverse inclusion, let \( x \in B \alpha A \). By assumption, there exist \( y_{i}, z_{i} \in A, \alpha_{i}, \beta_{i} \in \Gamma \) where \( i \in \Delta \) such that \( x \leq \sum_{i=1}^{n} x \alpha_{i} y_{i} z_{i} \beta_{i} \). Since \( x \in ([x \Gamma T \Gamma T]) \alpha A \), \( x \in ([x \Gamma T \Gamma T]) \alpha ([x \Gamma T \Gamma T]) \subseteq [x \Gamma A \Gamma A] \subseteq ([x \Gamma T \Gamma T]) \alpha A \). Hence \( A \) is right pure two-sided po-ternary \( \Gamma \)-ideal of \( T \).

Definition 4.3: An ordered ternary \( \Gamma \)-semiring \( T \) is said to be right weakly regular if for any \( x \in T, x \in ([x \Gamma T \Gamma T] \Gamma [x \Gamma T \Gamma T]) \).

Note: Every regular ordered ternary \( \Gamma \)-semiring is right weakly regular.

Theorem 4.4: Let \( T \) be an ordered ternary \( \Gamma \)-semiring. The following are equivalent.

(i) \( T \) is right weakly regular.
(ii) \([A \Gamma A \Gamma A] = A \) for all right po-ternary \( \Gamma \)-ideals \( A \) of \( T \).
(iii) \( B \alpha A = ([B \Gamma A \Gamma A]) \) for all right po-ternary \( \Gamma \)-ideals \( B \) and all two-sided po-ternary \( \Gamma \)-ideals \( A \) of \( T \).
(iv) \( B \subset A = ([B \Gamma A \Gamma A]) \) for all right po-ternary \( \Gamma \)-ideals \( B \) and all po-ternary \( \Gamma \)-ideals \( A \) of \( T \).
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Proof: (i) $\Rightarrow$ (ii). Assume that $T$ is right weakly regular.
Let A be a right po-ternary $\Gamma$-ideal of T.
Since $[A \Gamma^A A] \subseteq [A \Gamma T T] \subseteq A$, we have $([A \Gamma^A A] \subseteq A$.
Let $x \in A$. By assumption, $x \in \{\{x \Gamma T T\} \bigcup \{x \Gamma T T\} \bigcup \{x \Gamma T T\}\}$ $\subseteq ([A \Gamma^A A]$. 
Then A $\subseteq ([A \Gamma^A A]$, whence $([A \Gamma^A A] = A$.
(ii) $\Rightarrow$ (i). Assume that $([A \Gamma^A A] = A$ for all right po-ternary $\Gamma$-ideals A of T.
Let $x \in T$. Since $\{\{x \} \bigcup \{x \Gamma T T\}\}$ is a right po-ternary $\Gamma$-ideal of T, we have
$\{\{x \} \bigcup \{x \Gamma T T\}\} = \{\{x \} \bigcup \{x \Gamma T T\}\}$ $\subseteq ([A \Gamma^A A]$.
Hence T is right weakly regular.

(iii) Assume that T is right weakly regular. Let B and A be a right po-ternary $\Gamma$-ideal and a two-sided po-
ternary $\Gamma$-ideal of T, respectively. Since $[B \Gamma^A A] \subseteq [B \Gamma T T] \subseteq B$, $([B \Gamma^A A] \subseteq B$.
Similarly, $([B \Gamma^A A] \subseteq A$. Then $([B \Gamma^A A] \subseteq B \cap A$.
Let $x \in B \cap A$. We have $\{\{x \} \bigcup \{x \Gamma T T\}\} \subseteq ([B \Gamma^A A]$.
By assumption, we get $x \in ([\Gamma T T] \bigcup \{x \Gamma T T\} \bigcup \{x \Gamma T T\}\}$, hence $x \in ([B \Gamma^A A]$.
Thus B/$\cap A \subseteq ([B \Gamma^A A]$, whence B/$\cap A = ([B \Gamma^A A]$.
That (iii) $\Rightarrow$ (iv) is clear.
(iv) $\Rightarrow$ (i). Assume that $B \cap A = ([B \Gamma^A A]$ for all right po-ternary $\Gamma$-ideals B and all po-ternary $\Gamma$-ideals A of T.
To prove that T is right weakly regular, let $x \in T$.
We have $\{\{x \} \bigcup \{x \Gamma T T\}\}$ and $\{\{x \} \bigcup \{x \Gamma T T\}\} \subseteq ([B \Gamma^A A]$ and ideal of T, respectively.
Then $\{\{x \} \bigcup \{x \Gamma T T\}\}$ and $\{\{x \} \bigcup \{x \Gamma T T\}\}$ are right-poternary $\Gamma$- ideal and ideal of T, respectively.

Theorem 4.5. Let T be an ordered ternary $\Gamma$-semiring. The following are equivalent.
(i) T is right weakly regular.
(ii) Every two-sided po-ternary $\Gamma$-ideal A of T is right pure.
(iii) Every po-ternary $\Gamma$-ideal A of T is right pure.
Proof: This follows from Theorems 4.2 and 4.9.

Definition 4.6: An element $a$ of a po-ternary $\Gamma$-semigroup T is said to be zero of T provided $a b a c = a b a c = a b c b a = a$ and $a \leq b \forall \ c \in T, \ a, \ b \in \Gamma$.

Theorem 4.7. Let T be an ordered ternary $\Gamma$-semiring with zero 0.
(i) $\{0\}$ is a right pure po-ternary $\Gamma$-ideal of T.
(ii) Union of any right pure two-sided po-ternary $\Gamma$-ideals (respectively, po-ternary $\Gamma$-ideal) of T is a right pure two-sided po-ternary $\Gamma$-ideal (respectively, po-ternary $\Gamma$-ideals) of T.
(iii) Finite intersection of right pure two-sided ideals (respectively, ideal) of T is a right pure two-sided po-
ternary $\Gamma$-ideal (respectively, po-ternary $\Gamma$-ideals) of T.
Proof: (i) This is obvious.
(ii) Let $A_i, i \in I$ be right pure two-sided po-ternary $\Gamma$-ideals of T. We have $\bigcup_{i \in I} A_i$ is a
two-sided po-ternary $\Gamma$-ideal of T. Let $x \in \bigcup_{i \in I} A_i$. Then $x \in A_j$ for some $j \in I$.
Since $A_j$ is right pure two-sided po-ternary $\Gamma$-ideal, there exist $y, z \in A_j$, $\alpha, \beta \in \Gamma$ such that
$x \leq [x y \beta z]$. Since $\gamma, \delta \in A_j \subseteq \bigcup_{i \in I} A_i$, we have $\bigcup_{i \in I} A_i$ is right pure.
(iii) Let $A_1, A_2, \ldots, A_n$ be right pure two-sided po-ternary $\Gamma$-ideals of T.
Then $\bigcap_{i=1}^{n} A_i$ is a two-sided po-ternary $\Gamma$-ideal of $T$.

Let $x \in \bigcap_{i=1}^{n} A_i$. For $k \in \{1, 2, \ldots, n\}$, there exist $y_k, z_k \in A_k, \alpha, \beta \in \Gamma$ such that $x \leq [x\alpha y_k\beta z_k]$.

We have $x \leq [(x\alpha y_k\beta z_k) \ldots \ldots (y_n\alpha y_{n+1}\beta z_{n+1})]$, since $[(y_1\alpha y_2\beta y_3) \ldots \ldots (y_n\beta z_1)] \in \bigcap_{i=1}^{n} A_i$, we have $\bigcap_{i=1}^{n} A_i$ is a right pure two-sided po-ternary $\Gamma$-ideal of $T$.

**Theorem 4.8:** Let $T$ be an ordered ternary $\Gamma$-semiring with zero 0 and $A$ is a two-sided po-ternary $\Gamma$-ideal of $T$. Then $A$ contains the largest right pure two-sided po-ternary $\Gamma$-ideal of $T$, denoted by $S(A)$. $S(A)$ is called the pure part of $A$.

**Proof:** Since $\{0\}$ is a right pure two-sided po-ternary ideal of $T$ contained in $A$, it follows that the union of all right pure two-sided po-ternary $\Gamma$-ideals of $T$ contained in $A$ exists, and hence is the largest right pure two-sided po-ternary $\Gamma$-ideal of $T$ contained in $A$.

Similarly, we have the following.

**Theorem 4.9:** Let $T$ be an ordered ternary $\Gamma$-semiring with zero 0 and $A$ is a po-ternary $\Gamma$-ideal of $T$. Then $A$ contains the largest right pure po-ternary $\Gamma$-ideal of $T$.

**Theorem 4.10:** Let $T$ be an ordered ternary $\Gamma$-semiring with zero 0. Let $A$, $B$, and $A_0$ be po-two-sided po-ternary $\Gamma$-ideals of $T$.

(i) $S(A \cap B) = S(A) \cap S(B)$.

(ii) $\bigcup_{i \in I} S(A_i) \subseteq S(\bigcup_{i \in I} A_i)$.

**Proof:** (i) Since $S(A) \subseteq A$ and $S(B) \subseteq B$, we have $S(A) \cap S(B) \subseteq A \cap B$.

Hence $S(A) \cap S(B) \subseteq S(A \cap B)$. Since $S(A \cap B) \subseteq A \cap B \subseteq S(A)$, we get $S(A \cap B) \subseteq S(A)$.

Similarly, $S(A \cap B) \subseteq S(A \cap B)$.

Hence $S(A \cap B) = S(A)$.

Similarly, $S(A \cap B) = S(B)$.

(ii) Since $S(A_i) \subseteq A_i$ for all $i \in I$, we have $\bigcup_{i \in I} S(A_i) \subseteq \bigcup_{i \in I} A_i$. Then $\bigcup_{i \in I} S(A_i) \subseteq S(\bigcup_{i \in I} A_i)$.

**Definition 4.11:** A right pure two-sided po-ternary $\Gamma$-ideal $A$ of an ordered ternary $\Gamma$-semiring $T$ is said to be purely maximal if for any proper right pure two-sided po-ternary $\Gamma$-ideal $B$ of $T$, $A \subseteq B$ implies $A = B$.

**Definition 4.12:** A proper right pure two-sided po-ternary $\Gamma$-ideal $A$ of an ordered ternary $\Gamma$-semiring $T$ is said to be purely prime if for any right pure two-sided po-ternary $\Gamma$-ideals $B_1, B_2$ of $T$, $B_1 \cap B_2 \subseteq A$ implies $B_1 \subseteq A$ or $B_2 \subseteq A$.

**Theorem 4.13:** Every purely maximal two-sided po-ternary $\Gamma$-ideal of an ordered ternary $\Gamma$-semiring $T$ is purely prime.

**Proof:** Let $A$ be a purely maximal two-sided po-ternary $\Gamma$-ideal of $T$. Let $B$ and $C$ be right pure two-sided po-ternary $\Gamma$-ideals of $T$ such that $B \cap C \subseteq A$ and $B \not\subseteq A$. Since $B \cup A$ is a right pure two-sided po-ternary $\Gamma$-ideal such that $A \subseteq B \cup A$, so $T = B \cup A$.

We have $C = C \cap T = C \cap (B \cup A) = (C \cap B) \cup (C \cap A) \subseteq A$. Then $A$ is purely prime.

**Theorem 4.14:** The pure part of any maximal two-sided po-ternary $\Gamma$-ideal of an ordered ternary $\Gamma$-semiring $T$ with zero is purely prime.

**Proof:** Let $A$ be a maximal two-sided po-ternary $\Gamma$-ideal of $T$. To show that $S(A)$ is purely prime, let $B, C$ be right pure two-sided po-ternary $\Gamma$-ideals of $T$ such that $B \cap C \subseteq S(A)$. If $B \subseteq A$, then $B \subseteq S(A)$. Suppose that $B \not\subseteq A$.

We have $B \cup A$ is a two-side po-ternary $\Gamma$-ideal of $T$. By maximality of $A$, $T = B \cup A$, and hence $C \subseteq A$.

Thus $C \subseteq S(A)$.

**Theorem 4.15:** Let $T$ be an ordered ternary $\Gamma$-semiring and $A$ a right pure two-sided po-ternary $\Gamma$-ideal of $T$. If $x \in T \setminus A$, then there exists a purely prime two-sided po-ternary $\Gamma$-ideal $B$ of $T$ such that $A \subseteq B$ and $x \not\in B$.

**Proof:** Let $P = \{B \mid B$ is a right pure two-sided po-ternary $\Gamma$-ideal of $T, A \subseteq B$ and $x \not\in B\}$. Since $A \in P$, $P \neq \emptyset$.

Under the usual inclusion, $P$ is a partially ordered set. Let $B_0, k \in K$ be a totally ordered subset of $P$. By Theorem 4.7, $\bigcup_{i \in K} B_i$ is a right pure two-sided po-ternary $\Gamma$-ideal of $T$.

Since $A \subseteq \bigcup_{i \in K} B_i$ and $x \not\in \bigcup_{i \in K} B_i$, we have $\bigcup_{i \in K} B_i \subseteq P$.

By Zorn’s Lemma, $P$ has a maximal element, say $M$. Then $M$ is a right pure two-sided po-ternary $\Gamma$-ideal, $A \subseteq M$ and $x \not\in M$. We shall show that $M$ is purely prime. Let $A_1$ and $A_2$ be right pure two-sided po-ternary $\Gamma$-ideals of $T$ such that $A_1 \subseteq M$ and $A_2 \subseteq M$.

Since $A_1, A_2$ and $M$ are right pure two-sided po-ternary $\Gamma$-ideals of $T$, we obtain $A_1 \cup M$ and $A_2 \cup M$ are right pure two-sided po-ternary $\Gamma$-ideals of $T$ such that $M \subseteq A_1 \cup M$ and

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Let \( M \subset A_2 \cup M \). Thus \( x \in A_2 \cup M \) (\( k = 1, 2 \)). Since \( x \notin M \), \( x \notin A_2 \cap M \). Hence \( A_2 \cap M \subseteq M \). This shows that \( M \) is purely prime.

**Theorem 4.16:** Any proper right pure two-sided po-ternary \( \Gamma \)-ideal \( A \) of an ordered ternary \( \Gamma \)-semiring \( T \) is the intersection of all the purely prime two-sided po-ternary \( \Gamma \)-ideals of \( T \) containing \( A \).

**Proof:** By Theorem 4.15, there exists purely prime po-ternary \( \Gamma \)-ideals containing \( A \).

Let \( \{ B_i : i \in I \} \) be the set of all purely prime two-sided po-ternary \( \Gamma \)-ideals of \( T \) containing \( A \). We have \( A \subseteq \bigcap_{i \in I} B_i \).

To show that \( \bigcap_{i \in I} B_i \subseteq A \). Let \( x \in A \). By Theorem 4.15, there exists purely prime po-ternary \( \Gamma \)-ideal \( B_j \) such that \( A \subseteq B_j \) and \( x \notin B_j \). Hence \( x \notin \bigcap_{i \in I} B_i \).

**V. Weakly pure ideals in ordered ternary po-semirings**

In this section, we introduce the concept of weakly pure po-ternary \( \Gamma \)-ideal in ordered ternary \( \Gamma \)-semiring.

**Definition 5.1.** Let \( T \) be an ordered ternary \( \Gamma \)-semiring. A two-sided po-ternary \( \Gamma \)-ideal \( A \) of \( T \) is called left (respectively, right) weakly pure if \( A \cap B = ([\Gamma A \Gamma] B) \) (respectively, \( A \) \( \cap B = (\{ B \Gamma A \Gamma \}) \)) for all two-sided po-ternary \( \Gamma \)-ideals \( B \) of \( T \).

In an ordered ternary \( \Gamma \)-semiring, every left (right) pure two-sided po-ternary \( \Gamma \)-ideals is left(right) weakly pure.

**Theorem 5.2.** Let \( T \) be an ordered ternary semigroup with zero 0. If \( A \) and \( B \) are two-sided po-ternary \( \Gamma \)-ideals of \( T \), then

\[
B \Gamma A = \{ (x, y, z) \in T \mid x \in A, \alpha \beta \in \Gamma, [x \alpha \beta \in e B] \}
\]

are two-sided po-ternary \( \Gamma \)-ideals of \( T \).

**Proof:** We shall show that \( B \Gamma A \) is a two-sided po-ternary \( \Gamma \)-ideal of \( T \). That \( A \beta \Gamma B \) is a two-sided po-ternary \( \Gamma \)-ideal of \( T \) can be proved similarly. Clearly, \( 0 \in B \Gamma A \). Let \( u, v \in T, \alpha, \beta \in \Gamma \) and \( \alpha \beta \in B \Gamma A \). To show that \( [u \alpha \beta] \in B \Gamma A \), let \( y \in A \). Since \( [x y \mu \nu] \in A \) for \( x, y, \mu, \nu \in A \), we have \( [x y u \alpha \beta v] = [x y u \alpha \beta v] \beta \in B \Gamma A \). Thus \( [u \alpha \beta] \in B \Gamma A \).

Conversely, assume that \( (B \Gamma A) \cap A = A \cap B \) for all po-ternary \( \Gamma \)-ideals \( B \) of \( T \). To show that \( A \) is left weakly pure two-sided po-ternary \( \Gamma \)-ideal.

Let \( C \) be any po-ternary \( \Gamma \)-ideal of \( T \). To show that \( \Lambda \cap C = ([\Gamma A \Gamma] C) \cap A \). By assumption, \( \Lambda \cap C = \Gamma A \Gamma \cap A \). Since \( [\Gamma A \Gamma] C \subseteq \Gamma A \Gamma \cap A \), \( ([\Gamma A \Gamma] C) \cap A \subseteq \Gamma A \Gamma \cap A \).

**Theorem 5.3.** Let \( T \) be ordered ternary \( \Gamma \)-semiring and \( A \) two-sided po-ternary \( \Gamma \)-ideal of \( T \). Then \( A \) is left (right) weakly pure two-sided po-ternary \( \Gamma \)-ideal if and only if \( (B \Gamma A^{-1}) \cap A = A \cap B \) for all po-ternary \( \Gamma \)-ideals \( B \) of \( T \).

**Proof:** Suppose that \( A \) is left weakly pure two-sided po-ternary \( \Gamma \)-ideal. Let \( B \) be a po-ternary \( \Gamma \)-ideal of \( T \). By Theorem 5.2, \( B \Gamma A^{-1} \) is a two-sided po-ternary \( \Gamma \)-ideal of \( T \), and thus \( A \cap B = ([\Gamma A \Gamma] B) \cap A \).

Conversely, assume that \( (B \Gamma A^{-1}) \cap A = A \cap B \) for all po-ternary \( \Gamma \)-ideals \( B \) of \( T \). To show that \( A \) is left weakly pure two-sided po-ternary \( \Gamma \)-ideal.

Let \( C \) be any po-ternary \( \Gamma \)-ideal of \( T \). To show that \( \Lambda \cap C = ([\Gamma A \Gamma] C) \cap A \). By assumption, \( \Lambda \cap C = \Gamma A \Gamma \cap A \). Since \( [\Gamma A \Gamma] C \subseteq \Gamma A \Gamma \cap A \), \( ([\Gamma A \Gamma] C) \cap A \subseteq \Gamma A \Gamma \cap A \).

**Theorem 5.4:** Let \( T \) be an ordered ternary \( \Gamma \)-semiring. The following are equivalent.

(i) Every two-sided po-ternary \( \Gamma \)-ideal is left weakly pure two-sided po-ternary \( \Gamma \)-ideal.

(ii) For every two-sided po-ternary \( \Gamma \)-ideal \( A \) of \( T \), \( [\Gamma A \Gamma] A = A \). i.e. each two-sided po-ternary \( \Gamma \)-ideal is idempotent.

(iii) Every two-sided po-ternary \( \Gamma \)-ideal is right weakly pure two-sided po-ternary \( \Gamma \)-ideal.

**Proof:** (i) \( \Rightarrow \) (ii) Suppose that each two-sided po-ternary \( \Gamma \)-ideal of \( T \) is left weakly pure. Let \( A \) be the two-sided po-ternary \( \Gamma \)-ideal of \( T \), then for each two-sided po-ternary \( \Gamma \)-ideal \( B \) of \( T \) we have \( A \cap B = (\Gamma A \Gamma) B \). In particular \( A = A \cap A = (\Gamma A \Gamma) A \). Therefore each two-sided po-ternary \( \Gamma \)-ideal of \( T \) is idempotent.

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(ii) ⇒ (i) Suppose that each two-sided po-ternary \( \Gamma \)-ideal of \( T \) is idempotent. Let \( A \) be a two-sided po-ternary \( \Gamma \)-ideal of \( T \), then for any two-sided po-ternary \( \Gamma \)-ideal \( B \) of \( T \) we always have \( \Gamma \alpha_{\Gamma}B = \alpha \cap B \). On the other hand, \( A \cap B = (A \cap B) \cap (A \cap B) \cap (A \cap B) \subseteq \alpha_{\Gamma}B. \) Hence we have \( A \cap B = \alpha_{\Gamma}B \). Thus \( A \) is left weakly pure.

(ii) ⇒ (iii) Similarly as (ii) ⇒ (i)

(iii) ⇒ (ii) Suppose that each two-sided po-ternary \( \Gamma \)-ideal of \( T \) is right weakly pure two-sided po-ternary \( \Gamma \)-ideal. Let \( A \) be any two-sided po-ternary \( \Gamma \)-ideal of \( T \). Then \( A \) is right weakly pure. Therefore for each two-sided po-ternary \( \Gamma \)-ideal \( B \) of \( T \), we have \( A \cap B = B \cap A \). In particular \( A \cap A = A \cap A \). Thus each two-sided po-ternary \( \Gamma \)-ideal of \( T \) is idempotent.

6. Pure spectrum of an ordered ternary \( \Gamma \)-semiring

**Notation 6.1:** Let \( T \) be an ordered ternary \( \Gamma \)-semiring with zero such that \([T \cap TT] = T\). The set of all right pure po-ternary \( \Gamma \)-ideals of \( T \) and the set of all proper purely prime po-ternary \( \Gamma \)-ideals of \( T \) will be denoted by \( P(T) \) and \( P'(T) \), respectively. For \( \alpha \in P(T) \), let

\[ I_{\alpha} = \{ J \in P'(T) \mid \alpha \not\subseteq J \} \quad \text{and} \quad \tau(T) = \{ I_{\alpha} \mid \alpha \in P(T) \}. \]

**Theorem 6.2:** \( \tau(T) \) forms a topology on \( P'(T) \).

**Proof:** Since \( \{ \} \) is a right pure po-ternary \( \Gamma \)-ideal of \( T \) and \( I_{\{\}} = \emptyset \), we have \( \emptyset \in \tau(T) \). Since \( T \) is a right pure po-ternary \( \Gamma \)-ideal of \( T \) such that \( I_T = P(T) \), we get \( P'(T) \in \tau(T) \).

Let \( \{ I_{\alpha_{\alpha}} \mid \alpha \in \Lambda \}\subseteq \tau(T) \). We have \( \bigcup_{\alpha \in \Lambda} I_{\alpha_{\alpha}} = \{ J \in P'(T) \mid \alpha \not\subseteq J \} \) \( \subseteq \bigcup_{\alpha \in \Lambda} I_{\alpha_{\alpha}} \). Whence \( \bigcup_{\alpha \in \Lambda} I_{\alpha_{\alpha}} \in \tau(T) \). Let \( I_{\alpha}, I_{\beta} \in \tau(T) \). We shall show that \( I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} = I_{\alpha_{\alpha} \cap \beta_{\beta}} \). Therefore let \( J \in I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} \). We have \( J \in P'(T), A \not\subseteq J \) and \( A \not\subseteq J \). Suppose that \( A \cap A \not\subseteq J \). Since \( J \) is purely prime, \( A \subseteq J \) or \( A \subseteq J \). A contradiction. Then \( J \in I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} \), hence \( I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} \subseteq I_{\alpha_{\alpha} \cap \beta_{\beta}} \). For the reverse inclusion, let \( J \in I_{\alpha_{\alpha} \cap \beta_{\beta}} \). Since \( A \cap A \not\subseteq J \), \( A \not\subseteq J \) and \( A \not\subseteq J \). This implies that \( J \in I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} \), thus \( I_{\alpha_{\alpha}} \not\subseteq I_{\alpha_{\alpha}} \). Consequently, \( I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} = I_{\alpha_{\alpha} \cap \beta_{\beta}} \), which implies \( I_{\alpha_{\alpha}} \cap I_{\beta_{\beta}} \in \tau(T) \). Therefore \( \tau(T) \) forms a topology on \( P'(T) \).

VI. Conclusion

In this paper mainly we start the study of pure po-ternary \( \Gamma \)-ideals, weakly pure po-ternary \( \Gamma \)-ideals and purely prime po-ternary \( \Gamma \)-ideals in po-ternary \( \Gamma \)-semirings. We characterize po-ternary \( \Gamma \)-semirings by the properties of pure and weakly pure po-ternary \( \Gamma \)-ideals.

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On Pure PO-Ternary $\Gamma$-Ideals in Ordered Ternary $\Gamma$-Semirings

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