On Q*g closed sets in Supra Topological Spaces

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Abstract: The aim of this paper is to introduce and study some properties of supra topological spaces. We introduce the concepts of supra Q*g closed sets, Supra Q*g open sets, Supra Q*closed sets and supra Q*open sets.

Keywords: supra Q*g closed sets, supra Q*g open sets, supra Q*closed sets, supra Q* open sets.

I. Introduction


II. Preliminaries

Definition 2.1 : [2] A topological Space ( X, τ ) is said to be generalized closed (briefly g-closed) set if cl(A) ⊆ U whenever A ⊆ U and U is open ( X, τ ).

Definition 2.2 : [8] A topological space ( X, τ ) is said to be generalized star closed (briefly g* - closed) set if cl(A) ⊆ U whenever A ⊆ U and U is g-open( X, τ ).

Definition 2.3 : [7] A topological space ( X, τ ) is said to be generalized star star closed (briefly g**-closed) set if cl(A) ⊆ U whenever A ⊆ U and U is g*-open( X, τ ).

Definition 2.4 : [2] Let X be a non empty set. The subfamily μ ⊆ 𝒫(X) where 𝒫(X) is the power set of X is said to be a supra topology on X if X ∈ μ and μ is closed under arbitrary unions. The pair ( X, μ ) is called a supra topological space. The elements of μ are said to be supra open in ( X, μ ). Complements of supra open sets are called supra closed sets.

Definition 2.5 : [9] Let A be a subset of ( X, μ ). Then the supra closure of A is denoted by clμ(A) = ∩ { B / B is a supra closed set and A ⊆ B }.

Definition 2.6 : [9] Let A be a subset of ( X, μ ). Then the supra interior of A is denoted by intμ(A)= ∪ { B / B is a supra open set and A ⊇ B }.

Definition 2.7 : [1] Let ( X, μ ) be a topological space and μ be a supra topology on X. μ is supra topology associated with τ if τ ⊆ μ.

III. On Q*G Closed Sets In Supra Topological Spaces

Definition 3.1 : A subset A of a supra topological space ( X, μ ) is called
(1) a supra Q* - closed if intμ(A) = ∅ and A is closed.
(2) a supra Q* - open if clμ(A) = X and A is open.

Definition 3.2 : A subset A of supra topological space ( X, μ ) is called a supra Q*g closed if clμ(A) ⊆ U. whenever A ⊆ U and U is Q* - open in ( X, μ ). The complement of a supra Q*g closed set is called supra Q*g open set.

Theorem 3.3 : Every supra closed set is supra Q*g closed.

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Therefore $A$ is supra closed.

**Converse part:** Every supra $Q^*g$ closed set is not supra closed.

**Proof:**
Let $A$ be supra $Q^*g$ closed set. Since $\text{cl}^p(A) \subseteq U$, whenever $A \subseteq U$. And $U$ is $Q^*$-open in $(X, \mu)$. Since the elements of $\mu$ are called supra open in $(X, \mu)$. Therefore every supra $Q^*g$ closed set is not supra closed.

**Remark 3.3:** The converse of the theorem is not true as shown in the following example.

**Example 3.4:** Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, X, \{a, b\}, \{a, b, d\}, \{b, c, d\}, \{c, d, e\}\}$

$A = \{a, b, c\}$ is supra $Q^*g$ closed but not supra closed.

**Corollary 3.5:** Every closed set is supra $Q^*g$ closed.

**Proof:** Every closed set is supra closed. By the theorem “Every Supra closed set is supra $Q^*g$ closed”.

**Theorem 3.6:** A subset $A$ of $X$ is supra $Q^*g$ closed if and only if $\text{cl}^p(A) \setminus A$ contains no non-empty supra $Q^*$-closed set.

**Proof:**

**Necessity:** Let $F$ be a supra $Q^*$ closed set of $\text{cl}^p(A) \setminus A$ that is $F \subseteq \text{cl}^p(A) \setminus A$.
Now $A \subseteq F^c$. Where $F^c$ is supra $Q^*$ open. Since $A$ is supra $Q^*g$ closed, $\text{cl}^p(A) \subseteq F^c \Rightarrow F \subseteq [\text{cl}^p(A)]^c$.
Therefore $F \subseteq \text{cl}^p(A) \cap [\text{cl}^p(A)]^c = \emptyset$.
Hence $F^c$ is supra $Q^*$ open, $\text{cl}^p(A) = X$ and $A$ is open. Therefore $F^c \neq \emptyset$.

**Sufficiency:** Suppose $A \subseteq U$ and $U$ is supra $Q^*$ open. Suppose $\text{cl}^p(A) \not\subseteq U$. Then $\text{cl}^p(A) \cap U$ is supra $Q^*$ closed subset of $\text{cl}^p(A) \setminus A$.
Hence $\text{cl}^p(A) \cap U = \emptyset$. And hence $\text{cl}^p(A) \subseteq U$.
Therefore $A$ is supra $Q^*g$ closed.

**Corollary 3.7:** A supra $Q^*g$ closed set $A$ of $X$ is supra closed if and only if $\text{cl}^p(A) \setminus A$ is supra $Q^*$ closed.

**Proof:**
The supra $Q^*g$ closed set $A$ is supra closed.
Then $\text{cl}^p(A) = A$, and hence $\text{cl}^p(A) \setminus A = \emptyset$ is supra $Q^*$ closed.

**Conversely:** Suppose that $\text{cl}^p(A) \setminus A$ is supra $Q^*$ closed.
Since $A$ is supra $Q^*g$ closed $\text{cl}^p(A) \setminus A$ does not contain any non-empty $Q^*$ closed set.
But since $\text{cl}^p(A) \setminus A$ is itself supra $Q^*$ closed,$\text{cl}^p(A) \setminus A = \emptyset$. Which implies $\text{cl}^p(A) = A$.
Therefore $A$ is supra closed.

**References**


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