Probability Distribution of Maximum Temperature in Adamawa State, Nigeria

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Abstract: *Normal body temperature varies by person, age, activity, and time of day. The average normal body temperature is generally accepted as 98.6°F (37°C). Some studies have shown that the "normal" body temperature can have a wide range, from 97°F (36.1°C) to 99°F (37.2°C). A temperature over 100.4°F (38°C) is usually an indication of a fever caused by an infection or illness. A temperature over 100.4°F (38°C) is usually an indication of a fever caused by an infection or illness, hyperthermia and hyperpyrexia. Hyperthermia occurs when the body produces or absorbs more heat than it can dissipate. It is usually caused by prolonged exposure to high temperatures, Axelrodand Diringer (2008), and Laupland (2009).The Easy Fit 5.5 Standard version was used to obtain the parameter estimates and also in determining the distribution that best fits the data on maximum temperature recorded in Adamawa state, Nigeria. Four distributions namely: Johnson S_B, Beta distribution, Kumaraswamy and the Generalized Pareto were adopted and based on the Kolmogorov-Smirnov, Anderson-Darling and the Chi-Square criteria, the Johnson S_B distribution was identified to best fit the data. It reported a 95% confidence interval of [29.82, 42.332] for the maximum temperature in the state.*

Keywords: *Maximum temperature, Johnson S_B, Beta, Kumaraswamy, Generalized Pareto.*

I.   Introduction

Temperatures have risen during the last 30 years, and 2001 to 2010 was the warmest decade ever recorded. As the Earth warms up, heat waves are becoming more common in some places, including the United States. Heat waves happen when a region experiences very high temperatures for several days and nights, EPA's Climate Change Indicators (2014).

The effect of temperature variations and or increase can be seen in virtually all areas of human engagements. In health related matters, heat waves are uncomfortable for everyone, but for infants and young children, the elderly, and people who are already sick, they can be especially dangerous. Extreme heat can cause illnesses such as heat cramps, heat stroke, and even death. A 2003 heat wave in Europe caused about 50,000 deaths, and a 1995 heat wave in Chicago caused more than 600 deaths. In fact, heat waves cause more deaths in the United States every year than hurricanes, tornadoes, floods, and earthquakes combined. In Nigeria, several ailments such as meningitis, low comprehension for students studying under extreme temperature, heat rashes e.t.c. Climate change might allow some infectious diseases to spread. One big concern is malaria, a deadly disease spread by mosquitoes in many hot, humid parts of the world like Adamawa state, Nigeria. Denissen et al. (2008) found that weather's daily influence has more of an impact on a person's negative mood, rather than helping one's positive mood. Higher temperatures raise a person with a low mood up, while things like wind or not enough sun made a low person feel even lower. Hsiang et al. (2013) found a link between human aggression and higher temperatures. They also stated that as temperatures rose, that intergroup conflicts also tended to jump — by 14 percent (a significant increase) and also found that interpersonal violence rose by 4 percent.

Body temperature is a measure of the body's ability to generate and get rid of heat. The body is very good at keeping its temperature within a narrow, safe range in spite of large variations in temperatures outside the body. When the body is too hot, the blood vessels in the skin expand (dilate) to carry the excess heat to the skin's surface. One may begin to sweat, and as the sweat evaporates, it helps cool your body. The body temperature can be measured in many locations on our body. The mouth, ear, armpit, and rectum are the most commonly used places. Temperature can also be measured on your forehead. Also, our normal body temperature changes by as much as 1°F (0.6°C) throughout the day, depending on how active you are and the time of day. Body temperature is very sensitive to hormone levels and may be higher or lower when a woman is ovulating or having her menstrual period, Karakitsos and Karabinis (2008).

Heat stress occurs when the body cannot cool itself enough to maintain a healthy temperature. Heat-related illnesses include heat rash, heat cramps, dizziness or fainting, heat exhaustion, heat stroke, and a worsening of existing medical conditions. Overexertion in hot weather, sun or bushfire exposure, and exercising or working in hot, poorly ventilated or confined areas can increase your risk of heat stress.

The heat-regulating mechanisms of the body eventually become overwhelmed and unable to deal effectively with the heat, causing the body temperature to climb uncontrollably. Hyperthermia at or above about 40 °C (104 °F) is a life-threatening medical emergency that requires immediate treatment. Common symptoms include headache, confusion, and fatigue. If sweating has resulted in dehydration, then the affected person may have dry, red skin.

In a medical setting, mild hyperthermia is commonly called heat exhaustion or heat prostration; severe hyperthermia is called heat stroke. Heat stroke may come on suddenly, but it usually follows the untreated milder stages, Axelrodand Diringer (2008), and Laupland (2009.

II. Methodology

The following distributions were considered based on their rank as the first four (4) best fit probability distributions for the data on maximum temperature in Adamawa State from 1984 to 2013.

Johnson S_B Distribution

Johnson's S_B PDF is part of a system of distributions proposed by Johnson (1949) generated by methods of translation on a standard normal variate that permits representation over the whole possible region of the plane (β_1, β_2), where β_1 is thesquare of the standardized measure of skewness and β_2 is the standardized measure of kurtosis. The general form of normalizing transformation is given as (George and Ramachandran, 2011):

$$Z = \gamma + \delta f\left(\frac{X-\xi}{\lambda}\right) \tag{1}$$

where $f(.)$ denotes the transformation function, Z is a standard normal random variable, γ and δ are shape parameters, λ is a scale parameter and ξ is a location parameter. It is assumed that $\delta > 0$ and $\lambda > 0$. The system consists of three distribution families: S_U, S_L, and S_B, defined, respectively, for unbounded variates, variates bounded at one end, and bounded from above and below.
The bounded system of distribution S_B is defined by

$$Z = \gamma + \delta \ln\left(\frac{X-\xi}{\xi+\lambda-X}\right), \xi \leq X \leq \xi + \lambda \tag{2}$$

If X follows theJohnson S_B distribution and $Y = \frac{X-\xi}{\lambda}$ then, the pdfcan be expressed as,

$$f(y) = \frac{\delta}{[y/(1-y)]\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\gamma + \delta \ln\left(\frac{y}{1-y}\right)\right]^2\right\} \tag{3}$$

It is a continuous distribution defined on bounded range $\xi \leq X \leq \xi + \lambda$, and the distribution can be symmetric or asymmetric.Where $\lambda, \delta > 0$, $-\infty < \xi, \gamma < \infty$. The parameter λ gives the range, ξ is the location parameter (lower bound), δ and γ are shape parameters as earlier stated, and $\gamma = 0$ indicates symmetry (Fonseca et al, 2009).
Generally, the pdf of X is given by(George and Ramachandran, 2011):

$$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}} \cdot g\left(\frac{x-\xi}{\lambda}\right) \cdot \exp\left\{-\frac{1}{2}\left[\gamma + \delta \cdot g\left(\frac{x-\xi}{\lambda}\right)\right]^2\right\}, \quad x \epsilon D \tag{4}$$

Where $g(y) = \frac{1}{[y(1-y)]}$ and $g(y) = \ln(y/(1-y))$ for the S_B family. $\qquad(5)$

The support D of the distribution is: $D = [\xi, \xi + \lambda]$ for the S_B family.

Kumaraswamy Distribution

Kumaraswamy's double bounded distribution is a family of continuous probability distributions defined on the interval [a, b] differing in the values of their two non-negative shape parameters, α_1 and α_2(Cordeiro and Castro, 2009).
The probability density function of the Kumaraswamy distribution is

$$f(x; \alpha_1, \alpha_2, a, b) = \frac{\alpha_1\alpha_2 Z^{\alpha_1-1}(1-Z^{\alpha_1})^{\alpha_2-1}}{(b-a)} \tag{5}$$

Where $Z \equiv \frac{x-a}{b-a}$

Beta Distribution

The probability density function of the beta distribution, for $a \leq x \leq b$, (i.e a, b are boundary parameters) and shape parameters $\alpha_1, \alpha_2 > 0$, is a power function of the variable X as follows:

$$f(x; \alpha_1, \alpha_2, a, b) = \frac{1}{B(\alpha_1, \alpha_2)} \frac{(x-a)^{\alpha_1-1}(b-a)^{\alpha_1-1}}{(b-a)^{\alpha_1+\alpha_2-1}} \tag{6}$$

$$\text{Where } Z \equiv \frac{x-a}{b-a} \quad \text{and} \quad B(\alpha_1, \alpha_2) = \int_0^1 t^{\alpha_1-1}(1-t)^{\alpha_2-1}dt, \quad (\alpha_1, \alpha_2 > 0)$$

The beta function, $B(.)$, is a normalization constant to ensure that the total probability integrates to 1. In the above equations x is a realization—an observed value that actually occurred—of a random process X, Walck and Fysikum (2007).

Generalized Pareto Distribution

The generalized Pareto distribution (GPD) is a family of continuous probability distributions. It is often used to model the tails of another distribution. It is specified by three parameters: location μ, scale $\sigma > 0$, and shape k (Coles 2001, Dargahi-Noubary 1989). Sometimes it is specified by only scale and shape (Hosking and Wallis, 1987) and sometimes only by its shape parameter. Some references give the shape parameter as $k = -\xi$, Davison(1984)

The standard cumulative distribution function (CDF) of the GPD is defined byEmbrechts et al(1997)

$$F(x; \mu, \sigma, k) = \begin{cases} 1 - \left[1 + k\frac{(x-\mu)}{\sigma}\right]^{-\frac{1}{k}} & , k \neq 0 \\ 1 - \exp\left[-\frac{(x-\mu)}{\sigma}\right] & , k = 0 \end{cases} \tag{7}$$

And the PDF is

$$f(x; \mu, \sigma, k) = \begin{cases} \frac{1}{\sigma}\left[1 + k\frac{(x-\mu)}{\sigma}\right]^{-(1+\frac{1}{k})} & , k \neq 0 \\ \frac{1}{\sigma}\exp\left[-\frac{(x-\mu)}{\sigma}\right] & , k = 0 \end{cases} \tag{8}$$

$$\text{Domain: } \mu \leq x \leq \infty \text{ for } k \geq 0, \quad \mu \leq x \leq \mu - \frac{\sigma}{k} \text{ for } k < 0.$$

Parameter Estimation

There are many methods available for estimating the true value(s) of the parameter(s) of interest. These methods include; Method of Moments, Method of Least Squares, Minimum Chi-Squares Method, Minimum Distance Method, Method of Maximum Likelihood, ZSEstimation (ZSE) proposed by Zhang and Stephens (2009), SSEstimation (SSE) by Song and Song (2012). The method of maximum likelihood estimation (MLE) would be used because it is easy to use and produces good statistical properties (MLE is approximately a Minimum Variance Unbiased Estimator (MVUE) and it also has an asymptotic normal distribution).

Maximum Likelihood Method

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population X with probability density function $f(x; \theta)$, where θ is an unknown parameter. The likelihood function, $L(\theta)$, is the distribution of the sample. That is

$$L(\theta) = \prod_{i=1}^n f(x; \theta). \tag{9}$$

This definition says that the likelihood function of a random sample $X_1, X_2, \ldots, X_n$ is the joint density of the random variables $X_1, X_2, \ldots, X_n$. The θ that maximizes the likelihood function $L(\theta)$ is called the maximum likelihood estimator of , and it is denoted by $\hat{\theta}$. Hence

$$\hat{\theta} = Arg\,max_{\theta \in \Theta} L(\theta), \tag{10}$$

where θ is the parameter space of θ so that $L(\theta)$ is the joint density of the sample. The method of maximum likelihood in a sense picks out of all the possible values of θ the one most likely to have produced the given observations $x_1, x_2, \ldots, x_n$, Sahoo(2013).

The MLE-Least Square approach is used to the four parameters of the Johnson family of distributions, George and Ramachandran(2011). Using the general form of the Johnson densities (equation 4), the likelihood function is:

$$l(x) = \frac{\delta^n}{\lambda^n (2\pi)^{n/2}} \prod_{i=1}^n g\left(\frac{x-\xi}{\lambda}\right) exp\left\{-\frac{1}{2}\sum_{i=1}^n \left(\gamma + \delta g\left(\frac{x-\xi}{\lambda}\right)\right)^2\right\}, \tag{11}$$

And the log-likelihood is,

$$log\,l(x) = nlog\delta - nlog\lambda - \frac{n}{2}log(2\pi) + \sum_{i=1}^n g\left(\frac{x-\xi}{\lambda}\right) - \frac{1}{2}\sum_{i=1}^n \left(\gamma + \delta g\left(\frac{x-\xi}{\lambda}\right)\right)^2 \tag{12}$$

Setting the partial derivatives; with respect to δ to zero, and with respect to γ to zero and solving, the estimators will be obtained as:

$$\hat{\gamma} = \frac{-\delta \sum_{i=1}^{n} g\left(\frac{x-\xi}{\lambda}\right)}{n} = -\delta \bar{g} \tag{13}$$

and

$$\hat{\delta}^2 = \frac{n}{\sum_{i=1}^{n}\left[g\left(\frac{x-\xi}{\lambda}\right)\right]^2 - \frac{1}{n}\sum_{i=1}^{n}\left[g\left(\frac{x-\xi}{\lambda}\right)\right]^2} = \frac{1}{var(g)} \tag{14}$$

Where $\bar{g}$ is the mean and $var(g)$ is the variance of the values of g defined in (5).

The partial derivative of (12) with respect to ξ and λ are cumbersome, hence the Least Squares Method is applied here to estimate parameters ξ and λ (George and Ramachandran, 2011). Using (1),

$$x = \xi + \lambda f^{-1}\left(\frac{z-\gamma}{\delta}\right)$$

would be obtained. For fixed values of ξ and δ, the equation is considered as a linear equation with parameters ξ and λ. The sum of squares of error is,

$$S(\xi,\lambda) = \sum\left[x - \xi + \lambda f^{-1}\left(\frac{z-\gamma}{\delta}\right)\right]^2. \tag{15}$$

To determine the value of ξ and λ that minimizes $S(\xi,\lambda)$, the partial derivatives of $S(\xi,\lambda)$ with respect to ξ and λ are calculated and these partial derivatives are equated to zero. The normal equations are then obtained:

$$\sum x = n\xi + \lambda \sum f^{-1}\left(\frac{z-\gamma}{\delta}\right) \tag{16}$$

$$\sum x f^{-1}\left(\frac{z-\gamma}{\delta}\right) = \xi \sum f^{-1}\left(\frac{z-\gamma}{\delta}\right) + \lambda \sum\left[f^{-1}\left(\frac{z-\gamma}{\delta}\right)\right]^2 \tag{17}$$

Solving the normal equations results in

$$\hat{\lambda} = \frac{n\sum x f^{-1}\left(\frac{z-\gamma}{\delta}\right) - \sum f^{-1}\left(\frac{z-\gamma}{\delta}\right)\sum x}{n\sum\left[f^{-1}\left(\frac{z-\gamma}{\delta}\right)\right]^2 - \left[\sum f^{-1}\left(\frac{z-\gamma}{\delta}\right)\right]^2} \tag{18}$$

And

$$\hat{\xi} = \bar{x} - \lambda\left[mean\left[f^{-1}\left(\frac{z-\gamma}{\delta}\right)\right]\right] \tag{19}$$

Goodness of Fit Test

Goodness of fit tests are performed to validate the researcher's opinion about the distribution of the population from where the sample is drawn. In this work, Kolmogorov-Smirnov (K-S) test, the Pearson's Chi-Square (χ^2) test and Anderson-Darling (A-D) test were used to fit the distributions. The results are as presented below:

Hypothesis: H_0: The data follows the said distribution Vs H_1: Not H_0

Table1

DISTRIBUTION	TEST						α				
		n	Statistic	P-Value	Rank	d.f	0.2	0.1	0.05	0.02	0.01
BETA	K-S	360	0.0486	0.3511	3		.05655	.06446	.07157	.08001	.08586
	A-D	360	0.7805	N/A	2		1.3749	1.9286	2.5018	3.2892	3.9074
	χ^2		7.837	0.4496	3	8	11.03	13.362	15.507	18.168	20.09
JOHNSON S_B	K-S	360	0.0408	0.5731	1		0.0565	0.0644	0.0715	0.0800	0.0859
	A-D	360	0.6626	N/A	1		1.3745	1.9286	2.5018	3.2892	3.9074
	χ^2		7.142	0.5214	1	8	11.03	13.362	15.507	18.168	20.09
KUMARAS-WAMY	K-S	360	0.0488	0.3470	4		0.0566	0.0645	0.0716	0.0800	0.0859
	A-D	360	0.7875	N/A	3		1.3749	1.9286	2.5018	3.2892	3.9074
	χ^2		7.8359	0.4497	2	8	11.03	13.362	15.507	18.168	20.09
GENERALIZED PARETO	K-S	360	0.0418	0.5406	2		0.0566	0.0645	0.0716	0.0800	0.0859
	A-D*	360	20.133	N/A	4		1.3749	1.986	2.5018	3.2892	3.9074

Summary of results from the goodness-of-fit test
*The hypothesis that the data follows a Generalized Pareto Distribution is rejected at the various levels of significance using the A-D criterion.

Parameter Estimates
These are as obtained using the EasyFit 5.5 Standard version.

> **Beta Distribution**
> $\alpha_1 = 1.1371, \alpha_2 = 1.631, a = 29.983, b = 42.384$
>
> **Generalized Pareto Distribution**
> $k = -0.70797, \sigma = 8.3772, \mu = 30.193$
>
> **Johnson S_B Distribution**
> $\gamma = 0.31197, \delta = 0.7292, \lambda = 12.512, \xi = 29.82$
>
> **Kumaraswamy Distribution**
> $\alpha_1 = 1.1241, \alpha_2 = 1.6436, a = 29.985, b = 42.387$

III.   Discussion Of Findings And Conclusion

From the Table 1, it can be seen that the estimates of the statistics are less than their respectivecritical values at various levels of significance thus implying that the null hypothesis that the data on maximum temperature follows the stated distribution be accepted except for Anderson Darling statistic for the Generalized Pareto with a value of about 20.132 which clearly exceeds the critical values at the various levels of significance.From our results, we find that the lower and upper bounds for the maximum temperature recorded in Adamawa state, Nigeria exceeds that of the normal body temperature. The lower and upper bounds as reported by the Johnson S_B, Beta, Kumaraswamy and Generalized Pareto distributions are [29.82, 42.332], [29.98, 42.38], [29.985, 42.387] and [30.193, 42.026] respectively. These all indicate that there exist times when extreme temperature periodsare experienced in the state. Thus, one can infer that adverse conditions would be prevalent such as health disorders, agricultural limitations (certain crops can't survive such high temperatures; limited rainfall etc). New-born, aged persons and persons with health problems may have difficulty surviving such periods. Academic institutions and learning would also be affected greatly.As such, policy makers need to look into possible ways of cushioning the effect of extreme temperature conditions in the state.

Figure1: Graph of the various probability distribution functions.

a: Johnson S_B b: Generalized Pareto
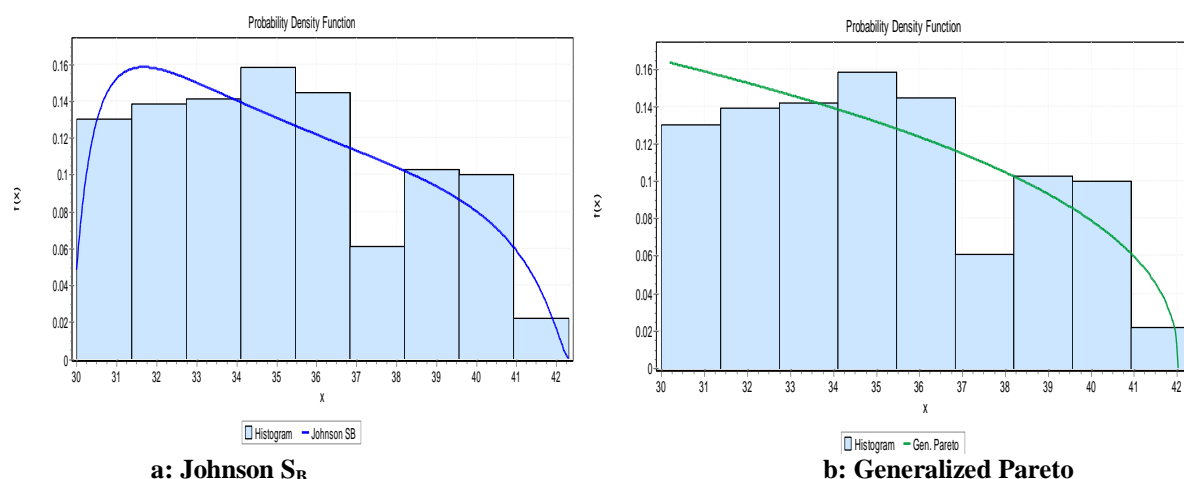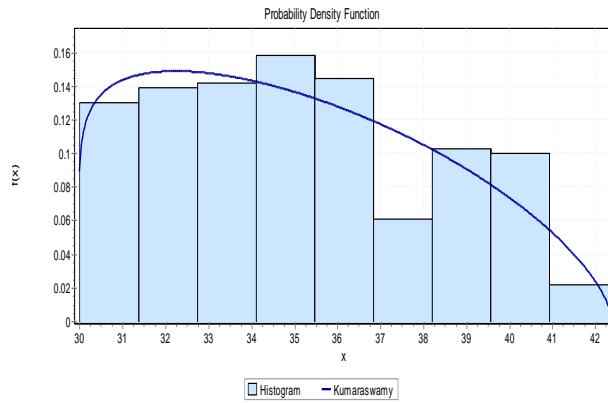
c: Kumaraswamy
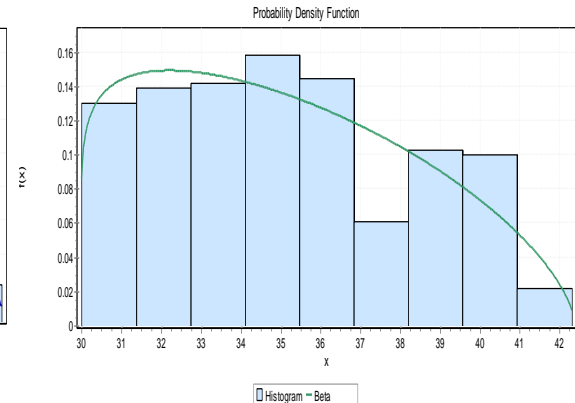
d: Beta

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