

## S\*- Hyperconnectedness

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**Abstract:** The aim of this paper is to introduce and study hyperconnectedness in supra topological spaces, which we called, supra hyperconnectedness and denoted by  $S^*$ -hyperconnectedness. Several characterizations of supra hyperconnectedness are provided. We give conditions under which supra hyperconnectedness is preserved. Further, we introduce and study strong forms of supra hyperconnected spaces and obtain some characterizations of hyperconnected spaces.

**Keywords:** supra topology, supra open set, hyperconnectedness,  $S^*$ -hyperconnectedness,  $S^*$ -dense.

### I. Introduction And Preliminaries

In 1983 A. S. Mashhour et al. [1] developed the notion of supra topological spaces and studied the notions of supra- continuity. Devi et al. [2] introduce and studied a class of sets and maps called supra  $\alpha$ - open sets and supra  $\alpha$ - continuous maps. Arockiarani [3] introduced the class of supra semi- open (resp. supra regular open) sets. In 2010, Sayed et al. [4] introduced supra b- open sets and supra b- continuity and studied some of its properties. In [5], the concepts of supra  $\beta$ - open sets, supra  $\beta$ - continuity were introduced and studied. Levine [6] called a topological spaces X as a D- space if every non-empty open set of X is dense in X. Sharma [7] has shown that D- space is equivalent to hyperconnected space due to Steen and Seebach [8]. Noiri [9, 10] studied hyperconnected spaces. Ekici [11] introduced and studied hyperconnectedness in generalized topological spaces. In this paper we introduce and study the concept of supra hyperconnectedness. Several characterizations of supra hyperconnected spaces are investigated and some preservation theorems are given. Further, we obtain some characterizations of hyperconnected topological spaces. Let us recall the following definitions and results which are useful in the sequel.

**Definition 1.1.** Let  $(X, \tau)$  be a topological space and A be a subset of X. The closure and the interior of A will be denoted by  $cl(A)$  and  $int(A)$ . A set A is called  $\alpha$ - open [12] (resp. preopen [13], semi- open, [14], b- open [15],  $\beta$ -open [16]) if  $A \subset int(cl(int(A)))$  (resp.  $A \subset int(cl(A))$ ,  $A \subset cl(int(A))$ ,  $A \subset int(cl(A)) \cup cl(int(A))$ ,  $A \subset cl(int(cl(A)))$ ). The complement of the above sets are called there respective closed sets.

The family of all  $\alpha$ - open ( resp. pre- open, semi-open, b- open,  $\beta$ -open) sets are denoted by  $\alpha(X)$ ,  $PO(X)$ ,  $SO(X)$ ,  $BO(X)$  and  $\beta O(X)$  respectively. Let  $P(X)$  be the power set of X, for a nonempty set X.

**Definition 1.2.** [1] Let X be a nonempty set. A subset  $S^* \subset P(X)$  where  $P(X)$  is the power set of X, is called a supra topology if (1)  $X, \emptyset \in S^*$ . (2)  $S^*$  is closed under arbitrary union. The pair  $(X, S^*)$  is called a supra topological space. The members of  $S^*$  are called supra -open sets and denoted  $s^*$ -open sets. The complement of supra- open set is the supra closed sets, denoted by  $s^*$ -closed

**Remark 1.3.** [1] Let  $(X, \tau)$  be a topological space. Then,  $\alpha O(X)$ ,  $PO(X)$ ,  $SO(X)$ ,  $BO(X)$  and  $\beta O(X)$  are supra topologies associated with  $\tau$ .

**Definition 1.4.** [1] Let  $(X, S^*)$  be a supra topological space and  $A \subset X$ . Then  $S^*$ - interior and  $S^*$ - closure of A in  $(X, S^*)$  defined as  $\bigcup \{U : U \subseteq A, U \in S^*\}$  and  $\bigcap \{F : A \subseteq F, X - F \in S^*\}$  respectively. The  $S^*$ -interior and  $S^*$ -closure of A in  $(X, S^*)$  are denoted as  $int_{s^*}(A)$  and  $cl_{s^*}(A)$  respectively.

**Theorem 1.5.** [17] Let  $(X, S^*)$  be a supra topological space and  $A \subset X$ . Then

- (1)  $int_{s^*}(A) \subseteq A$ .
- (2)  $int_{s^*}(A) = A$  if and only if  $A \in S^*$ .
- (3)  $A \subseteq cl_{s^*}(A)$ .

- (4)  $cl_{s^*}(A) = A$  if and only if  $A$  is supra closed.
- (5)  $x \in cl_{s^*}(A)$  if and only if every supra open set  $U_x$  containing  $x$ ,  $U_x \cap A \neq \emptyset$ .

**Definition 1.6.** A subset  $A$  of a supra topological space  $(X, S^*)$  is said to be

- (1)  $S^*$ - $\alpha$ -open[2] (briefly  $\alpha_{s^*}$ -open) if  $A \subset \text{int}_{s^*}(cl_{s^*}(\text{int}_{s^*}(A)))$ .
- (2)  $S^*$ -pre-open (briefly  $p_{s^*}$ -open) if  $A \subset \text{int}_{s^*}(cl_{s^*}(A))$ .
- (3)  $S^*$ -semi-open[3] (briefly  $s_{s^*}$ -open) if  $A \subset cl_{s^*}(\text{int}_{s^*}(A))$ .
- (4)  $S^*$ -b-open[4] (briefly  $b_{s^*}$ -open) if  $A \subset cl_{s^*}(\text{int}_{s^*}(A)) \cup \text{int}_{s^*}(cl_{s^*}(A))$ .
- (5)  $S^*$ - $\beta$ -open[5] (briefly  $\beta_{s^*}$ -open) if  $A \subset cl_{s^*}(\text{int}_{s^*}(cl_{s^*}(A)))$ .
- (6)  $S^*$ -regular-open [3] (briefly  $r_{s^*}$ -open) if  $A = \text{int}_{s^*}(cl_{s^*}(A))$ .

The set of all  $\alpha_{s^*}$ -open (resp.  $p_{s^*}$ -open,  $s_{s^*}$ -open,  $b_{s^*}$ -open,  $\beta_{s^*}$ -open,  $r_{s^*}$ -open) sets is denoted by  $\alpha^*(X)$  (resp.  $P^*O(X)$ ,  $S^*O(X)$ ,  $B^*O(X)$ ,  $\beta^*O(X)$ ,  $R^*O(X)$ ).

It is clear that  $\alpha^*(X) \Rightarrow S^*O(X) \Rightarrow B^*O(X) \Rightarrow \beta^*O(X)$  and

$$\alpha^*(X) \Rightarrow P^*O(X) \Rightarrow B^*O(X) \Rightarrow \beta^*O(X)$$

The complements of the above mentioned sets are called their respective closed sets.

## II. $S^*$ – Hyperconnected Spaces

In this section we introduce and study the notion of supra hyperconnectedness. Several characterizations are given.

**Definition 2.1.** A subset  $A$  of a supra topological space  $(X, S^*)$  is said to be

- (1)  $s^*$ -dense if  $cl_{s^*}(A) = X$ .
- (2)  $s^*$ -nowhere dense if  $\text{int}_{s^*}(cl_{s^*}(A)) = \emptyset$ .

**Definition 2.2.** A supra topological space  $(X, S^*)$  is said to be

- (1) Hyperconnected (equivalently  $X$  is  $S^*$ -hyperconnected) if  $A$  is  $s^*$ -dense for every  $s^*$ -open subset  $A \neq \emptyset$  of  $X$ .
- (2) Connected (equivalently  $X$  is  $S^*$ -connected) if  $X$  cannot be written as the union of nonempty and disjoint  $s^*$ -open sets  $A$  and  $B$  in  $(X, S^*)$ .
- (3) Irreducible (equivalently  $X$  is  $S^*$ -irreducible) if  $A \cap B \neq \emptyset$  for every nonempty  $s^*$ -open subsets  $A$  and  $B$  of  $(X, S^*)$ .

**Remark 2.3.** From Definitions 2.2 we have the following relation.

$$(X, S^*) \text{ is hyperconnected} \Rightarrow (X, S^*) \text{ is connected}$$

The above implication is not reversible as shown in the following example:

**Example 2.4.** Let  $X = \{a, b, c\}$ ,  $S^* = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ . Then  $(X, S^*)$  is connected but not hyperconnected.

**Theorem 2.5.** Let  $(X, S^*)$  be a supra topological space. The following are equivalent:

- (1)  $(X, S^*)$  is hyperconnected.
- (2)  $A$  is  $s^*$ -dense or  $s^*$ -nowhere dense, for every subset  $A$  of  $X$ .
- (3)  $A \cap B \neq \emptyset$ , for every nonempty  $s^*$ -open subsets  $A$  and  $B$  of  $X$ .

**Proof.** (1)  $\Rightarrow$  (2): Let  $(X, S^*)$  be a hyperconnected and  $A$  be a subset of  $X$  and suppose that  $A$  is not  $s^*$ -nowhere dense. Then  $cl_{s^*}(X \setminus cl_{s^*}(A)) = X \setminus \text{int}_{s^*}(cl_{s^*}(A)) \neq X$ . Since  $\text{int}_{s^*}(cl_{s^*}(A)) \neq \emptyset$ , so by (1),  $cl_{s^*}(\text{int}_{s^*}(cl_{s^*}(A))) = X$ . Since  $cl_{s^*}(\text{int}_{s^*}(cl_{s^*}(A))) = X \subset cl_{s^*}(A)$ . Then  $cl_{s^*}(A) = X$ . Hence  $A$  is  $s^*$ -dense.

(2)  $\Rightarrow$  (3): Suppose that  $A \cap B = \emptyset$  for some nonempty  $s^*$ -open subsets  $A$  and  $B$  of  $X$ . Then  $cl_{s^*}(A) \cap B = \emptyset$  and  $A$  is not  $s^*$ -dense. Since  $A \in S^*$ , so  $\emptyset \neq A \subset \text{int}_{s^*}(cl_{s^*}(A))$ . Hence  $A$  is not  $s^*$ -nowhere dense. This is a contradiction.

(3)  $\Rightarrow$  (1): Let  $A \cap B \neq \emptyset$  for every nonempty  $s^*$ -open subsets  $A$  and  $B$  of  $X$ . Suppose that  $(X, S^*)$  is not hyperconnected. Then there is a nonempty  $s^*$ -open subset  $U$  of  $X$  such that  $U$  is not  $s^*$ -dense in  $X$ , thus  $cl_{s^*}(U) \neq X$ . Hence  $X \setminus cl_{s^*}(U)$  and  $U$  are disjoint nonempty  $s^*$ -open subsets of  $X$ . This is a contradiction. Hence  $(X, S^*)$  is hyperconnected.

By Definition 2.2(3) and Theorem 2.5, we have,

**Corollary 2.6.** The supra topological space  $(X, S^*)$  is hyperconnected if and only if it is irreducible.

**Definition 2.7.** The  $S^*$ -semi-closure (resp.  $S^*$ -pre-closure,  $S^*$ - $\beta$ -closure) of a subset  $A$  of a supra topological space  $(X, S^*)$ , denoted by  $c_{s^*}^s(A)$  (resp.  $c_{s^*}^p(A)$ ,  $c_{s^*}^\beta(A)$ ) is the intersection of all  $s_{s^*}$ -closed (resp.  $p_{s^*}$ -closed,  $\beta_{s^*}$ -closed) sets of  $X$  containing  $A$ .

The following Lemma is needed in the next result.

**Lemma 2.8.** Let  $A$  be any subset of a supra topological  $(X, S^*)$ . Then  $A \cup \text{int}_{s^*}(cl_{s^*}(\text{int}_{s^*}(A))) = c_{s^*}^\beta(A)$ .

**Theorem 2.9.** For a supra topological space  $(X, S^*)$ , the following are equivalent.

- (1)  $(X, S^*)$  is hyperconnected.
- (2)  $A$  is  $s^*$ -dense for every  $\beta_{s^*}$ -open subset  $\phi \neq A \subset X$ .
- (3)  $A$  is  $s^*$ -dense for every  $b_{s^*}$ -open subset  $\phi \neq A \subset X$ .
- (4)  $A$  is  $s^*$ -dense for every  $p_{s^*}$ -open subset  $\phi \neq A \subset X$ .
- (5)  $c_{s^*}^s(A) = X$  for every  $p_{s^*}$ -open subset  $\phi \neq A \subset X$ .
- (6)  $c_{s^*}^p(A) = X$  for every  $s_{s^*}$ -open subset  $\phi \neq A \subset X$ .
- (7)  $c_{s^*}^\beta(A) = X$  is for every  $s_{s^*}$ -open subset  $\phi \neq A \subset X$ .

**Proof.** (1)  $\Rightarrow$  (2): Suppose that  $A$  is a nonempty  $\beta_{s^*}$ -open subset of  $X$ . Hence  $\text{int}_{s^*}(cl_{s^*}(A)) \neq \emptyset$ . Then  $cl_{s^*}(A) = cl_{s^*}(\text{int}_{s^*}(cl_{s^*}(A))) = X$ .

(2)  $\Rightarrow$  (3): Since every  $b_{s^*}$ -open is  $\beta_{s^*}$ -open, we have (3).

(3)  $\Rightarrow$  (4): Since every  $p_{s^*}$ -open is  $b_{s^*}$ -open, we have (4).

(4)  $\Rightarrow$  (5): Suppose that  $A \neq \emptyset$  is a  $p_{s^*}$ -open set such that  $c_{s^*}^s(A) \neq X$ . Then there is a nonempty  $s_{s^*}$ -open set  $U$  such that  $U \cap A = \emptyset$ . Hence  $\text{int}_{s^*}(U) \cap A = \emptyset$ . Then, by (4),  $\emptyset = \text{int}_{s^*}(U) \cap cl_{s^*}(A) = \text{int}_{s^*}(U)$  which is a contradiction.

(5)  $\Rightarrow$  (6): Suppose that there is a nonempty  $s_s^*$ -open set  $A$  such that  $c_{s_s^*}^p(A) \neq X$ . Hence there is a  $p_{s_s^*}$ -open set  $U \neq \phi$  such that  $U \cap A = \phi$ . So  $U \cap (\text{int}_{s_s^*} A) = \phi$ . Hence, by (5),  $\text{int}_{s_s^*}(A) = c_{s_s^*}^s(U) \cap \text{int}_{s_s^*}(A) \subset cl_{s_s^*} U \cap \text{int}_{s_s^*}(A) = \phi$ . This is a contradiction.

(1)  $\Rightarrow$  (7): Let  $A$  be a non empty  $s_s^*$ -open subset of  $X$ . So  $\text{int}_{s_s^*}(A) \neq \phi$ , hence, by assumption,  $cl_{s_s^*}(\text{int}_{s_s^*}(A)) = X$ . By Lemma 2.8,  $c_{s_s^*}^\beta(A) \supset A \cup \text{int}_{s_s^*}(cl_{s_s^*}(\text{int}_{s_s^*}(A))) = A \cup \text{int}_{s_s^*}(X) = X$ . Hence  $c_{s_s^*}^\beta(A) = X$ .

(7)  $\Rightarrow$  (6): This is obvious.

(6)  $\Rightarrow$  (1): Let  $A$  be a nonempty  $s_s^*$ -open subset of  $X$ . So  $A$  is  $s_s^*$ -open. Hence by (6),  $c_{s_s^*}^p(A) = X$ . Since  $c_{s_s^*}^p(A) \subset cl_{s_s^*}(A)$ . Then  $cl_{s_s^*}(A) = X$ . So we have (1).

**Corollary 2.10.** Let  $(X, S^*)$  be a supra topological space. Then the following are equivalent.

- (1)  $(X, S^*)$  is hyperconnected.
- (2)  $A \cap B \neq \phi$  for every nonempty  $s_s^*$ -open set  $A$  and nonempty  $p_{s_s^*}$ -open set  $B$ .
- (3)  $A \cap B \neq \phi$  for every nonempty  $s_s^*$ -open sets  $A$  and  $B$ .

**Proof.** (1)  $\Rightarrow$  (2): Assume that  $X$  is hyperconnected. Let  $A$  be a nonempty  $s_s^*$ -open set and  $B$  be a nonempty  $\beta_{s_s^*}$ -open  $\beta_{s_s^*}$ -open set such that  $A \cap B = \phi$ . Then by Theorem 2.9(5),  $c_{s_s^*}^s(B) = X$ . But  $X = c_{s_s^*}^s(B) \subset c_{s_s^*}^s(X - A) = X - A$ . Hence  $A = \phi$ , this is a contradiction.

(2)  $\Rightarrow$  (1): Suppose that  $A$  and  $B$  are any two nonempty  $s_s^*$ -open subsets of  $X$ . Hence, by (2),  $A \cap B \neq \phi$ . Thus, by Proposition 2.5,  $X$  is hyperconnected. Similarly we can prove (1) is equivalent to (3).

### III. Preservation Theorems

In this section, some types of functions under which supra hyperconnectedness is preserved are introduced and some theorems are given.

**Definition 3.1.** The  $S^*$ -semi-interior of a subset  $A$  of a supra topological space  $(X, S^*)$ , denoted by  $i_{s_s^*}^s(A)$  is the union of all  $s_s^*$ -open sets of  $X$  contained in  $A$ .

**Definition 3.2.** Let  $(X, S_1^*)$  and  $(Y, S_2^*)$  be two supra topological spaces. A function  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is called rs-continuous if for each nonempty  $r_{s_2^*}$ -open set  $V$  of  $Y$ ,  $f^{-1}(V) \neq \phi$  then  $i_{s_1^*}^s(f^{-1}(V)) \neq \phi$ .

**Definition 3.3.** A function  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is called  $S^*$ -semi-continuous function if  $f^{-1}(V)$  is  $s_{s_1^*}$ -open in  $X$  for each  $S_2^*$ -open set  $V$  of  $Y$ .

**Theorem 3.4.** Let  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  be an  $S^*$ -semi-continuous function, then  $f$  is rs-continuous function.

**Proof.** Suppose that  $V$  is a  $r_{s_2^*}$ -open subset of  $Y$  such that  $f^{-1}(V) \neq \phi$ . Since every  $r_{s_2^*}$ -open set is  $S_2^*$ -open and  $f$  is  $S^*$ -semi-continuous. So  $f^{-1}(V)$  is a nonempty  $s_{s_1^*}$ -open in  $X$ . Hence  $f^{-1}(V) = i_{s_1^*}^s(f^{-1}(V))$  and  $i_{s_1^*}^s(f^{-1}(V)) \neq \phi$ . Thus  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is rs-continuous.

**Theorem 3.5.** If  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is an rs-continuous surjection and  $(X, S_1^*)$  is hyperconnected, then  $(Y, S_2^*)$  is hyperconnected.

**Proof.** Assume that  $(Y, S_2^*)$  is not hyperconnected. Then there are disjoint  $S_2^*$ -open sets  $A$  and  $B$ . Put  $U = \text{int}_{S_2^*}(cl_{S_2^*}(A))$  and  $V = \text{int}_{S_2^*}(cl_{S_2^*}(B))$ . Hence  $U = \text{int}_{S_2^*}(cl_{S_2^*}(U))$  and  $V = \text{int}_{S_2^*}(cl_{S_2^*}(V))$ . Thus  $U$  and  $V$  are disjoint nonempty  $r_{S_2^*}$ -open sets. Hence  $i_{S_1^*}^s(f^{-1}(U)) \cap i_{S_1^*}^s(f^{-1}(V)) \subset f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Since  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is an rs-continuous surjection, then  $i_{S_1^*}^s(f^{-1}(U)) \neq \emptyset$  and  $i_{S_1^*}^s(f^{-1}(V)) \neq \emptyset$ . Hence, by Corollary 2.10, that  $(X, S_1^*)$  is not a hyperconnected. This is a contradiction.

Combining Theorem 3.4 and Theorem 3.5, we get the following result.

**Corollary 3.6.** If  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is  $S^*$ -semi-continuous surjection and  $(X, S_1^*)$  is hyperconnected, then  $(Y, S_2^*)$  is hyperconnected.

**Definition 3.7.** [1] A function  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is called  $S^*$ -continuous function if  $f^{-1}(V) \in S_1^*$  for every  $V \in S_2^*$ .

**Remark 3.8.** It is clear that every  $S^*$ -continuous function is  $S^*$ -semi-continuous but not conversely as shown by the following example.

**Example 3.9.** Let  $X=Y= \{a,b,c,d\}$  and  $S_1^* = S_2^* = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}\}$ . Define  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  as follows,  $f(a)= a, f(b)= d, f(c)= d, f(d)= b$ . Then  $f$  is  $S^*$ -semi-continuous but not  $S^*$ -continuous.

From Theorem 3.4 and Remark 3.8 we have the following implications.

**Remark 3.10.**  $S^*$ -continuous  $\Rightarrow S^*$ -semi-continuous  $\Rightarrow$  rs-continuous.

From Theorem 3.5 and Remark 3.10 we obtained the following result.

**Corollary 3.11.** If  $f : (X, S_2^*) \rightarrow (Y, S_1^*)$  is  $S^*$ -continuous surjection and  $(X, S_1^*)$  is hyperconnected, then  $(Y, S_2^*)$  is hyperconnected.

#### IV. Strong Form Of Supra Hyperconnected Spaces.

In this section, we introduce some strong forms of supra hyperconnectedness. Relationships among them are discussed and characterizations of hyperconnectedness are obtained.

**Definition 4.1.** A supra topological space  $(X, S^*)$  is called  $\alpha^*$ -hyperconnected (resp.  $P^*$ -hyperconnected, semi $^*$ -hyperconnected,  $b^*$ -hyperconnected,  $\beta^*$ -hyperconnected) if  $(X, \alpha^*(X))$  (resp.  $(X, P^*O(X)), (X, S^*O(X)), (X, B^*O(X)), (X, \beta^*O(X))$ ) is hyperconnected.

As an immediate consequence of the Definition 4.1, we have the following implications.

- (1)  $\beta^*$ -hyperc.  $\Rightarrow b^*$ -hyperc.  $\Rightarrow$  semi $^*$ -hyperc.  $\Rightarrow \alpha^*$ -hyperc.  $\Rightarrow$  hyperc.
- (2)  $b^*$ -hyperc.  $\Rightarrow p^*$ -hyperc.  $\Rightarrow \alpha^*$ -hyperc.

The converses of these implications is not true in general, the following example shows some of them.

**Example 4.2.** Let  $X = \{a,b,c\}$   $S^* = \{X, \emptyset, \{a,b\}\}$  then  $B^*O(X) = P(X) - \{c\} = P^*(X)$  and  $S^*O(X) = \{X, \emptyset, \{a,b\}\}$ . So  $X$  is supra hyperconnected, semi $^*$ -hyperconnected space but not  $b^*$ -hyperconnected (hence not  $\beta^*$ -hyperconnected). Also it is not  $p^*$ -hyperconnected

**Notation 4.3.** If we assume that  $S^*$  is a topology, it clear that the notions given in Definition 4.1, are equivalent to the known notions:  $\alpha$ -hyperconnected [18] (resp. Pre-hyperconnected [18], semi-hyperconnected [18],  $b$ -hyperconnected,  $\beta$ -hyperconnected [18]) respectively. We say that  $(X, \tau)$  is  $b$ -hyperconnected if there is no disjoint nonempty  $b$ -open sets. In this section we give more results of these spaces.

Recall that  $(X, \tau)$  is a partition space [12] if every open subset is closed or equivalently if every subset is preopen.

It is easy to prove the following,

**Theorem 4.4.** Let  $(X, \tau)$  be a partition space then

- (1)  $\tau = \alpha O(X) = SO(X)$ .
- (2)  $\beta O(X) = BO(X) = PO(X)$ .

**Corollary 4.5.** Let  $(X, \tau)$  be a partition space. Then

- (1)  $X$  is hyperconnected if and only if  $X$  is  $\alpha$ -hyperconnected if and only if  $X$  is s-hyperconnected.
- (2)  $X$  is  $\beta$ -hyperconnected if and only if  $X$  is b-hyperconnected if and only if  $X$  is p-hyperconnected.

Mathew in [19], proved that  $(X, \tau)$  a topological space  $(X, \tau)$  is hyperconnected if and only if  $SO(X)/\{\emptyset\}$  is a filter on  $X$ . Now, we give the following result.

**Lemma 4.6.** [19] If  $(X, \tau)$  is hyperconnected space. Then a nonempty set is s-open if and only if it contains a nonempty open set.

**Theorem 4.7.** A topological space  $X$  is s-hyperconnected space if and only if  $SO(X)/\{\emptyset\}$  is a filter on  $X$ .

**Proof.** Necessity. Suppose that  $X$  is s-hyperconnected, hence  $X$  is hyperconnected. Therefore  $SO(X)/\{\emptyset\}$  be a filter on  $X$ , by (Theorem 1) of [19].

Sufficiency. Let  $SO(X)/\{\emptyset\}$  be a filter on  $X$ . Let  $A, B \in SO(X)/\{\emptyset\}$ . So  $A \cap B \neq \emptyset$ . Thus  $X$  is s-hyperconnected space.

Elkin[20] defined a topological space  $(X, \tau)$  to be globally disconnected if every set which can be placed between an open set and its closure is open, equivalently if every semi open set is open.

We have the following characterization of hyperconnected topological space.

**Theorem 4.8.** If  $(X, \tau)$  is globally disconnected then,  $(X, \tau)$  is hyperconnected if and only if  $(X, \tau)$  is semi-hyperconnected space.

## V. Conclusion

Topology is important in many fields of applied sciences as well as branches of mathematics, such as, computational topology for geometric design, algebraic geometry, digital topology, particle physics and quantum physics. Hyperconnectedness is an important property which depends on the classes of open and dense sets. In this work we study hyperconnectedness in supra topological spaces and strong forms of hyperconnected spaces are defined. Several characterizations of hyperconnectedness are provided, preservation theorems are given. Finally we may mention that, this work help researcher to study several generalizations of hyperconnectedness in topological spaces.

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