Wet Dark Fluid Cosmological Model In Ruban’s Background

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Abstract: We have obtained Ruban’s cosmological model with wet dark fluid in general theory of relativity. For solving the Einstein field equations the relation between metric coefficients is used. Also, some physical and kinematical properties of the model are discussed.

Key words: Ruban’s space time, Wet dark fluid.

I. Introduction

Even today, one of the basic problems in cosmology is to known the formation of large scale structure of the universe. Cosmological models play a vital role in the understanding of the universe around us. In view of its importance in explaining the observational cosmology many authors have consider cosmological models with dark energy. The wet dark fluid (WDF) as a model for dark energy. This model was in the spirit of the generalized chaplygin gas (GCG), where a physically motivated equation of state was offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait (1988) and Hayword (1967) to treat water and aqueous solution. The equation of state for WDF is very simple.

\[ p_{WDF} = \gamma (\rho_{WDF} - \gamma) \]  

and is motivated by the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures possible.

To find the WDF energy density, we use the energy conservation equation

\[ \dot{\rho}_{WDF} + 3H (p_{WDF} + \rho_{WDF}) = 0 \]  

\[
\Rightarrow \quad \rho_{WDF} = \frac{\gamma}{1 + \gamma} \rho^* + \frac{D}{V^{(1+\gamma)}} ,
\]

where \( D \) is the constant of integration and \( V \) is the volume expansion. WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece that red shifts as a standard fluid with an equation of state \( p = \gamma \rho \). We can show that it we take \( D > 0 \), this fluid will never violate the strong energy condition \( p + \rho \geq 0 \). Thus, we get

\[ p_{WDF} + \rho_{WDF} = (1 + \gamma) \rho_{WDF}^* - \gamma \rho^* = D(1 + \gamma) \frac{D}{V^{(1+\gamma)}} \geq 0 . \]


The purpose of the present work is to obtain Ruban’s cosmological model in presence of wet dark fluid. Our paper is organized as follows. In section 2, we derive the field equation in presence wet dark fluid with the aid of Ruban’s space time. Section 3, deals with solution of field Equations, section 4 is mainly concerned with the physical and kinematical properties of the model. The last section contain some conclusion.
II. The Metric and Field equations

We consider the space-time of Ruban’s in the form
\[ ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2dz^2) \]  
where \( h(y) = \frac{\sin \sqrt{k}y}{\sqrt{k}} \) if \( k = 1 \)
\( = y \) if \( k = 0 \)
\( = \sin y \) if \( k = -1 \)

and \( k \) is the curvature parameter of the homogeneous 2-spaces \( t \) and \( x \) constants. The functions \( Q \) and \( R \) are free and will be determined.

The energy-momentum tensor for wet dark fluid is
\[ T_{ij} = (\rho_{WDF} + p_{WDF})u_iu_j - p_{WDF} \]  

Where, \( u^i \) is flow vector satisfying \( g_{ij}u_iu_j = 1 \)
\[ x^i = (1,0,0,0) \]  
\[ u^i = (0,0,0,1) \]  
\[ x^1 = -1 \] and \( u^4u_4 = 1 \)

The Einstein’s field equation (with gravitational unit \( 8\pi G = C = 1 \))
\[ R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \]  

where \( R^i_{\cdot j} \) is the Ricci tensor and \( R = g^{ij}R_{ij} \) is the Ricci scalar

The field equations (7) for the metric (4) with matter distribution (5) yield
\[ 2 \frac{\dddot{R}}{R} + \left( \frac{\ddot{R}}{R} \right)^2 + \frac{k}{R^2} = -p_{WDF} \]  
\[ \frac{\dddot{Q}}{RQ} + \frac{\dddot{R}}{R} + \frac{\dddot{Q}}{Q} = -p_{WDF} \]  
\[ 2 \frac{\dddot{R}}{RQ} + \left( \frac{\ddot{R}}{R} \right)^2 + \frac{k}{R^2} = \rho_{WDF} \]

Here the over head dot at the symbol \( Q \) and \( R \) denotes ordinary differentiation with respectively to time \( t \).

III. Solution of field Equations

Here we have three independent field equations (8-10) connecting four unknown \( R, Q, \rho_{WDF} \) and \( p_{WDF} \). Therefore in order to obtain exact solutions, we must need one more relation connecting the unknown quantities. We assume the relations as the equation of state for WDF \( \rho_{WDF} = p_{WDF} \). Also, the field equations are highly nonlinear, we assume the relation between metric coefficients \( Q = x^nR^n \).

Using this relation, the field equation (8-10) admit the exact solution.
\[ R = M(c_1t + c_2)^{\frac{1}{n+2}} \]  
\[ Q = Nc_n(c_1t + c_2)^{\frac{n}{n+2}} \]
\[ M = (n + 2)^{\frac{1}{n+2}} \]  
\[ N = m^n \]
Through a Proper choice of coordinates and constants Ruban’s cosmological model can be written as

\[ ds^2 = \frac{dT^2}{c_1^2} - N^2 x^{2n} t^{2n} dx^2 - M^2 T^{2n} \left( dy^2 + h^2 dz^2 \right) \]  

(13)

where \( T = c_1 t + c_2 \).

IV. The Physical And Kinematical Properties

In this section we discuss some Physical and Kinematical Properties of the WDF model. The energy density \( \rho_{WDF} \), the pressure density \( P_{WDF} \) for the mode (13) are given by

\[ P_{WDF} = \frac{(2n+1) c_1^2}{(n+2)^2} T = \rho_{WDF} \]  

(14)

Also, the physical quantities that are important in cosmology are spatial volume, scalar expansion, shear scalar and Hubble parameter for the model (13) given by

The spatial volume is given by \( V = x^n N^2 T \)  

(15)

The scalar expansion \( \theta = \frac{Q}{Q} + 2 \frac{R}{R} = \frac{c_1}{T} \)  

(16)

shear scalar \( \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{c_1}{T^2} \)  

(17)

where \( c_3 = \frac{(n-1)^2 c_1^2}{3 (n+2)^2} \)

Hubble parameter \( H = \frac{1}{3} \frac{V}{V} = \frac{\theta}{3} = \frac{c_1}{3T} \)  

(18)

It may be observed that initial moment, when \( T=0 \) the spatial volume will be zero while the energy density \( \rho_{WDF} \) and pressure density \( P_{WDF} \) diverge.

when \( T \to 0 \), then scalar expansion \( \theta \), shear scalar \( \sigma^2 \) and Hubble parameter \( H \) tends to \( \infty \).

For large values of \( T (T \to \infty) \) we observe that spatial volume, scalar expansion \( \theta \), shear scalar \( \sigma^2 \), Hubble parameter \( H \), energy density \( \rho_{WDF} \) and pressure density \( P_{WDF} \) becomes zero.

Also \( T \xrightarrow{\text{lim}} \infty \left( \frac{\sigma}{\theta} \right)^2 \neq 0 \) and hence the model does not approach isotropy.

V. Conclusion

In this paper, we have considered Einstein field equation in the presence of wet dark fluid in Ruban’s background. For solving the field equation the relation between metric coefficients is used. The model are free from initial singularities and they are expanding, shearing and non rotating in the standard way. Also, we find that all physical quantities like energy density of the wet dark fluid diverge at the initial moment of creation. It is interesting to note that the result of Einstein field equation in Ruban’s back ground with Wet dark fluid is similar to the result of N-Dimensional Kaluza-Klein Type Cosmological model with wet dark fluid (13).

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References