An EOQ model for Price Discount Linked to Order Quantity under Fuzzy Environment in Quadratic Demand Pattern

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Abstract: In this paper a study has been carried out using crisp and fuzzy inventory model for the deteriorating items under trapezoidal fuzzy numbers when the supplier offered price discount to the retailer at the time of replenishment. In this model the deterioration rate is constant. Many researchers suggested, demand rate in the inventory models are constant, exponential (increase/decrease) and linearly increasing / decreasing demand patterns.. In practical situation, quadratic demand rate is more realistic,. In this paper, an inventory model is developed in crisp and fuzzy environment. This paper investigates the feasibility of regular order and special order offered by the supplier in which to maximize the total cost saving. Numerical example and sensitivity analysis carried out in fuzzy environment

Keywords: Fuzzy inventory, Inventory, Deterioration, Price linked, Holding Cost

I. Introduction


Assumptions and Notations:
(i) \( D(t) = (a_1 + b_1t + c_1t^2) \) \( a_1 \geq 0, b_1 \neq 0, c_1 \neq 0 \). Here a is the initial rate of demand, b is the rate with which the demand rate increases and c is the rate with which the change in the rate demand rate itself increases
(ii) \( I_1(t) \) is the inventory at any time ‘t’
(iii) Replenishment rate is infinite
(iv) Lead time is zero
(v) \( \theta \) is the deterioration rate which is constant
(vi) \( C \), is the cost per unit
(vii) \( h \) is the holding cost

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(viii) A is the ordering cost  
(ix) $T^*$ is the optimal length of the replenishment cycle  
(x) $Q^*$ is the optimal order quantity  
(xi) $I_{sd}(t)$ Inventory level during $0 \leq t \leq T_{sp}$

**II. Mathematical Model**

The inventory level depletes as the time passes due to selling rate and deterioration. The differential equation which describes the inventory level at time t can be written as

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \quad 0 \leq t \leq T.$$  

Where $D(t) = (a_1 + b_1 t + c_1 t^2)$  

(1)

The solution of equation (1) for the boundary condition $I(T) = 0$, is

$$I_1(t) = \left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3}\right)(1 - \theta(T-t)) + \frac{b_1}{\theta} (-t + Te^{\theta(T-t)}) + \frac{c_1}{\theta} (-t^2 + T^2 e^{\theta(T-t)}) + \frac{2c_1}{\theta^2} (t - Te^{\theta(T-t)})$$  

(2)

Carrying cost/holding cost per cycle =

(3)

Material cost per cycle

(including Deterioration Loss) = $QC = I(0)C$

(4)

Total Cost (TC) = Carrying cost + Material cost + Ordering cost

$$= \frac{h}{0} \int I_1(t) dt + I(0)C + A$$  

(5)

$$Total \cos \; t(TC) = \frac{h}{T} \left[ -\left(\frac{a_1 T + b_1 T^2}{2} + \frac{c_1 T^3}{3}\right) - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T + c_1 T^2}{\theta^3} - \frac{b_1 + 2c_1 T}{\theta^4} + \frac{2c_1}{\theta^5}\right) e^{\theta T} \right]$$  

$$+ \frac{C}{T} \left[ -\frac{a_1}{\theta} + \frac{b_1}{\theta^2} + \frac{2c_1}{\theta^3}\right] \left(\frac{a_1 + b_1 T + c_1 T^2}{\theta} - \frac{b_1 + 2c_1 T}{\theta^2} + \frac{2c_1}{\theta^3}\right) e^{\theta T} + \frac{A}{T}$$  

(6)

From the above (6) the unique value of $T^*$, optimal length of replenishment cycle time (say $T^*$) can be obtained. Similarly the optimal order quantity $Q^*$ can be found out from $I(t)$, i.e $I(0) = Q$. The purpose of this paper is to study optimal order quantity by maximizing the total cost saving during the length of depletion time for the special order quantity.

Special order occurs (Retailers replenishment)  

If the retailer order $Q_{sp}$ units under special order policy, the inventory level at time ‘t’ is

$$I_{sp}(t) = \left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3}\right)(1 - e^{\theta(T-t)}) + \frac{b_1}{\theta} (-t + Te^{\theta(T-t)}) + \frac{c_1}{\theta} (-t^2 + T^2 e^{\theta(T-t)}) + \frac{2c_1}{\theta^2} (t - Te^{\theta(T-t)})$$  

, $0 \leq t \leq T_{sp}$

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Page 21
An EOQ model for Price Discount Linked to Order Quantity under Fuzzy...

Similarly

$$Q_{sp} = \frac{-a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^3} + \left( \frac{a_1 + b_1T + c_1T^2}{\theta} - \frac{b_11 + 2c_1t}{\theta^2} + \frac{2c_1}{\theta^3} \right) \theta^T$$

Since the price discount rate being dependent on special order let price discount rate be $\delta_t$ in $(0,T_{sp})$ denoted by $TC_{sp}(T_{sp})$

$$TC_{sp}(T_{sp}) = h(1-\delta_t) \left[ -\frac{a_1T_{sp}^2 + b_1T_{sp}^3 + c_1T_{sp}^4}{2} \theta^2 + \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^3} + \left( \frac{a_1 + b_1T_{sp} + c_1T_{sp}^2}{\theta} - \frac{b_11 + 2c_1t}{\theta^2} + \frac{2c_1}{\theta^3} \right) \theta^T \right]$$

$$+ C(1-\delta_t) \left[ -\frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^3} + \left( \frac{a_1 + b_1T_{sp} + c_1T_{sp}^2}{\theta} - \frac{b_11 + 2c_1t}{\theta^2} + \frac{2c_1}{\theta^3} \right) \theta^T + A \right]$$

(7)

On the other hand, if the retailer adopts $Q^*$ (regular order policy) in place of a large special order policy the TC(Total Cost) during $[0,T_{sp}]$ can be obtained by average cost approach. i.e. in the time interval $[0,T_{sp}]$ the total cost of regular order is $TC_{ro}(T_{sp})$

$$TC_{ro}(T_{sp}) = \frac{T_{sp}}{\theta^T} \left[ -\frac{a_1T_{sp}^2 + b_1T_{sp}^3 + c_1T_{sp}^4}{2} \theta^2 + \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^3} + \left( \frac{a_1 + b_1T_{sp}^* + c_1T_{sp}^{*2}}{\theta} - \frac{b_11 + 2c_1t^*}{\theta^2} + \frac{2c_1}{\theta^3} \right) \theta^T \right]$$

$$+ C \left[ -\frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^3} + \left( \frac{a_1 + b_1T_{sp}^* + c_1T_{sp}^{*2}}{\theta} - \frac{b_11 + 2c_1t^*}{\theta^2} + \frac{2c_1}{\theta^3} \right) \theta^T + A \right]$$

(8)

Comparing (7) and (8) for the fixed price discount rate $\delta_t$, the total cost saving can be formulated as follows.

\[ T_1(t) \]

Fig (1): Regular order vs. special order policies when the special order time coincides with the retailer's
replenishment time.

\[
G_s(T_{sp}) = \frac{Ts_p}{T}
\]

\[
= \frac{1}{h} \left[ \left( -\frac{a_1 T^* + b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3} \right) - \frac{a_1}{\theta^1} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left( \frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^4} \right) e^{\theta t} \right] \\
+ C \left[ \left( -\frac{a_1}{\theta} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} \right) + \left( \frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^4} \right) e^{\theta t} \right] + A
\]

\[
- h(1 - \delta_1) \left[ \left( -\frac{a_1}{\theta} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} \right) + \left( \frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^4} \right) e^{\theta t} \right] - A
\]

\[
- C(1 - \delta_1) \left[ \left( -\frac{a_1}{\theta} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} \right) + \left( \frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^4} \right) e^{\theta t} \right] - A
\]

The necessary condition for a maximize \(G_s(T_s)\),

\[
\frac{d(G_s(T_s))}{dT_s} = 0 \quad \frac{d^2(G_s(T_s))}{dT_s^2} < 0
\]

III. Fuzzy Model and solution

Let us consider the model in fuzzy environment. Due to fuzziness, precisely defining all parameter is not easy. Hence Let \(\alpha\) = (A1, A12, A13, A14), \(C_{i}=(0, 02, 03, 04), C_{DC}=(C_{DC1}, C_{DC2}, C_{DC3}, C_{DC4})\) be trapezoidal fuzzy numbers in LR form. Now, in the fuzzy sense the total cost of the system is given by

\[
F_s(T_{sp}) = \frac{Ts_p}{T}
\]
An EOQ model for Price Discount Linked to Order Quantity under Fuzzy...
An EOQ model for Price Discount Linked to Order Quantity under Fuzzy...

\[
Z = \frac{T_{sp}}{T}
\]

\[
\left[ -\left( \frac{aT^* + bT^2}{2} + cT^3}{3} \right) \right] - \left[ \frac{a_1}{a_4^2} + \frac{b_1}{a_4^2} - \frac{2c_1}{a_4^2} + \left( \frac{a_1 + b_1T^* + c_1T^2}{a_4^2} \right) \right]^{\delta T^2}
\]

\[
- C\left[ \frac{aT_{sp}}{a_4^2} + \frac{bT_{sp}^2}{a_4^2} + \frac{cT_{sp}^3}{a_4^2} \right] - \left[ \frac{a_1}{a_4^2} + \frac{b_1}{a_4^2} - \frac{2c_1}{a_4^2} + \left( \frac{a_1 + b_1T_{sp} + c_1T_{sp}^2}{a_4^2} \right) \right]^{\delta T_{sp}^2}
\]

The \(a\)-cuts, \(C_L(\alpha)\) and \(C_R(\alpha)\) of the Trapezoidal fuzzy number \(F_3(T_{sp})\) are given by

\[
C_L(\alpha) = W+(X-W) \alpha \quad \text{and} \quad C_R(\alpha) = Z-(Z-Y) \alpha
\]

By using signed distance method, the defuzzified value of fuzzy number \(F_3(T)\), is given by

\[
F_3(T_{sp})_{SD} = \frac{1}{2} \left[ C_L(\alpha) + C_R(\alpha) \right] d\alpha
\]

The necessary condition for minimizing the total cost is

\[
\frac{\partial F_3(T_{sp})_{SD}}{\partial T_{sp}^2} = 0 \quad \text{and} \quad \frac{\partial^2 F_3(T_{sp})_{SD}}{\partial T_{sp}^2} < 0
\]

The optimal value of \(T_{sp}^*\) and the total cost \((F_3(T_{sp})_{SD})\) is obtained using mathematical software MATHCAD

3.1. Numerical example.

To illustrate the effectiveness of the model, we consider the following values for the parameters

\(a = 100, b = 70\) and \(c = 3\), \(A = 300, h = 1\), \(C = 10\), \(\Theta = 0.01, T^* = 1.003, Q = 135.993\) (T find \(T^*\))

**Crisp Model:**

When \(a = 100, b = 70\) and \(c = 3\), \(A = 300, h = 1\), \(C = 10\), \(\Theta = 0.01, T^* = 1.003, G_s = 124.835, Q_{sp}^* = 136.929\)

\((T^* < T_{sp}^*)\)

**Case I:**

When all \(A, \delta, \Theta\) are trapezoidal numbers, solution of fuzzy model is \(T_{sp}^* = 1.003, G_s(T_{sp}) = 126.658\), \(Q_{sp}^* = 136.929\)

**Case II:**

When all \(\delta, \Theta\) are trapezoidal numbers then solution of fuzzy model is \(T_{sp}^* = 1.003, G_s(T_{sp}) = 126.658, Q_{sp} = 136.929\)

**Case III:**

When \(\Theta\) is fuzzy trapezoidal numbers then solution of fuzzy model is \(T_{sp}^* = 1.001, G_s(T_{sp}) = 125.76, Q_{sp}^* = 136.581\)

When none of these parameters are fuzzy trapezoidal number, then \(T^* = 1.001, G_s(T_{sp}) = 125.76, Q_{sp}^* = 136.581\)

DOI: 10.9790/5728-11412026 www.iosrjournals.org 25 | Page
3.2 Sensitivity Analysis:

### Sensitivity analysis for the parameter $A$

<table>
<thead>
<tr>
<th>Defuzzify value of $A$</th>
<th>Fuzzify value of $A$</th>
<th>$T_0$</th>
<th>$E(T_0)$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>50,150,250,350</td>
<td>0.827</td>
<td>13.538</td>
<td>107.492</td>
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<td>250</td>
<td>100,200,300,400</td>
<td>0.932</td>
<td>15.637</td>
<td>124.771</td>
</tr>
<tr>
<td>300</td>
<td>150,250,350,450</td>
<td>1.001</td>
<td>125.762</td>
<td>136.581</td>
</tr>
<tr>
<td>350</td>
<td>200,300,400,500</td>
<td>1.069</td>
<td>140.128</td>
<td>148.577</td>
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</tbody>
</table>

### Sensitivity analysis for the parameter $\Theta$

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<tr>
<th>Defuzzify value of $\Theta$</th>
<th>Fuzzify value of $\Theta$</th>
<th>$T_0$</th>
<th>$E(T_0)$</th>
<th>$Q_0$</th>
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<tbody>
<tr>
<td>004</td>
<td>0.001,003,005,007</td>
<td>1.008</td>
<td>126.498</td>
<td>137.8</td>
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<td>006</td>
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<td>008</td>
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<td>126.003</td>
<td>136.929</td>
</tr>
<tr>
<td>010</td>
<td>0.007,009,011,013</td>
<td>1.001</td>
<td>125.762</td>
<td>136.58</td>
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</table>

### Sensitivity analysis for the parameter $\delta$

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<th>Defuzzify value of $\delta$</th>
<th>Fuzzify value of $\delta$</th>
<th>$T_0$</th>
<th>$E(T_0)$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.04,0.08,012,016</td>
<td>1.003</td>
<td>126.658</td>
<td>136.929</td>
</tr>
<tr>
<td>02</td>
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<td>179.742</td>
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<td>03</td>
<td>0.0,0.2,0.4,0.6</td>
<td>1.533</td>
<td>523.333</td>
<td>240.009</td>
</tr>
</tbody>
</table>

### IV. Conclusion

This paper investigates an EOQ model with time dependent quadratic demand pattern approach is derived. Here the deterioration rate is constant. This paper an contains the analysis of temporary price discount offered by a supplier on a retailer replenishment policy for deteriorating items. Numerical example and sensitivity analysis also carried out.

**Scope For Further Research**

This paper can be extended by incorporating with shortages. Instead of special order vs regular order policy, this paper can be modified, when special order time occurs during the retailer’s sales period.

### References