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Successive Approximation Method for Rayleigh Wave Equation

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Abstract: *In this paper, Rayleigh wave equation has been solved numerically for finding an approximate solution by Successive approximation method and Finite difference method. Example showed that Successive approximation method is much faster and effective for this kind of problems than Finite difference method.*
Keywords - *Rayleigh wave equation, SAM, FDM.*

I. Introduction

Nonlinear first-order partial differential equations arise in a variety of physical theories, primarily in dynamics (to generate canonical transformations), continuum mechanics (to record conservation of mass, momentum, energy, etc.) and optics (to describe wave fronts). Although the strong nonlinearity generally precludes our deriving any simple formulas for solutions, we can, remarkably, often employ There is an approximation method for solving integral equations and differential equations. This method starts by using the constant function as an approximation to a solution. We substitute this approximation into the right side of the given equation and use the result as a next approximation to the solution. Then we substitute this approximation into the right side of the given equation to obtain what we hope is a still better approximation and we continuing the process. Our goal is to find a function with the property that when it is substituted in the right side of the given equation the result is the same function. This procedure is known as successive approximation method [1].

II. Indentations And EQUATIONS

2.1 Mathematical model:

In the physical and mathematical literature [2]-[4] we find the Rayleigh wave equation

$$u_{tt} = u_{xx} + \varepsilon(u_t - u_t^3) \tag{1}$$

Related to Rayleigh wave equation of Van Der Pol type

$$u_{tt} - u_{xx} = \varepsilon(1 - u^2)u_t. \tag{2}$$

Each of these has been used to model physical phenomena. Now , the PDE (1) serves as a model for large amplitude vibrations of wind-blown , ice-laden power transmission lines, in time that, the PDE (2) describes plan electromagnetic wave propagation between two parallel planes in a region where the conductivity varies quadratically with electric field.

2.2 Finite Difference Method (FDM)

A finite difference method proceeds by replacing the derivatives in the differential equations by finite difference approximations. We assume the function whose derivatives are to be approximated by using Taylor's expansion. This gives a large algebraic system of equations to be solved in place of the differential equation, something that is easily solved on a computer. So we can get [5]

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{p,q} = \frac{u(p+1,q) - 2u(p,q) + u(p-1,q)}{h^2} \tag{3}$$

Similarly $$\left(\frac{\partial^2 u}{\partial t^2}\right)_{p,q} = \frac{u(p,q+1) - 2u(p,q) + u(p,q-1)}{k^2} \tag{4}$$

With the notation the forward difference approximation $\frac{\partial u}{\partial t}$ is

$$\left(\frac{\partial u}{\partial t}\right)_{p,q} = \frac{u(p,q+1) - u(p,q)}{k} \tag{5}$$

Similarly, for central difference approximation $\frac{\partial u}{\partial x}$ is

$$\left(\frac{\partial u}{\partial x}\right)_{p,q} = \frac{u(p+1,q)-u(p-1,q)}{2h} \tag{6}$$

2.3 Derivation of FDM for solving nonlinear Rayleigh wave equation

Substituting equations (3-5) in equation (1) we get

$$\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{k^2} = \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2} + \frac{0.1}{k}\left(u_{i,j}-u_{i,j-1}\right) - \frac{0.1}{k^3}\left(u_{i,j}-u_{i,j-1}\right)^3$$

$$\left(u_{i,j+1}-2u_{i,j}+u_{i,j-1}\right) = \frac{k^2}{h^2}\left(u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right) + 0.1k\left(u_{i,j}-u_{i,j-1}\right) - \frac{0.1}{k}\left(u_{i,j}-u_{i,j-1}\right)^3$$

$$u_{i,j+1} = (2-2r+s)u_{i,j} + r\left(u_{i+1,j}+u_{i-1,j}\right) - (1+s)u_{i,j-1} + o\left(u_{i,j}-u_{i,j-1}\right)^3 \tag{7}$$

$$r = \frac{k^2}{h^2}; \qquad s = 0.1k; \qquad o = \frac{0.1}{k}$$

2.4 Basic Idea of the Successive Method (SAM)

The method of SAM provides a method that can, in principle, be used to solve any initial value problem [6]

$$u' = f(t,u); \qquad u(t_0) = u_0 \tag{8}$$

It starts by observing that any solution to (8) must also be a solution to

$$u(t) = u_0 + \int_{t_0}^{t} f(s,u(s))ds \tag{9}$$

And then iteratively constructs a sequence of solutions that get closer and closer to the actual (exact) solutions of (9). The SAM is based on the integral equation (9) as follows:

$$u_0(t) = u_0$$

$$u_1(t) = u_0 + \int_{t_0}^{t} f(s,u_0)ds$$

$$u_2(t) = u_0 + \int_{t_0}^{t} f(s,u_1(s))ds$$

$$u_3(t) = u_0 + \int_{t_0}^{t} f(s,u_2(s))ds$$

This process can be continued to obtain the nth approximation,

$$u_n(t) = u_0 + \int_{t_0}^{t} f(s,u_{n-1}(s))ds, \qquad n = 1,2,\ldots.$$

Then determine whether $u_n(x)$ approaches the solution u(x) as n increases. This will be done by proving the following:

- The sequence $\{u_n(x)\}$ converges to a limit u(x), that

$$\lim_{n\to\infty} u_n(x) = u(x), \quad a \le x \le b$$

- The limiting function $u(x)$ is a solution of (9) on the interval $a \le x \le b$.
- The solution $u(x)$ of (9) is unique.

A proof of these results can be constructed along the lines of the corresponding proof for ordinary differential equations [7]

2.5 Derivation of SAM for solving nonlinear Rayleigh wave equation

The general successive approximation method for equation (R)

$$u_n(x,t) = u_0(x,0) + \iint_0^t \frac{\partial^2 u_{n-1}(x,s)}{\partial x^2} dsds + \varepsilon \iint_0^t \frac{\partial u_{n-1}(x,s)}{\partial s} dsds - \varepsilon \iint_0^t \frac{\partial u_{n-1}(x,s)^3}{\partial s} \quad (10)$$

To approximate solution for equation (R), we start with putting n=1 in equation (s) to obtain $u_1(x,t)$

$$u_1(x,t) = u_0(x,t) + \iint_0^t \frac{\partial^2 u_0(x,s)}{\partial x^2} dsds + \varepsilon \iint_0^t \frac{\partial u_0(x,s)}{\partial s} dsds - \varepsilon \iint_0^t \frac{\partial u_0(x,s)^3}{\partial s} \quad (11)$$

$$u_1(x,t) = u_0(x,0) + \left(\frac{\partial^2 u_0(x,t)}{\partial x^2} + \varepsilon \frac{\partial u_0(x,t)}{\partial t} - \varepsilon \frac{\partial u_0(x,t)^3}{\partial t} \right) \frac{t^2}{2}$$

Let

$$v_0(x,t) = \left(\frac{\partial^2 u_0(x,t)}{\partial x^2} + \varepsilon \frac{\partial u_0(x,t)}{\partial t} - \varepsilon \frac{\partial u_0(x,t)^3}{\partial t} \right)$$

So

$$u_1(x,t) = u_0(x,t) + v_0(x,t) \frac{t^2}{2} \quad (12)$$

Put n=2 in (10) to obtain a second approximation $u_2(x,t)$ as follow :

$$u_2(x,t) = u_0(x,t) + \iint_0^t \frac{\partial^2 u_1(x,s)}{\partial x^2} dsds + \varepsilon \iint_0^t \frac{\partial u_1(x,s)}{\partial s} dtdt - \varepsilon \iint_0^t \frac{\partial u_1(x,s)^3}{\partial s} \quad (13)$$

Substituting equation (12) in equation (13) we get

$$u_2(x,t) = u_0(x,t) + \iint_0^t \frac{\partial^2}{\partial x^2} \left(u_0(x,s) + v_0(x,s) \frac{t^2}{2} \right) dsds + \varepsilon \iint_0^t \frac{\partial}{\partial t} (u_0(x,s) + v_0(x,s) \frac{t^2}{2}) dsds -$$
$$\varepsilon \iint_0^t \left(\frac{\partial}{\partial t} \left(u_0(x,s) + v_0(x,s) \frac{t^2}{2} \right) \right)^3 dsds$$

$$u_2(x,t) = u_0(x,t) + \left(\frac{\partial^2}{\partial x^2} u_0(x,t) \frac{t^2}{2} + \frac{\partial^2}{\partial x^2} v_0(x,t) \frac{t^4}{4!} \right) + \varepsilon v_0(x,t) \frac{t^3}{6}) - \varepsilon (v_0(x,t))^3 \frac{t^5}{20} \quad (14)$$

By the same way for **n=3,4,...**

III. Figures And Tables

We apply successive approximations method and finite difference method to solve non linear Rayleigh wave equation, and present numerical result to verify the effectiveness of these method ,we take the following example:

$$u_{tt} = u_{xx} + 0.1(u_t - u_t^3), \;\; u(x,0) = x + \sin x, \;\; u_t(x,0) = x(\pi - x)$$

The results are given in the following figures and tables

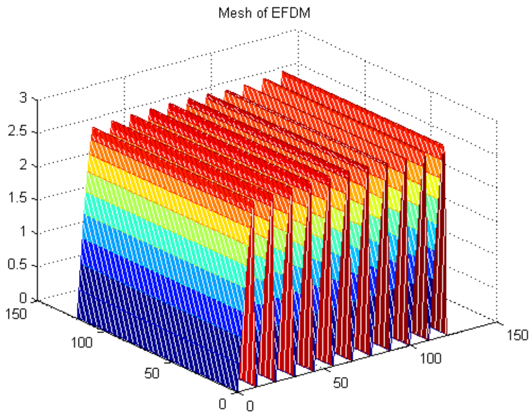

Fig. 1:mesh using finite difference method

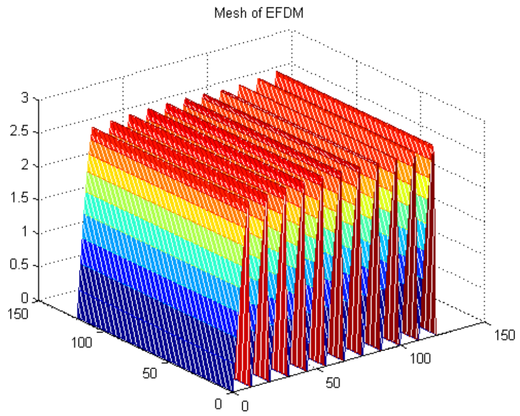

Fig. 2: mesh using successive approximation method

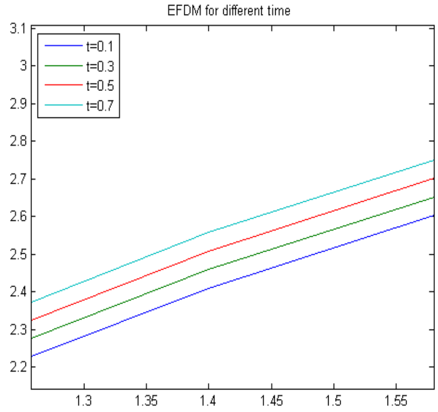

Fig. 3: Explicit Finite Difference method for different t

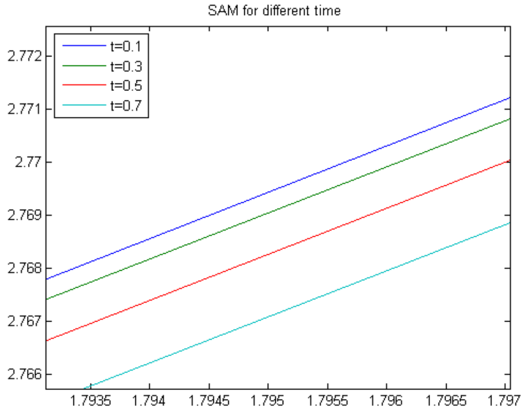

Fig4: Successive approximation method for different t

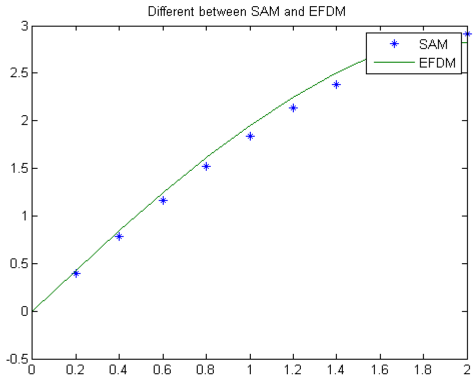

Fig.5: Comparison between Explicit Finite Difference and successive approximation method

Table 1: Explicit Finite Difference method for different t

	t=0.01	t=0.03	t=0.05	t=0.07
x=0	0.00E+00	0.00E+00	0.00E+00	0.00E+00
x=0.2	0.4045525	0.416275195	0.427926508	0.439491096
x=0.4	0.800384682	0.822265341	0.844063097	0.865761991
x=0.6	1.179891984	1.210366276	1.240794057	1.271159198
x=0.8	1.536088775	1.573585194	1.611110546	1.648649727
x=1	1.862886844	1.905818369	1.94887281	1.992037624
x=1.2	2.155338123	2.202099781	2.24907228	2.29624643
x=1.4	2.409831949	2.458803013	2.508044288	2.557546378
x=1.6	2.624239006	2.673770112	2.723346998	2.772436446
x=1.8	2.797990402	2.824360026	2.821120493	2.788684095
x=2	2.797990407	2.824360031	2.821120498	2.788684100

Table 2: successive approximation method for different t

	t=0.01	t=0.03	t=0.05	t=0.07
x=0	-8.33E-11	-6.75E-09	-5.21E-08	-2.00E-07
x=0.2	0.398659394	0.398579775	0.398420023	0.398179227
x=0.4	0.789398863	0.789242828	0.78893001	0.788459217
x=0.6	1.164614228	1.164387982	1.163934634	1.16325303
x=0.8	1.517320203	1.517032745	1.516456912	1.515591746
x=1	1.841428885	1.841091657	1.840416247	1.839401934
x=1.2	2.131992453	2.131618894	2.130870796	2.129747649
x=1.4	2.385400423	2.385005434	2.384214458	2.383027124
x=1.6	2.599523589	2.599122932	2.598320617	2.597116308
x=1.8	2.773798904	2.773408571	2.77262691	2.771453519
x=2	2.909251932	2.908887497	2.908157653	2.907061835

IV. Conclusion

From numerical example we showed that successive approximation method easier, faster and more accurate than finite difference method as shown in figures (1-5) and tables (1-2) .

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