Generalized Common Fixed Theorem in Sequentially Compact Intuitionistic Fuzzy Metric Spaces

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Abstract: The aim of this paper is to introduce the notion of sequentially compact intuitionistic fuzzy metric spaces and prove a generalized common fixed point theorem for pairs of weakly compatible self mappings in this newly defined space.

Keywords: Intuitionistic fuzzy metric space, sequentially compact intuitionistic fuzzy metric space, compatible mappings, weakly compatible mappings, common fixed point.

I. Introduction

The concept of Fuzzy sets was initially investigated by Zadeh [17] as a new way to represent vagueness in everyday life. As a generalization of fuzzy sets introduced Zadeh [17], Atanassov [2], introduced the concept of intuitionistic fuzzy sets. Recently, using the idea of intuitionistic fuzzy sets, in 2004, Park [12] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms. In 2006, Turkoglu [16] proved Jungck’s [6], common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings. Recently, in 2006, Alaca et al. [1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [10]. Jungck and Rhoades [6] gave more generalized concept weak compatibility then compatibility. Recently, many authors have studied fixed point theorem in intuitionistic fuzzy metric spaces ([11], [12], [14,16]). Recently, Rao, K.P.R., Rao, K.R.K. and Rao, T.Ranga [13] introduced the concept of sequentially compact fuzzy metric space. Using this concept, we introduce the notion sequentially compact intuitionistic fuzzy metric spaces and prove a generalized common fixed point theorem for pairs of weakly compatible self mappings in this newly defined space.

II. Preliminaries:

Definition 2[1]:
A binary operation \([0,1] \times [0,1] \rightarrow [0,1]\) is a continuous t-norm if it satisfies the following conditions:
1. \(\ast\) is associative and commutative,
2. \(\ast\) is continuous,
3. \(a \ast 1 = a\) for all \(a \in [0,1]\),
4. \(a \ast b \leq c \ast d\) whenever \(a \leq c\) and \(b \leq d\), for each \(a, b, c, d \in [0,1]\).

Example :- Two typical examples of continuous t-norm are
\[a \ast b = ab\] and \[a \ast b = \min(a, b)\]

Definition 2[2]:
A binary operation \(\ominus: [0,1] \times [0,1] \rightarrow [0,1]\) is a continuous t-conorm if satisfies the following conditions:
1. \(\ominus\) is associative and commutative,
2. \(\ominus\) is continuous,
3. \(a \ominus 0 = a\) for all \(a \in [0,1]\),
4. \(a \ominus b \leq c \ominus d\) whenever \(a \leq c\) and \(b \leq d\), for each \(a, b, c, d \in [0,1]\).

Example :- Two typical examples of continuous t-conorm are
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\[ a \odot b = \min(a + b, 1) \text{ and } a \odot b = \max(a, b) \]

**Definition 2[3]**: A 5-tuple \((X, M, N, *, \odot)\) is called an intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm, \(\odot\) is a continuous t-conorm and M, N are fuzzy sets on \(X \times [0, \infty)\), satisfying the following conditions:

i. \(M(x, y, t) + N(x, y, t) \leq 1\),

ii. \(M(x, y, 0) = 0\),

iii. \(M(x, y, t) = 1\) if and only if \(x = y\),

iv. \(M(x, y, t) = M(y, x, t)\),

v. \(M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)\),

vi. \(M(x, y, .): [0, \infty) \rightarrow [0, 1]\) is left continuous,

vii. \(\lim_{t \rightarrow \infty} M(x, y, t) = 1\),

viii. \(N(x, y, 0) = 1\),

ix. \(N(x, y, t) = 0\) if and only if \(x = y\),

x. \(N(x, y, t) = N(y, x, t)\),

xi. \(N(x, y, t) \odot N(y, z, s) \geq N(x, z, t + s)\),

xii. \(N(x, y, .): [0, \infty) \rightarrow [0, 1]\) is right continuous,

xiii. \(\lim_{t \rightarrow \infty} N(x, y, t) = 0\),

Then \((M, N)\) is called intuitionistic fuzzy metric on X. The function \(M(x, y, t)\) and \(N(x, y, t)\) denote the degree of nearness and the degree of non–nearness between x and y with respect to t, respectively.

**Definition 2[4]**: Let \((X, M, N, *, \odot)\) be an intuitionistic fuzzy metric space. Then a sequence \(\{x_n\}\) in \(X\) is said to be Cauchy sequence if, for all \(t > 0\) and \(p > 0\),

\[ \lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1, \quad \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0. \]

**Definition 2[5]**: Let \((X, M, N, *, \odot)\) be an intuitionistic fuzzy metric space. Then a sequence \(\{x_n\}\) in \(X\) is said to be convergent to a point \(x \in X\) if for all \(t > 0\)

\[ \lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} N(x_n, x, t) = 1. \]

Since * and \(\odot\) are continuous, the limit is uniquely determined from (v) and (xi), respectively.

**Definition 2[6]**: An intuitionistic fuzzy metric space \((X, M, N, *, \odot)\) is said to be complete if every Cauchy sequence is convergent.

**Definition 2[7]**: Let A and B be mappings from an intuitionistic fuzzy metric space \((X, M, N, *, \odot)\) in to itself. The mappings A and B are said to be compatible if

\[ \lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1, \]

\[ \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0 \quad \text{for all} \quad t > 0, \]

whenever \(\{x_n\}\) is a sequence in \(X\).
Such that \( \lim_{n \to \infty} A x_n = \lim_{n \to \infty} B x_n = z \) for some \( z \in X \).

**Definition 2[8]:** Self mappings \( A \) and \( B \) be mappings from an intuitionistic fuzzy metric space \((X, M, N, *,\emptyset)\) is said to be weakly compatible if they commute at their coincidence point, that is \( A x = B x \) implies that \( AB x = BA x \) for some \( x \in X \).

It is easy to see that if self mappings \( A \) and \( B \) of an intuitionistic fuzzy metric space \((X, M, N, *,\emptyset)\) is compatible then they are weakly compatible.

**Definition 2[9]:** \((X, M, N, *,\emptyset)\) is said to be sequentially compact intuitionistic fuzzy metric space if every sequence in \( X \) has a convergent subsequence in it.

Let \( \Phi \) be the set of all function \( \phi \), \( \phi : [0,1]^2 \to [0,1] \) such that

i. \( \phi \) is non decreasing and non increasing in all coordinates respectively,

ii. \( \phi(t_1,t_2,t_3,t_4,t_5) \) and \( \phi(t_1,t_2,t_3,t_4,t_5) \) are continuous in \( t_4 \) and \( t_5 \) and

iii. \( \phi(t,t,t,t,t) > t, \phi(t,t,t,t,t) < t \) for every \( t \in [0,1] \).

### III. Main Result:

Here afterwards, assume that \((X, M, N, *,\emptyset)\) be a sequentially compact intuitionistic fuzzy metric space with \( t^* t \geq t \), \( s \leq s \) \( \forall t, s \in [0,1] \).

**Theorem 3.1:** Let \( P, Q, A, B, S \) and \( T \) be self mappings on \((X, M, N, *,\emptyset)\) such that:

i. \( P(X) \subset ST(X) \) and \( Q(X) \subset AB(X) \).

ii. \( P \) and \( AB \) are continuous or \( Q \) and \( ST \) are continuous.

iii. \( AB = BA, ST = TS, PB = BP, TQ = QT \).

iv. The pairs \((P, AB)\) and \((Q, ST)\) are weakly compatible.

v. There exists \( q \in (0,1) \) such that for every \( x, y \in X \) and \( t > 0 \)

\[
M(Px, Qy, qt) \geq \min \left\{ M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t) * M(Px, STy, t) \right\}
\]

\[
N(Px, Qy, qt) \leq \max \left\{ N(ABx, STy, t) \odot N(Px, ABx, t) \odot N(Qy, STy, t) \odot N(Px, STy, t) \right\};
\]

vi. for all \( x, y \in X \), \( \lim_{t \to \infty} M(x, y, t) = 1 \), \( \lim_{t \to \infty} N(x, y, t) = 0 \).

If the pair of maps \((P, AB)\) is reciprocal continuous and semi compatible maps then \( P \), \( Q \), \( S \), \( T \), \( A \) and \( B \) have a unique common fixed point in \( X \).

**Proof:** Let \( x_0 \in X \), from (1) there exist \( x_1, x_2 \in X \) such that \( Px_0 = STx_1 \) and \( Qx_1 = ABx_2 \).

Inductively, we can construct sequences \( \{x_n\} \) and \( \{y_n\} \) in \( X \) such that

\[
P_{x_{2n-2}} = ST_{x_{2n-1}} = y_{2n-1}
\]

And

\[
Q_{x_{2n+1}} = AB_{x_{2n}} = y_{2n} \quad \text{for } n = 1, 2, 3, \ldots
\]

By using contractive condition (v), we obtain

\[
M(Px_{2n}, Qx_{2n+1}, qt) \geq \min \left\{ M(ABx_{2n}, STx_{2n+1}, t) * M(Px_{2n}, ABx_{2n}, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Px_{2n}, STx_{2n+1}, t) \right\}
\]

\[
N(Px_{2n}, Qx_{2n+1}, qt) \leq \max \left\{ N(ABx_{2n}, STx_{2n+1}, t) \odot N(Px_{2n}, ABx_{2n}, t) \odot N(Qx_{2n+1}, STx_{2n+1}, t) \odot N(Px_{2n}, STx_{2n+1}, t) \right\}
\]

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\[ M(P_{x_{2n}}, Q_{x_{2n+1}}, qt) \geq \min \{ M(y_{2n}, y_{2n+1}, t) \ast M(y_{2n+1}, y_{2n+2}, t) \ast M(y_{2n+2}, y_{2n+1}, t) \} \]

\[ N(P_{x_{2n}}, Q_{x_{2n+1}}, qt) \leq \max \{ N(y_{2n}, y_{2n+1}, t) \ast N(y_{2n+1}, y_{2n+2}, t) \ast N(y_{2n+2}, y_{2n+1}, t) \} \]

The only t-norm \( \ast \) satisfying \( t \ast t \geq t \) for all \( t \in [0,1] \) is the minimum t-norm, that is

\( a \ast b = \min \{ a, b \} \) for all \( a, b \in [0,1] \) and

by the definition 2[2] of continuous t-conorm then

\( a \otimes b = \max \{ a, b \} \) for all \( a, b \in [0,1] \).

By the conditions, we have

\[ M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t) \]

\[ N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t) \]

Similarly we have

\[ M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t) \]

\[ N(y_{2n+2}, y_{2n+3}, qt) \leq N(y_{2n+1}, y_{2n+2}, t) \]

Thus, we have

\[ M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t) \] for \( n = 1, 2, 3, \ldots \)

\[ N(y_{n+1}, y_{n+2}, qt) \leq N(y_n, y_{n+1}, t) \] for \( n = 1, 2, 3, \ldots \)

\[ M(y_n, y_{n+1}, t) \geq M(y_{n-2}, y_{n-1}, \frac{t}{q^2}) \geq M(y_{n-2}, y_{n-1}, \frac{t}{q^2}) \]

\[ \geq M(y_1, y_2, \frac{t}{q^n}) \rightarrow 1 \text{ as } n \rightarrow \infty \]

and

\[ N(y_n, y_{n+1}, t) \leq N(y_{n-2}, y_{n-1}, \frac{t}{q^2}) \]

\[ \leq N(y_1, y_2, \frac{t}{q^n}) \rightarrow 0 \text{ as } n \rightarrow \infty \].

And hence \( M(y_n, y_{n+1}, t) \rightarrow 1 \) as \( n \rightarrow \infty \) for \( t > 0 \)

and \( N(y_n, y_{n+1}, t) \rightarrow 0 \) as \( n \rightarrow \infty \) for \( t > 0 \)

for such \( \varepsilon > 0 \) and \( t > 0 \), we can choose \( n_0 \in N \) such that

\[ M(y_n, y_{n+1}, t) > 1-\varepsilon \text{ for all } n > n_0 \]

\[ N(y_n, y_{n+1}, t) < 1-\varepsilon \text{ for all } n > n_0 \]

For \( m, n \in N \), we suppose \( m \geq n \) then we have

\[ M(y_n, y_m, t) \geq M(y_n, y_{n+1}, \frac{t}{m-n}) \ast M(y_{n+1}, y_{n+2}, \frac{t}{m-n}) \ast \ldots \ast M(y_{m-1}, y_m, \frac{t}{m-n}) \]
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\[ \geq (1-\varepsilon)^*(1-\varepsilon)^*\ldots\ldots\ldots(1-\varepsilon)(m-n)\text{times} \]

and for \( m, n \in \mathbb{N} \), we suppose \( m \leq n \), then we have

\[ N\left(y_n, y_{n+1}, t/m_n\right) \preceq N\left(y_{n+1}, y_{n+2}, t/m_n\right) \preceq \ldots \preceq N\left(y_{m-1}, y_m, t/m_n\right) \preceq (1-\varepsilon)^*(1-\varepsilon)^*\ldots\ldots(1-\varepsilon)(m-n)\text{times} \]

\[ \leq (1-\varepsilon)^*(1-\varepsilon)^*\ldots\ldots(1-\varepsilon) \]

And hence \( \{y_n\} \) is a Cauchy sequence in \( X \).

Since \( (X, M, *) \) and \( (X, N, \hat{\circ}) \) is complete, \( \{y_n\} \) converges to some point \( z \in X \). Also its subsequences converges to the same point i.e. \( z \in X \).

\[ \{Qx_{n+1}\} \rightarrow z \quad \text{and} \quad \{STx_{n+1}\} \rightarrow z \]
\[ \{Px_{n}\} \rightarrow z \quad \text{and} \quad \{ABx_{n}\} \rightarrow z \]

Since the pair \( (P, AB) \) is reciprocally continuous mapping then we have

\[ \lim_{n \rightarrow \infty} PABx_{n} = Pz \]

\[ \lim_{n \rightarrow \infty} ABPx_{n} = ABz \]

And semi compatibility of \( (P, AB) \) gives

\[ \lim_{n \rightarrow \infty} ABPx_{n} \rightarrow ABz \]

Therefore \( Pz = ABz \).

We claim \( Pz = ABz = z \).

**Step 1:** Put \( x = z \) and \( y = x_{2n+1} \), gives in condition (v),

\[ M\left(Pz, Qx_{2n+1}, qt\right) \geq \min\left\{ M\left(ABz, STx_{2n+1}, t\right) \hat{\circ} M\left(Pz, ABz, t\right) \hat{\circ} M\left(Pz, STx_{2n+1}, t\right)\right\} \]

\[ N\left(Pz, Qx_{2n+1}, qt\right) \leq \max\left\{ N\left(ABz, STx_{2n+1}, t\right) \hat{\circ} N\left(Pz, ABz, t\right) \hat{\circ} N\left(Pz, STx_{2n+1}, t\right)\right\} \]

Taking \( n \rightarrow \infty \) and using equation (i), we get

\[ M\left(Pz, z, qt\right) \geq \min\left\{ M\left(z, z, t\right) \hat{\circ} M\left(Pz, z, t\right) \hat{\circ} M\left(Pz, z, t\right)\right\} \]

\[ N\left(Pz, z, qt\right) \leq \max\left\{ N\left(z, z, t\right) \hat{\circ} N\left(Pz, z, t\right) \hat{\circ} N\left(Pz, z, t\right)\right\} \]

i.e

\[ M\left(Pz, z, qt\right) \geq M\left(Pz, z, t\right) \]

Then we get, \( Pz = z \).

Therefore, \( ABz = Pz = z \). \ldots \ldots \ldots (3) \]

and

\[ N\left(Pz, z, qt\right) \leq N\left(Pz, z, t\right) \]

Then we get, \( Pz = z \).

Therefore .
Step 2:- Putting \( x = Bz \) and \( y = x_{2n+1} \) in condition (v), we get

\[
M(PBz, Qx_{2n+1}, qt) \geq \min \left\{ M(ABBz, STx_{2n+1}, t) \ast M(PBz, ABz, t) \ast M(Qx_{2n+1}, STx_{2n+1}, t) \ast M(PBz, STx_{2n+1}, t) \right\}
\]

\[
N(PBz, Qx_{2n+1}, qt) \leq \max \left\{ N(ABBz, STx_{2n+1}, t) \odot N(PBz, ABz, t) \odot N(Qx_{2n+1}, STx_{2n+1}, t) \odot N(PBz, STx_{2n+1}, t) \right\}
\]

As \( BP = PB, AB = BA \),
so we have
\[
P(Bz) = B(Pz) = Bz
\]
and
\[
(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz
\]
Taking \( n \to \infty \) and using (i), we get
\[
M(Bz, z, qt) \geq \min \left\{ M(Bz, z, t) \ast M(Bz, Bz, t) \ast M(z, z, t) \ast M(Bz, z, t) \right\}
\]
\[
N(Bz, z, qt) \leq \max \left\{ N(Bz, z, t) \odot N(Bz, Bz, t) \odot N(z, z, t) \odot N(Bz, z, t) \right\}
\]
i.e.
\[
M(Bz, z, qt) \geq M(Bz, z, t)
\]
then we get
\[
Bz = z
\]
and also we have
\[
ABz = z
\]
\[
\Rightarrow \quad Az = \ast z
\]
Therefore \( Az = Bz = Pz = z \) \hspace{1cm} \text{.........}(5)

and
\[
N(Bz, z, qt) \leq N(Bz, z, t)
\]
then we get
\[
Bz = z
\]
and also we have
\[
ABz = z
\]
\[
\Rightarrow \quad Az = z
\]
Therefore \( Az = Bz = Pz = z \) \hspace{1cm} \text{.........}(6)

Step 3:- As \( P(x) \subseteq ST(x) \), there exists \( u \in X \) such that
\[
z = Pz = STu
\]
Putting \( x = x_{2n} \) and \( y = u \) in (v), we get

\[
M(Px_{2n}, Qu, qt) \geq \min \left\{ M(ABx_{2n}, STu, t) \ast M(Px_{2n}, ABx_{2n}, t) \ast M(Qu, STu, t) \ast M(Px_{2n}, STu, t) \right\}
\]
\[
N(Px_{2n}, Qu, qt) \leq \max \left\{ N(ABx_{2n}, STu, t) \odot N(Px_{2n}, ABx_{2n}, t) \odot N(Qu, STu, t) \odot N(Px_{2n}, STu, t) \right\}
\]
Taking \( n \to \infty \) and using (1) and (2), we get
\[
M(z, Qu, qt) \geq \min \left\{ M(z, z, t) \ast M(z, z, t) \ast M(Qu, z, t) \ast M(z, z, t) \right\}
\]
\[
N(z, Qu, qt) \leq \max \left\{ N(z, z, t) \odot N(z, z, t) \odot N(Qu, z, t) \odot N(z, z, t) \right\}
\]
i.e.
\[
M(z, Qu, qt) \geq M(z, Qu, t)
\]
Then we get
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\[ Qu = z \]
\[ STu = z = Qu \]

\[ M \left( STu, STQu, t \right) \geq M \left( STu, Qu, \frac{1}{r} \right) = 1 \]

i.e.
\[ STu = STQu \]
\[ \Rightarrow z = STz \]

and

\[ N \left( z, Qu, qt \right) \leq N \left( z, Qu, t \right) \]

Then we get,
\[ Qu = z \]
\[ STu = z = Qu \quad ...........(7) \]

Hence
\[ N \left( STu, STQu, t \right) \leq N \left( STu, Qu, \frac{1}{r} \right) = 0 \]

i.e
\[ STu = STQu \]
\[ \Rightarrow z = STz \quad ...........(8) \]

**Step-4 :-** Putting \( x = x_{2n} \) and \( y = z \) in equation (v) , we get

\[ M \left( Px_{2n}, Qz, qt \right) \geq \min \left\{ M \left( ABx_{2n}, STz, t \right) \ast M \left( Px_{2n}, ABx_{2n}, t \right) \ast M \left( Qz, STz, t \right) \ast M \left( Px_{2n}, STz, t \right) \right\} \]

\[ N \left( Px_{2n}, Qz, qt \right) \leq \max \left\{ N \left( ABx_{2n}, STz, t \right) \diamond N \left( Px_{2n}, ABx_{2n}, t \right) \diamond N \left( Qz, STz, t \right) \diamond N \left( Px_{2n}, STz, t \right) \right\} \]

Taking \( n \to \infty \) and using (2) and step-3 , we get

\[ M \left( z, Qz, qt \right) \geq \min \left\{ M \left( z, Qz, t \right) \ast M \left( z, z, t \right) \ast M \left( Qz, Qz, t \right) \ast M \left( z, Qz, t \right) \right\} \]

\[ N \left( z, Qz, qt \right) \leq \max \left\{ N \left( z, Qz, t \right) \diamond N \left( z, z, t \right) \diamond N \left( Qz, Qz, t \right) \diamond N \left( z, Qz, t \right) \right\} \]

i.e.
\[ M \left( z, Qz, qt \right) \geq M \left( z, Qz, t \right) \]

Then we get ,
\[ Qz = z \]

So
\[ z = Qz = STz \quad ...........(9) \]

and

\[ N \left( z, Qz, qt \right) \leq N \left( z, Qz, t \right) \]

Then we get
\[ Qz = z \]

So
\[ z = Qz = STz \quad ...........(10) \]

**Step-5 :-** Putting \( x = x_{2n} \) and \( y = Tz \) in (v) , we get

\[ M \left( Px_{2n}, QTz, qt \right) \geq \min \left\{ M \left( ABx_{2n}, STTz, t \right) \ast M \left( Px_{2n}, ABx_{2n}, t \right) \ast M \left( QTz, STTz, t \right) \ast M \left( Px_{2n}, STTz, t \right) \right\} \]

\[ N \left( Px_{2n}, QTz, qt \right) \leq \max \left\{ N \left( ABx_{2n}, STTz, t \right) \diamond N \left( Px_{2n}, ABx_{2n}, t \right) \diamond N \left( QTz, STTz, t \right) \diamond N \left( Px_{2n}, STTz, t \right) \right\} \]

As \( AT = TQ \) and \( ST = TS \) we have

\[ QTz = TQz = Tz \quad \text{and} \]

\[ ST(Tz) = T(STz) = TQz = Tz \]

Taking \( n \to \infty \) we get,
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\[ M(z,Tz,qt) \geq \min \{ M(z,Tz,t) \ast M(z,z,t) \ast M(Tz,Tz,t) \ast M(z,Tz,t) \} \]
\[ N(z,Tz,qt) \leq \max \{ N(z,Tz,t) \circ N(z,z,t) \circ N(Tz,Tz,t) \circ N(z,Tz,t) \} \]
i.e. \[ M(z,Tz,qt) \geq M(z,Tz,t) \]
then we get,
\[ Tz = z \]
Now \[ STz = Tz = z \]
\[ \Rightarrow Sz = z \]
Hence \[ Sz = Tz = Qz = z \] \[ \ldots \ldots (11) \]
Using (5) and (11) we get
\[ Az = Bz = Pz = Qz = Tz = Sz = z \]
And
\[ N(z,Tz,qt) \leq N(z,Tz,t) \]
then we get,
\[ Tz = z \]
Now \[ STz = Tz = z \]
\[ \Rightarrow Sz = z \]
Hence \[ Sz = Tz = Qz = z \] \[ \ldots \ldots (12) \]
Combining (5) and (12), we get
\[ Az = Bz = Pz = Qz = Tz = Sz = z \]
Hence, \( z \) is the common fixed point of self maps \( A, B, S, T, P \) and \( Q \).

Uniqueness :-
Let \( u \) be another common fixed point of \( A, B, S, T, P \) and \( Q \).
Then \[ Au = Bu = Pu = Qu = Su = Tu = u \].
Put \( x = z \) and \( y = u \) in (v), we get
\[ M(Pz,Qu,qt) \geq \min \{ M(ABz,STu,t) \ast M(Pz,ABz,t) \ast M(Qu,STu,t) \ast M(Pz,STu,t) \} \]
\[ N(Pz,Qu,qt) \leq \max \{ N(ABz,STu,t) \circ N(Pz,ABz,t) \circ N(Qu,STu,t) \circ N(Pz,STu,t) \} \]
Taking \( n \to \infty \), we get
\[ M(z,u,qt) \geq \min \{ M(z,u,t) \ast M(z,z,t) \ast M(u,u,t) \ast M(z,u,t) \} \]
\[ N(z,u,qt) \leq \max \{ N(z,u,t) \circ N(z,z,t) \circ N(u,u,t) \circ N(z,u,t) \} \]
i.e. \[ M(z,u,qt) \geq M(z,u,t) \]
then we get,
\[ z = u \]
And
\[ N(z,u,qt) \leq N(z,u,t) \]
then we get,
\[ z = u \]
Therefore \( z \) is the unique common fixed point of self maps \( A, B, S, T, P \) and \( Q \). Hence proved

References
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