An Application of Interval Valued Fuzzy Soft Matrix In Medical Diagnosis

Dr.N.Sarala¹ and M.prabhavathi²

¹Department of mathematics, A.D.M.college for women (Auto),Nagai, India.
²Department of mathematics,E.G.S.PillayArts&sciencecollege,Nagai,India.

Abstract: Today is a world of uncertainty with its associated problems, which can be well handled by soft set theory. In this paper, we extend Sanchez’s approach for medical diagnosis using the representation of an interval valued fuzzy soft matrices. we introduce the definition of union and intersection of Interval valued fuzzy soft matrices with examples. Finally, we extend our approach in application of these matrices in medical Diagnosis.

Keywords: Soft set, fuzzy soft set, Interval valued fuzzy soft matrix, Union and intersection of Interval valued fuzzy soft matrix, Interval valued fuzzy soft matrix medical diagnosis.

I. Introduction

The concept of interval valued fuzzy matrix (IVFM) is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations, the parameterization tool of interval valued fuzzy matrices enhances the flexibility of its applications. Most of our real life problems in medical sciences, engineering, management environment and social sciences often involve data which are not necessarily crisp. Precise and deterministic in character due to various uncertainties associated with these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy set, intuitionistic fuzzy sets, interval mathematics and rough sets etc. The concept of IVFM as a generalization of fuzzy matrix was introduced and developed by shyamal and pal [8], by extending the max. min operations on fuzzy algebra \(F=[0,1]\), for elements \(a, b \in F\) \(a+b = \max \{a, b\}\) and \(a \cdot b = \min \{a, b\}\). Let \(F_{mn}\) be the set of all \(m \times n\) Fuzzy Matrices over the Fuzzy algebra with support \([0,1]\), that is matrices entries are intervals and all the intervals are subintervals of the interval \([0,1]\).

De et al. [2] have studied Sanchez’s [5,6] method of medical diagnosis using intuitionistic fuzzy set. Saikia et al.[7] have extended the method in [2] using intuitionistic fuzzy soft set theory. In [1], Chetia and Das have studied Sanchez’s approach of medical diagnosis through IVFSS obtaining an improvement of the same presented in De et al.[2 and 7]. In our earlier work [3], we have represented an IVFM \(A = (a_{ij}) = \{a_{ij}, a_{ij}\}\) where each \(a_{ij}\) is a subinterval of interval \([0,1]\), as the Interval matrix \(A = [A_1, A_2]\) whose \(ij^{\text{th}}\) entry is the interval \([a_{ij}, a_{ij}]\), where the lower limit \(A_1 = (a_{ij})\) and the upper limit \(A_2 = (a_{ij})\) are fuzzy matrices such that \(A_1 \leq A_2\). By using this representation we have discussed the consistency of Interval valued fuzzy relational equations in [4]. In [7] P.Rajarajeswari and P.Dhanalakshmi have introduced interval valued fuzzy soft matrix, its types with examples and some new operations on the basis of weights.

In this paper, we extend Sanchez’s approach for medical diagnosis using the representation of an interval valued fuzzy soft matrix. We introduce the definition of union and intersection of an interval valued fuzzy soft matrix with examples. Finally, we extend approach is application of these matrices in medical Diagnosis.

II. Preliminaries

Soft set 2.1 [9]

Suppose that \(U\) is an initial Universe set and \(E\) is a set of parameters, let \(P(U)\) denotes the power set of \(U\). A pair \((F, E)\) is called a soft set over \(U\) where \(F\) is a mapping given by \(F : E \rightarrow P(U)\). Clearly a soft set is a mapping from parameters to \(P(U)\) and it is not a set, but a parameterized family of subsets of the Universe.

Fuzzy soft set 2.2 [10]

Let \(U\) be an initial Universe set and \(E\) be the set of parameters, let \(A \subseteq E\). A pair \((F, A)\) is called fuzzy soft set over \(U\) where \(F\) is a mapping given by \(F : A \rightarrow I^U\), where \(I^U\) denotes the collection of all fuzzy subsets of \(U\).

Fuzzy soft Matrices 2.3 [12]

Let \(U = \{e_1, e_2, \ldots, e_n\}\) be the Universe set and \(E\) be the set of parameters given by \(E = \{c_1, c_2, \ldots, c_m\}\). Let \(A \subseteq E\) and \((F, A)\) be a fuzzy soft set in the fuzzy soft class \((U, E)\). Then fuzzy soft set \((F, A)\) in a matrix form as \(A_{\text{max}} = [a_{ij}]_{m \times n}\) or \(A = [a_{ij}]_{m \times n}\) for \(i = 1, 2, \ldots, m\) and \(j = 1, 2, 3, \ldots, n\)
Interval valued fuzzy soft set 2.4 [11]
Let $U$ be an initial Universe set and $E$ be the set of parameters, let $A \subseteq E$. A pair $(F,A)$ is called Interval valued fuzzy soft set over $U$ where $F$ is a mapping given by $F: A \to \mathbb{I}^U$, where $\mathbb{I}$ denotes the collection of all Interval valued fuzzy subsets of $U$.

Interval valued fuzzy soft matrix 2.5[13]
Let $U = \{c_1, c_2, c_3, \ldots, c_n\}$ be the Universe set and $E$ be the set of parameters given by $E = \{e_1, e_2, e_3, \ldots, e_n\}$. Let $A \subseteq E$ and $(F,A)$ be a interval valued fuzzy soft set over $U$, where $F$ is a mapping given by $F: A \to \mathbb{I}^U$, where $\mathbb{I}$ denotes the collection of all Interval valued fuzzy subsets of $U$. Then the Interval valued fuzzy soft set can expressed in matrix form as

$$\tilde{A}_{\max} = [a_{ij}]_{\max} \text{ or } \tilde{A} = [a_{ij}] \quad i=1,2,\ldots,m, \quad j=1,2,\ldots,n$$

where $a_{ij} = \begin{cases} \mu_{\tilde{F}(e_i)}(c_j), \mu_{\tilde{F}(e_i)}(c_j) & \text{if } e_i \in A \\ 0 & \text{if } e_i \notin A \end{cases}$

$[\mu_{\tilde{F}(e_i)}(c_j), \mu_{\tilde{F}(e_i)}(c_j)]$ represents the membership of $c_j$ in the Interval valued fuzzy set $F(e_i)$.

Note that if $\mu_{\tilde{F}(e_i)}(c_j)=\mu_{\tilde{F}(e_i)}(c_j)$ then the Interval valued fuzzy soft matrix (IVFSM) reduces to an FSM.

Example: 2.1
Suppose that there are four houses under consideration, namely the universes $U = \{h_1, h_2, h_3, h_4\}$, and the parameter set $E = \{e_1, e_2, e_3, e_4\}$ where $e_i$ stands for “beautiful”, “large”, “cheap”, and “in green surroundings” respectively. Consider the mapping $F$ from parameter set $A = \{e_1, e_2\} \subseteq E$ to all interval valued fuzzy subsets of power set $U$. Consider an interval valued fuzzy soft set $(F,A)$ which describes the “attractiveness of houses” that is considering for purchase. Then interval valued fuzzy soft set $(F,A)$ is $(F,A) = \{F(e_1) = \{(h_1,[0.6,0.8]),(h_2,[0.8,0.9]),(h_3,[0.6,0.7]),(h_4,[0.5,0.6])\} | F(e_2) = \{(h_1,[0.7,0.8]),(h_2,[0.6,0.7]),(h_3,[0.5,0.7]),(h_4,[0.8,0.9])\} \}$

We would represent this Interval valued fuzzy soft set in matrix form as

$$\begin{bmatrix} [0.6,0.8] & [0.7,0.8] & [0.0,0.0] & [0.0,0.0] \\ [0.8,0.9] & [0.6,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.6,0.7] & [0.5,0.7] & [0.0,0.0] & [0.0,0.0] \\ [0.5,0.6] & [0.8,0.9] & [0.0,0.0] & [0.0,0.0] \end{bmatrix}$$

Addition of interval valued fuzzy soft matrices 2.6 [13]
If $A = [a_{ij}] \in \text{IVFSM}_{\max}$, $B = [b_{ij}] \in \text{IVFSM}_{\max}$ then we define $\tilde{A} + \tilde{B}$, addition of $\tilde{A}$and $\tilde{B}$ s

$$\tilde{A} + \tilde{B} = [c_{ij}]_{\max} = \text{max} (\tilde{A}, \tilde{B})$$

Example: 2.2
Consider

$$\tilde{A} = \begin{bmatrix} [0.6,0.8] & [0.7,0.8] \\ [0.5,0.6] & [0.8,0.9] \end{bmatrix}_{2x2} \text{ and } \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2x2}$$

are two interval valued fuzzy soft matrices then sum of these two is

$$\tilde{A} + \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.7,0.8] \\ [0.6,0.7] & [0.8,0.9] \end{bmatrix}_{2x2}$$

Multiplication of interval valued fuzzy soft matrices 2.7 [13]
If $A = [a_{ij}] \in \text{IVFSM}_{\max}$, $B = [b_{ij}] \in \text{IVFSM}_{\max}$, then we define $\tilde{A} * \tilde{B}$,multiplication of $\tilde{A}$and $\tilde{B}$ as

$$\tilde{A} * \tilde{B} = [c_{ijk}]_{\max} = \text{max} (\mu_{\tilde{A}_i}(e_j), \mu_{\tilde{B}_j}(e_k)) \quad \forall i,j,k$$

Example: 2.3
Consider

$$\tilde{A} = \begin{bmatrix} [0.6,0.8] & [0.7,0.8] \\ [0.5,0.6] & [0.8,0.9] \end{bmatrix}_{2x2} \text{ and } \tilde{B} = \begin{bmatrix} [0.8,0.9] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2x2}$$

are two interval valued fuzzy soft matrices then product of these two matrices is

$$\tilde{A} * \tilde{B} = \begin{bmatrix} [0.6,0.8] & [0.6,0.7] \\ [0.6,0.7] & [0.5,0.7] \end{bmatrix}_{2x2}$$

Remark: $\tilde{A} * \tilde{B} = \tilde{A} * \tilde{B}$

Interval valued fuzzy soft complement matrix 2.8 [13]
Let $A = [a_{ij}] \in \text{IVFSM}_{mxn}$, when $a_{ij} = [\mu_{A}(c), \mu_{B}(c)]$ then $\tilde{A}$ is called interval valued fuzzy soft complement if $\tilde{A}^c = [b_{ij}]_{mxn}$ where $b_{ij} = [1-\mu_{A}(c), 1-\mu_{B}(c)], \forall ij$.

**Example 2.4**

Let $\tilde{A} = \begin{bmatrix} 0.6,0.8 & 0.7,0.8 \\ 0.5,0.6 & 0.8,0.9 \end{bmatrix}$

Be interval valued fuzzy soft matrix then complement of this matrix is $\tilde{A}^c = \begin{bmatrix} 0.2,0.4 & 0.2,0.3 \\ 0.4,0.5 & 0.1,0.2 \end{bmatrix}$

### III. Union and intersection of Interval valued fuzzy soft matrices

In this section, we introduce the definition of union and intersection of Interval valued fuzzy soft matrices with examples and its properties.

**Definition 3.1**

Let $A = [a_{ij}], B = [b_{ij}] \in \text{IVFSM}_{mxn}$ Then union of $A, B$ is defined by $\tilde{A}_{mn} \cup \tilde{B}_{mn} = \tilde{C}_{mn} = [c_{ij}]_{mn}$, where $c_{ij} = [\mu_{A}(c), \mu_{B}(c)]$.

For all $i$ and $j$.

**Example 3.1**

Let $\tilde{A} = \begin{bmatrix} 0.6,0.8 & 0.7,0.8 \\ 0.5,0.6 & 0.8,0.9 \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} 0.8,0.9 & 0.6,0.7 \\ 0.6,0.7 & 0.5,0.7 \end{bmatrix}$

are two interval valued fuzzy soft matrices than union of these two is $\tilde{A} \cup \tilde{B} = \begin{bmatrix} 0.92,0.98 & 0.88,0.94 \\ 0.80,0.88 & 0.90,0.97 \end{bmatrix}$

**Proposition 3.1**

Let $A = [a_{ij}], B = [b_{ij}] \in \text{IVFSM}_{mxn}$.

Then

(i) $A \cup \tilde{O} = \tilde{A}$

(ii) $\tilde{A} \cup \tilde{U} = \tilde{U}$

(iii) $A \cup \tilde{B} = \tilde{B} \cup A$

(iv) $(A \cup B) \cup C = A \cup (B \cup C)$.

**Proof:**

(i) Let $A = [a_{ij}], B = [b_{ij}]$

$A \cup \tilde{O} = [\mu_{A}, \mu_{O}]

= [(\mu_{A}+\mu_{O}, 0), (\mu_{A}+0, \mu_{O})]

= [(\mu_{A}, 0), (0, \mu_{O})]

= \tilde{A}$

(ii) $A \cup \tilde{U} = [\mu_{A}, 1] \cup [1, 1]$

$= [(\mu_{A}+1, \mu_{A}), (1+1, 1)]$

$= [1, 1] = \tilde{U}$.

(iii) $A \cup \tilde{B} = \tilde{B} \cup A$

$A \cup \tilde{B} = [(\mu_{A}+\mu_{B}, \mu_{A}x\mu_{B}), (\mu_{A}+\mu_{B}, \mu_{A}x\mu_{B})]

= [(\mu_{B}, 0), (0, \mu_{A})]

= \tilde{B} \cup A$

(iv) $(A \cup B) \cup C = [(\mu_{A}+\mu_{B}, \mu_{A}x\mu_{B}), (\mu_{A}+\mu_{B}, \mu_{A}x\mu_{B})] \cup [(\mu_{C}, \mu_{C})]

= [(\mu_{A}+\mu_{B}+\mu_{C}, \mu_{A}x\mu_{B}+\mu_{C}), (\mu_{A}+\mu_{B}+\mu_{C}, \mu_{A}x\mu_{B}+\mu_{C})]

= [(\mu_{A}+\mu_{B}+\mu_{C}, \mu_{A}x\mu_{B}+\mu_{C}), (\mu_{A}+\mu_{B}+\mu_{C}, \mu_{A}x\mu_{B}+\mu_{C})]

= [\mu_{A}+\mu_{B}+\mu_{C}, \mu_{A}x\mu_{B}+\mu_{C}]

$\Rightarrow A \cup (B \cup C) = (A \cup B) \cup C$. 

DOI: 10.9790/5728-11160106   www.iosrjournals.org 3 | Page
Definition: 3.2
Let $A = [a_i], B = [b_j] \in \text{IVFSM}_{\text{max}}$ then intersection of $A, B$ is defined by $\tilde{A} \cap \tilde{B} = \tilde{C}$ where $c_{ij} = [a_{ij}] \cap [b_{ij}] = [\mu_{A\cap B}, \mu_{B\cap A}] = [\mu_{A\cap B}, \mu_{B\cap A}]$

Example: 3.2
Let $\tilde{A} = \begin{bmatrix} [0.6, 0.8] & [0.7, 0.8] \\ [0.5, 0.6] & [0.8, 0.9] \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} [0.8, 0.9] & [0.6, 0.7] \\ [0.6, 0.7] & [0.5, 0.7] \end{bmatrix}$ are two interval valued fuzzy soft matrices then the intersection of these two is.

$\tilde{A} \cap \tilde{B} = \begin{bmatrix} [0.48, 0.72] & [0.42, 0.56] \\ [0.30, 0.42] & [0.40, 0.63] \end{bmatrix}$

Proposition: 3.2
Let $A, B, C \in \text{IVFSM}_{\text{max}}$

(i) $\tilde{A} \cap \tilde{B} = \tilde{O}$
(ii) $\tilde{A} \cap \tilde{U} = \tilde{A}$
(iii) $\tilde{A} \cap \tilde{B} = \tilde{A}$
(iv) $\tilde{A} \cap \tilde{C} = \tilde{A}$

Proof:

(iv) $\tilde{A} \cap \tilde{C} = [\mu_{A\cap C}] = [\mu_{A\cap B}, \mu_{B\cap A}]$

IV. Application of interval valued fuzzy soft matrix in medical diagnosis.

Suppose $S$ is a set of symptoms of certain diseases, $D$ is a set of diseases and $P$ is a set of patients.

Construct an interval-valued fuzzy soft set $(F,D)$ over $S$, where $F$ is a mapping $F:D \rightarrow \text{IVFSM}_{\text{max}}$. A relation matrix $R_1$ is constructed from the interval-valued fuzzy soft set $(F,D)$ and called symptom–diseases matrix. Similarly its complement $(F,D)^{\text{c}}$ gives another relation matrix $R_2$, called symptom–non-diseases matrix. Analogous to Sanchez’s notion of ‘Medical Knowledge’ we refer to each of the matrices $R_1$ and $R_2$ as ‘interval-valued soft Medical Knowledge’. Again we construct another interval-valued fuzzy soft set $(F_1,S)$ over $P$.

Where $F_1$ is a mapping given by $F_1: S \rightarrow \text{IVFSM}_{\text{max}}$. This interval-valued fuzzy soft set gives another relation matrix $Q$ called symptom–patient matrix. Then we obtain two new relation matrices $T_1 = Q \cap R_1$ and $T_2 = Q \cap R_2$, called symptom-patient matrix and non-symptom patient matrix respectively, in which the membership values are given by

$\mu_{T_1}(p,d) = \max \{\min(\mu_{T_1}(p,d)), \mu_{R_1}(p,d)\}$

$\mu_{T_2}(p,d) = \max \{\min(\mu_{T_2}(p,d)), \mu_{R_2}(p,d)\}$

We calculate

$S_{T_1} = \max_{ij} \{[\mu_{T_1}(p,d), \mu_{T_1}(p,d)] \}$

$S_{T_2} = \max_{ij} \{[\mu_{T_2}(p,d), \mu_{T_2}(p,d)] \}$
V. Algorithm

1. Input the interval valued fuzzy soft sets (F,D) and (F,D)^c over the sets S of symptoms, where D is the set of diseases. Also write the soft medical knowledge R_1 and R_2 reassessing the relation matrices of the IVFSS (F,D) and (F,D)^c respectively.

2. Input the IVFSS (F,S) over the set P of patients and write its relation matrix Q.

3. Compute the relation matrices T_1 = Q.R_1 and T_2 = Q.R_2.

4. Compute the diagnosis scores s_i = T_i(p_i,d_j)

5. Find S_k = max {s_i(p_i,d_j) - s_j(p_i,d_j)}

Then we conclude that the patient p_i is suffering from form the disease d_k.

6. If S_k has more than one value then go to step one and repeat the process by reassessing the symptoms for the patients.

VI. Case study

Suppose that there are three patients P_1, P_2 and P_3 in a hospital with symptoms fever, headache, generalized body pain (especially in the joint muscles) and rash problem respectively and the set D = {d_1, d_2} where d_1 and d_2 represent parameterized Dengue and Chikangunya respectively.

Suppose that F(d_1) = { (e_1, [0.5,0.6]), (e_2, [0.2,0.3]), (e_3, [0.8,0.9])}, F(d_2) = { (e_1, [0.8,0.9]), (e_2, [0.6,0.7]), (e_3, [0.3,0.4])}.

The interval valued fuzzy soft set (F,D) is a parameterized family {F(d_1), F(d_2)} of all interval valued fuzzy set over the set S and are determined from expert medical documentation. Thus the fuzzy soft set (F,D) gives an approximate description of interval valued fuzzy soft medical Knowledge of the two diseases and their symptoms. This interval valued fuzzy soft set (F,D) and its complement (F,D)^c are represented by two relation matrices R_1 and R_2 called symptom – disease matrix respectively, given by

\[ d_1 \quad d_2 \quad d_3 \]
\[ e_1 \begin{bmatrix} 0.5,0.6 \ 0.8,0.9 \end{bmatrix} \quad e_2 \begin{bmatrix} 0.4,0.5 \ 0.1,0.2 \end{bmatrix} \]
\[ e_3 \begin{bmatrix} 0.2,0.3 \ 0.6,0.7 \end{bmatrix} \quad e_4 \begin{bmatrix} 0.3,0.4 \ 0.5,0.6 \end{bmatrix} \]

Again we take P = {P_1, P_2, P_3} as the universal set where P_1, P_2 and P_3 represent patients respectively and S = {e_1, e_2, e_3, e_4} as the set of parameters. Suppose that,

\[ F_1(e_1) = \{ (p_1, [0.8,0.9]), (p_2, [0.2,0.3]), (p_3, [0.4,0.5]) \} \]
\[ F_1(e_2) = \{ (p_1, [0.6,0.8]), (p_2, [0.3,0.5]), (p_3, [0.5,0.6]) \} \]
\[ F_1(e_3) = \{ (p_1, [0.4,0.6]), (p_2, [0.5,0.7]), (p_3, [0.6,0.8]) \} \]
\[ F_1(e_4) = \{ (p_1, [0.7,0.9]), (p_2, [0.6,0.9]), (p_3, [0.3,0.6]) \} \]

The interval valued fuzzy soft set (F,S) is anther parameterized family of all interval valued fuzzy set and gives a collection of approximate description of the patient – symptoms in the hospital. This interval valued fuzzy soft set (F,S) represents a relation matrix Q called patient – symptom matrix given by

\[ P_1 \begin{bmatrix} 0.8,0.9 \ 0.6,0.8 \ 0.4,0.6 \ 0.7,0.9 \end{bmatrix} \]
\[ P_2 \begin{bmatrix} 0.2,0.3 \ 0.3,0.5 \ 0.5,0.7 \ 0.6,0.9 \end{bmatrix} \]
\[ P_3 \begin{bmatrix} 0.4,0.5 \ 0.5,0.6 \ 0.6,0.8 \ 0.3,0.6 \end{bmatrix} \]

Then combining the relation matrices R_1 and R_2 separately with Q we get two matrices T_1 and T_2 called patient – disease and patient – non disease matrices respectively, given by

\[ d_1 \quad d_2 \]
\[ T_1 = Q \cdot R_1 = \begin{bmatrix} 0.5,0.6 \ 0.8,0.9 \end{bmatrix} \]
\[ T_2 = Q \cdot R_2 = \begin{bmatrix} 0.5,0.7 \ 0.5,0.7 \end{bmatrix} \]
\[ T_3 = Q \cdot R_3 = \begin{bmatrix} 0.6,0.8 \ 0.6,0.8 \end{bmatrix} \]
An Application of Interval Valued Fuzzy Soft Matrix In Medical Diagnosis

\[ T_2 = Q \cdot R_2 = \begin{bmatrix} 0.6, 0.8 & 0.4, 0.5 \\ 0.6, 0.7 & 0.4, 0.5 \\ 0.5, 0.6 & 0.3, 0.5 \end{bmatrix} \]

Now we calculate

<table>
<thead>
<tr>
<th></th>
<th>( s_{T_1} )</th>
<th>( s_{T_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Now it clear that the patient \( P_1 \) is suffering from disease \( d_1 \) and patients \( P_2 \) and \( P_3 \) are both suffering from disease \( d_2 \).

VII. Conclusion

We have applied the notion of interval valued fuzzy soft matrices in Sanchez’s method of medical diagnosis. A case study have been taken to exhibit the simplicity of the technique.

References