Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and Translating Sphere

Indeevar Kumar¹, Khemlal Mahto²
¹(Department of Mathematics, Manipal University Jaipur, Rajasthan, India)
²(Department of Mathematics, University College of Engineering & Technology (UCET), VBU, Hazaribah, Jharkhand, India)

Abstract: Exact solutions are important not only in its own right as solution of particular flows, but also serve as accuracy check for numerical solution. Exact solution of the Navier-Stokes equation are, for example, those of steady and unsteady flows near a stagnation point. Stagnation point flows can either be viscous or inviscid, steady or unsteady, two dimensional or three dimensional, normal or oblique and forward or reverse. The classic problems of two dimensional and three dimensional stagnation point flow are associated with the names of Hiemenz and Homan. A novel radial stagnation point flow impinging axi symmetrically on a circular cylinder was reported by Wang. The present paper deals with the laminar boundary layer flow and heat transfer in the stagnation region of a rotating and translating sphere with uniform magnetic fields. The governing equations of flow are derived for ξ = 0 (t* = 0) and ξ = 1 (t* → ∞) and solutions in the closed form are obtained. The temperature and velocity fields for ξ = 0 are numerically computed. This shows that the thermal boundary layer thickness decreases as Prandtl number Pr increases. The surface heat transfer (28) increases with the Prandtl number Pr. The surface heat transfer (28) at the starting of motion is found to be strangely dependent on the Prandtl number Pr. But it is dependent of magnetic field, buoyancy force Bp and Rotation Parameter Ro.

Keywords: Temperature field, velocity field, uniform magnetic field, buoyancy force, Rotation Parameter.

I. Introduction

Exact solutions are important not only in its own right as solution of particular flows, but also serve as accuracy check for numerical solution.

Exact solution of the Navier-Stokes equation are, for example, those of steady and unsteady flows near a stagnation point. Stagnation point flows can either be viscous or inviscid, steady or unsteady, two dimensional or three dimensional, normal or oblique and forward or reverse. The classic problems of two dimensional and three dimensional stagnation point flow are associated with the names of Hiemenz and Homan. A novel radial stagnation point flow impinging axi symmetrically on a circular cylinder was reported by Wang.

Luthander and Rydberg measured drag coefficient on a rotating sphere in axial flow. Homan and Frossling first obtained the exact solution of the Navier–Stokes equations for rotationally symmetrical stagnation point flow and found that the boundary layer thickness was independent of the distance along the wall and the velocity profiles were similar. Mishra and Choudhary studied axi-symmetric stagnation point flow with uniform suction. Rott and Crabtree simplified the boundary layer calculations for bodies of revolution. Lok et al. studied the growth of the boundary layer of micropolar fluid started impulsively from rest near the forward stagnation point of a two dimensional plane surface.

We discussed axi-symmetric stagnation flow of a viscous and electrically conducting fluid near the blunt nose of a spinning body with pressure of magnetic field. Sparrow et. al investigated the effect of transpiration cooling in MHD stagnation point flow. Ece has investigated the initial boundary layer flow past an impulsively started translating and spinning body of revolution. Rajasekaran and Palekar studied the influence of buoyancy force on the steady forced convection flow over a spinning sphere. Lee et. al discussed heat transfer over rotating bodies in forced flows. Hatrikonstantinou studied the effects of a mixed convection and viscous dissipation on heat transfer about porous rotating sphere.

Bush analyzed the stagnation point boundary layer in the presence of an applied magnetic field. Ozturk and Ece investigated into unsteady force convection heat transfer from a translating and spinning body. Thakur et. al investigated hydromagnetic boundary layer flow and heat transfer in the stagnation region of a spinning and translating sphere in the presence of buoyancy forces.

The present paper deals with the laminar boundary layer flow and heat transfer in the stagnation region of a rotating and translating sphere with uniform magnetic fields. The governing equations of flow are derived for ξ = 0 (t* = 0) and ξ = 1 (t* → ∞) and solutions in the closed form are obtained. The temperature and velocity fields for ξ = 0 are numerically computed.
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and ....

**Notations**

- **u, v, w**: velocity components in the direction of X-axis, and Y-axis and Z-axis respectively
- **V**: Characteristic velocity
- **L**: Characteristic length
- **σ**: electrical conductivity
- **μ₀**: magnetic permeability
- **Rm= μ₀ σ vL<<1**: magnetic Reynold number
- **B**: constant magnetic field applied in the z-direction
- **T**: temperature
- **t**: time
- **ρ**: density
- **μ**: coefficient of viscosity
- **ν**: \( \mu/\rho = \text{kinematic viscosity} \)
- **K**: thermal conductivity
- **Ω**: angular velocity of the sphere
- **G**: acceleration due to gravity
- **β**: coefficient of thermal expansion
- **R**: radius of the sphere
- **C_p**: specific heat at a constant pressure
- **T_ω**: temperature on the surface
- **T_∞**: temperature in the free stream
- **Pr = μC_p / K**: Prandtl number
- **M = σB^2 / Pr**: magnetic parameter
- **Bp = μGr R / Re^2R**: Buoyancy parameter
- **GrR = g β (T_ω - T_∞) R^3 / ν^3**: Grashof number
- **Θ = T - T_∞ / T_ω - T_∞**: dimensionless temperature
- **b**: velocity gradient at the edge of the boundary layer.
- **ReR = b R^2 / ν**: Reynolds number
- **Ro = (Ω/b)^2**: rotation parameter
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and ....

\( u_e \) : velocity on the edge of the boundary layer, \( a > 0 \)

\( \eta = \left( \frac{2b}{v} \right)^{1/2} \frac{Z}{\xi}^{1/2} \) : dimensionless variable

\( \tau \) : dimensionless time

\( f^i \) : dimensionless velocity component along \( x \)-direction

\( s \) : dimensionless velocity component along \( y \)-direction

II. Formulation of the problems, assumptions and governing equations

Formulation

Suppose a sphere is at rest in an abient fluid with surface temperature \( T_{\infty} \) at \( t < 0 \) (i.e, prior to the time \( t = 0 \)). The sphere is suddenly spinning with the constant angular velocity \( \Omega \). When at \( t = 0 \) an impulsive motion is imposed to the fluid, and \( T_{\infty} \) is suddenly raised to \( T_\omega \) (\( T_\omega > T_{\infty} \)). The unsteadiness is caused by the impulsive motion of the fluid and the impulsive motion of sphere.

Flow Model

Consider the unsteady laminar boundary layer flow of a viscous, incompressible fluid of small electrical conductivity in the front stagnation region of this spinning sphere in the presence of uniform magnetic field and a buoyancy force. Take \( x \) the distance along a meridian from the front stagnation point, \( y \) the distance in the direction of spinning and \( z \) the distance normal to the surface.

Assumptions

Following assumptions are made.

i. A uniform magnetic field \( B \) is imposed in the direction of \( z \)-axis.

ii. The boundary layer flow under uniform magnetic field is axi-symmetric.

iii. The magnetic Reynolds number \( R_m \) is very small. i.e. \( R_m \ll 1 \).

iv. As \( R_m \ll 1 \), the effect of the induced magnetic field as compared to \( B \) is neglected.

v. The dissipation terms, Ohmic heating and surface curvature are neglected in the region of front stagnation point of the surface.

vi. The fluid has constant properties except the density changes which produce buoyancy forces.

vii. The effect of the buoyancy induced stream wise pressure gradient terms on the flow and temperature profile is negligible.

viii. \( T_w \) and \( T_{\infty} \) are taken as constants.
Governing Equations

Under the above assumptions the boundary layer equations governing the flow of the present problem after Lee et al., Ozturk et al, and Bush are

\[
\frac{\partial}{\partial x}(ux) + \frac{\partial}{\partial z}(wx) = 0 \quad (1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} - \frac{v^2}{2} = \frac{\mu}{e} \frac{\partial^2 u}{\partial x^2} + \frac{v}{\rho} \frac{\partial^2 w}{\partial z^2} + g \beta (T - T_\infty) - \frac{\sigma B^2}{\rho} (u - u_e) \quad (2)
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} = \frac{v}{\rho} \frac{\partial^2 w}{\partial z^2} - \frac{\sigma B^2 \rho}{2} \quad (3)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} = K \frac{\rho}{C_p} \frac{\partial^2 T}{\partial t^2} \quad (4)
\]

With initial conditions

\[
t < 0: \quad u(x, z, t) = 0 \quad v(x, z, t) = 0 \quad w(x, z, t) = 0 \quad T(x, z, t) = T_\infty \quad (5)
\]

And the boundary layer conditions

\[
t \geq 0 \quad u(x, 0, t) = 0 \quad v(x, 0, t) = \Omega x \quad w(x, 0, t) = Tw \quad u(x, \infty, t) = u_e(x) \quad v(x, \infty, t) = 0 \quad T(x, \infty, t) = T_\infty \quad (6)
\]

Application of Transformation

Following William and Rhyme, we apple the transformation given below for making the region of time integration finite:

\[
\tilde{t} = b t, \quad b > 0 \quad \xi = 1 - e^{-t} \quad \eta = \frac{2}{b} \sqrt{1 - \xi} \quad (7)
\]

\[
R_0 = \frac{\Omega}{b} \quad B_0 = \frac{Gr}{R_0^2 \beta \beta (T_\infty - T_\omega)} \quad Gr_R = \frac{b R_0^2}{R_0^2 \beta \beta (T_\infty - T_\omega)} \quad v^2 \quad (8)
\]

\[
S_0 = \frac{\mu}{b \beta} \quad M = \frac{\mu}{b \beta} \quad \xi \eta = \frac{u(x, z, t)}{b x} \quad \eta \frac{\partial}{\partial \xi} \quad (7)
\]

\[
\theta = \frac{T(x, x, t) - T_\infty}{T_\infty - T_\omega} \quad \eta \frac{\partial \theta}{\partial \xi} \quad (9)
\]

\[
f'(\xi, \eta) = \frac{u(x, z, t)}{b x} \quad S(\xi, \eta) = \frac{u(x, z, t)}{\frac{\partial}{\partial \xi} - \frac{\partial}{\partial \xi}} \quad \xi \eta = \frac{f'(\xi, \eta)}{b x} \quad \theta = \frac{T(x, z, t) - T_\infty}{T_\infty - T_\omega} \quad (10)
\]

These transformations (7) are used in the governing equations. Equation (1) is identically satisfied and equations (2), (3) and (4) are transformed into equations.

\[
f'''' + \frac{u}{4} (1 - \xi) f''' + \xi f'' + \xi \frac{1}{2} [1 - f'(2)] + \frac{5}{2} \eta M(1 - f') + \frac{1}{2} \xi B_0 \theta = \xi (1 - \xi) \frac{\partial f'}{\partial \xi} \quad (8)
\]

\[
s'' + \frac{s}{4} (1 - \xi) s' + \xi (f s' - f') - \frac{\mu s}{2} = \frac{1}{2} \xi (1 - \xi) \frac{\partial s}{\partial \xi} \quad (9)
\]

\[
\eta'' + \frac{u}{4} (1 - \xi) \eta'' + \xi f \theta' + \frac{1}{2} \xi (1 - \xi) \frac{\partial \theta}{\partial \xi} \quad (10)
\]

The boundary condition (6) become

\[
tf(\xi, 0) = f'(\xi, 0) = 0; \quad s(\xi, 0) = 0; \quad \theta(\xi, 0) = 0 \quad (11)
\]
Special forms of governing equation at time infinity and at time zero

When $\xi = 1 (t \to \infty)$ equation (8), (9) and (10) reduce to

\[
f''' + f f'' + \frac{1}{2} \left[ 1 - (f')^2 + R_0 S^2 \right] + \frac{M}{2} (1 - f') + \frac{\eta f}{2} = 0
\]

(12)

\[
s'' + s f' - f s - \frac{Me}{2} = 0
\]

(13)

\[
e^\eta + f \theta' = 0
\]

(14)

When $\xi = 0 (t = 0 \text{ i.e at the start of the motion})$, equations (8), (9) and (10) becomes.

\[
f''' + \frac{3}{4} f = 0
\]

(15)

\[
s'' + \frac{3}{4} s' = 0
\]

(16)

\[
\frac{1}{\xi^2} \theta^2 + \frac{3}{2} \theta'^2 = 0
\]

(17)

The boundary conditions (11), for the equations (12) – (17) changed to

\[f(0) = f'(0) = 0, \quad s(0) = \theta(0) = 1, \quad f'(\infty) = 1, \quad s(\infty) = \theta(\infty) = 0\]

(18)

Closed from solutions for the case $t = 0 (\xi = 0)$

From (17)

\[
\frac{1}{Pr} \theta'' = -\frac{\eta}{4} \theta'
\]

or, \[\int \frac{\theta''}{\theta'} d\eta = -\frac{1}{4} \int \eta \, d\eta\]

or \[\log \left( \frac{\theta'}{C} \right) = -\frac{1}{8} \eta^2\]

\[\frac{\theta'}{C} = e^{-\frac{1}{8} \eta^2}\]

(17a)

Or \[\frac{d\theta}{d\eta} = C \cdot e^{-\frac{1}{8} \eta^2}\]

\[\int d\theta = C \int e^{-\frac{1}{8} \eta^2} \, d\eta\]

\[\theta(\eta) = C \int_0^\eta e^{-\frac{1}{8} \eta^2} \, d\eta + D, \text{D is constant}\]

(18a)

Now, \[erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-x^2} \, dx\]

\[erfc(\eta) = 1 - erf(\eta)\]

(19)

(18) can be written as

\[\eta = 0 : f = f'' = 0, \quad s = 1, \quad \theta = 1\]

\[\eta \to \infty : f' = 1, \quad s = 0, \quad \theta = 0\]

(20)

Using first condition of (20) in (18), we get

\[\theta(0) = 0 + D\]

(21)

Or \[1 = D\]

Let \[x = \sqrt{\frac{Pr}{8} \eta}\]

Then \[dx = \frac{Pr}{8} \eta \, d\eta\]

\[\eta \to 0, \quad x \to 0\]

\[\eta \to \infty, \quad x \to \infty\]

So using second condition of (20) in (18), we get

\[0 = C \int_0^\infty e^{-x^2} \sqrt{\frac{8}{Pr}} \, dx + 1\]
or \[ -1 = C \int_{0}^{\infty} e^{-\frac{y^2}{8}} \frac{8}{P_r} \, dy \]

or \[ C = \frac{P_r 4 B}{\pi} \]

or \[ C = -\frac{P_r}{2\pi} \]

Putting C and D in (18), we get

\[ \theta(\eta) = 1 - \frac{P_r}{2\pi} \int_{0}^{P_r} e^{-\left(\frac{P_r}{2\pi}\eta\right)^2} \, d\eta \]

\[ \theta(\eta) = 1 - \frac{P_r}{2\pi} \int_{0}^{P_r} e^{-\left(\frac{P_r}{2\pi}\eta\right)^2} \, dy \]

Put \[ \frac{P_r}{8} y = t \]

Then \[ \frac{P_r}{8} \, dy = dt \]

So, \[ \theta(\eta) = 1 - \frac{P_r}{2\pi} \int_{0}^{\frac{P_r}{8}} e^{-t^2} \sqrt{\frac{8}{P_r}} \, dt \]

\[ = 1 - \frac{P_r}{2\pi} \sqrt{2} \int_{0}^{\frac{P_r}{8}} e^{-t^2} \, dt \]

\[ \theta(\eta) = 1 - \text{erf}\left(\frac{\sqrt{2}}{\sqrt{8}} \eta\right) \]

or, \[ \theta(\eta) = \text{erfc}\left(\frac{\sqrt{2}}{\sqrt{8}} \eta\right) \] (20a)

From (15), \[ f''(\eta) + \eta^4 f''(\eta) = 0 \]

or \[ \int f''(\eta) \, d\eta = -\int \frac{\eta^4}{4} \, d\eta \]

On integration

\[ \log \frac{f''(\eta)}{C_1} = -\frac{\eta^2}{8} \]

\[ f''(\eta) = C_1 e^{-\frac{\eta^2}{8}} , C_1 \text{ is constant of integration} \]

Again, on integration

\[ f'(\eta) = C_1 \int_{0}^{\eta} e^{-\frac{\eta^2}{8}} \, d\eta + D_1, D_1 \text{ is constant} \]

Using first condition of (18), \( D_1 = 0 \)

Then \[ f'(\eta) = C_1 \int_{0}^{\eta} e^{-\frac{\eta^2}{8}} \, d\eta \] (20b)

put \[ \frac{\eta^2}{2\eta^2} = x \] (20c)

Then \[ \frac{d\eta}{\sqrt{2\eta^2}} = dx \]
η → ∞, x → ∞
η → 0, x → 0

Using (20c) and 2nd condition of (18), we have

\[
f'(\eta) = C_1 \int_{0}^{\eta} e^{-\eta^2} dx
\]

\[
= 2\sqrt{2}C_1 \int_{0}^{\eta} e^{-x^2} dx
\]

\[
\therefore \quad 1 = 2\sqrt{2}C_1 \frac{\sqrt{\pi}}{2} = C_1\sqrt{2\pi}
\]

\[
\therefore \quad C_1 = \frac{1}{\sqrt{2\pi}}
\]

Putting \( C_1 \) in (20b), we get

\[
\therefore \quad f'(\eta) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\eta} e^{-\frac{\eta^2}{\pi}} d\eta
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{0}^{\xi/\sqrt{\pi}} e^{-\xi^2} d\xi
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{0}^{\eta/\sqrt{\pi}} e^{-\xi^2} \sqrt{8} dy \quad \text{(taking } \frac{t}{\sqrt{8}} = y)\n\]

\[
= \frac{\sqrt{\pi}}{\sqrt{2\pi}} \int_{0}^{\eta/\sqrt{\pi}} e^{-y^2} dy
\]

\[
f'(\eta) = \frac{2}{\sqrt{\pi}} \int_{0}^{\eta/\sqrt{\pi}} e^{-y^2} dy
\]

\[
\therefore \quad f'(\eta) = \text{erf} \left( \frac{\eta}{\sqrt{\pi}} \right)
\]

Integrating,

\[
f(\eta) = \int \text{erf} \left( \frac{\eta}{\sqrt{\pi}} \right) d\eta
\]

\[
= \int \left[ \frac{2}{\sqrt{\pi}} \int_{0}^{\eta/\sqrt{\pi}} e^{-\xi^2} d\xi \right] d\eta
\]

\[
= \frac{2}{\sqrt{\pi}} \int_{0}^{\eta/\sqrt{\pi}} \int_{0}^{\xi} e^{-\xi^2} d\xi d\eta
\]

\[
= \frac{2}{\sqrt{\pi}} \int_{0}^{\eta/\sqrt{\pi}} \int_{0}^{\xi} \left( 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \ldots \right) dx d\eta
\]

\[
= \frac{2}{\sqrt{\pi}} \left( \frac{x^3}{3} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \frac{x^9}{9.4!} - \ldots \right)_{0}^{\eta/\sqrt{\pi}} d\eta
\]

\[
f(\eta) = \frac{2}{\sqrt{\pi}} \left( \frac{(\eta/\sqrt{\pi})^3}{3} + \frac{(\eta/\sqrt{\pi})^5}{5.2!} - \frac{(\eta/\sqrt{\pi})^7}{7.3!} + \ldots \right) d\eta
\]

From (21),

\[
f'(\eta) = \text{erf} \left( \frac{\eta}{\sqrt{\pi}} \right)
\]

On integration
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and ....

\[ f'(\eta) = \int \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) d\eta \]

\[ = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \int \eta d \left[ \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) \right] d\eta \]

Using (22), we have

\[ f(\eta) = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \int \eta \left[ \frac{2}{\sqrt{\pi}} - \frac{3\eta^2}{8\sqrt{8}} - \frac{5\eta^4}{2!8^2\sqrt{8}} - \frac{7\eta^6}{3!8^3\sqrt{8}} - \frac{9\eta^8}{4!8^4\sqrt{8}} + \ldots \right] d\eta \]

\[ = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \frac{2}{\sqrt{\pi}} \left[ \frac{\eta}{\sqrt{B}} - \frac{\eta^3}{8\sqrt{8}} + \frac{\eta^5}{2!8^2\sqrt{8}} + \frac{\eta^7}{3!8^3\sqrt{8}} + \ldots \right] d\eta \]

\[ f(\eta) = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \frac{2}{\sqrt{\pi}} \left[ \frac{\eta}{\sqrt{B}} - \frac{\eta^3}{8\sqrt{8}} + \frac{\eta^5}{2!8^2\sqrt{8}} + \frac{\eta^7}{3!8^3\sqrt{8}} + \ldots \right] d\eta \]

\[ = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \frac{2}{\sqrt{\pi}} \left[ \frac{\eta}{\sqrt{B}} - \frac{\eta^3}{8\sqrt{8}} + \frac{\eta^5}{2!8^2\sqrt{8}} + \frac{\eta^7}{3!8^3\sqrt{8}} + \ldots \right] d\eta \]

\[ f(\eta) = \eta \text{erf} \left( \frac{\eta}{\sqrt{B}} \right) - \frac{2}{\sqrt{\pi}} \left[ 1 - e^{-\eta^2/8} \right] \]

From (16)

\[ s'' + \frac{\eta}{4} s' = 0 \]

or \[ s' = -\frac{\eta}{4} \]

On integration,

\[ \int s'(\eta) d\eta = -\frac{1}{4} \int \eta d\eta \]

or, \[ \log \frac{s'}{C} = -\frac{\eta^2}{8} \]

or \[ \frac{s'}{C} = e^{-\eta^2/8} \]

\[ s'(\eta) = C e^{-\eta^2/8} \]

(23a)

On integration

\[ s(\eta) = C \int e^{-\eta^2/8} d\eta + C_1 \]

Using first condition in (24) of (20)

\[ C_1 = 1 \]

\[ \therefore s(\eta) = C \int e^{-\eta^2/8} d\eta + 1 \]

\[ = 1 + C \int_0^\eta e^{-y^2/8} dy \]

\[ = 1 + C \int_0^\eta e^{-t^2} \left( \frac{\eta}{2\sqrt{2}} \right) dy \]

Put \[ t = \frac{y}{2\sqrt{2}} \]

\[ dt = \frac{dy}{2\sqrt{2}} \]

\[ \therefore s(\eta) = 1 + C \int_0^{\eta/2\sqrt{2}} e^{-t^2} 2\sqrt{2} dt \]
\[ s(\eta) = 1 + C 2\sqrt{2} \int_{0}^{n \sqrt{\pi}} \int_{0}^{\eta \sqrt{\pi}} e^{-t^2} dt \]  
(23b)

Using 2nd conditions of (20)

\[ s(\infty) = 01 + 2\sqrt{2} C \int_{0}^{\infty} e^{-t^2} dt \]

\[ 0 = 1 + 2\sqrt{2} C \sqrt{\pi} \]

\[ -1 = C\sqrt{2\pi} \]

\[ \therefore C = -\frac{1}{\sqrt{2\pi}} \]

Putting C, in (23a) we get

\[ s'(\eta) = -\frac{1}{\sqrt{2\pi}} e^{-\eta^2/\pi} \]

Putting \( C = \frac{1}{\sqrt{2\pi}} \) in (23b), we get

\[ s(\eta) = 1 + 2\sqrt{2} \left( -\frac{1}{\sqrt{2\pi}} \right) \int_{0}^{n \sqrt{\pi}} e^{-t^2} dt \]

\[ = 1 - \text{erf} \left( \eta / \sqrt{\pi} \right) \]

\[ \text{or, } s(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]

Similarly equation (17) is solved for \( \Theta'(\eta) \) and \( \Theta(\eta) \).

Equation (17) is solved for \( \Theta'(\eta) \) and \( \Theta(\eta) \) in the same way as (16) is solved for \( s'(\eta) \) and \( s(\eta) \)

Thus, the closed from solutions of (15) – (17) under boundary conditions (18) are

\[ f'(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]  
(24)

\[ f(\eta) = \eta e^{-r f c \left( \eta / \sqrt{\pi} \right)} - \frac{2\sqrt{2}}{\sqrt{\pi}} \left( 1 - e^{-n^2/\pi} \right) \]

\[ (25) \]

\[ s(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]

\[ s'(\eta) = -\frac{1}{\sqrt{2\pi}} e^{-\eta^2/\pi} \]

\[ \Theta'(\eta) = \frac{P_r}{2\pi} e^{-\frac{P_r \eta^2}{8}} \]

\[ \Theta(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]

\[ \Theta(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]

\[ \Theta(\eta) = e^{-r f c \left( \eta / \sqrt{\pi} \right)} \]

(28)

Surface heat transfer for \( \xi = 0 \) (i.e. at the start of the motion) is given by

\[ -\Theta'(0) = -\left[ \frac{P_r}{2\pi} e^{-\frac{P_r \eta^2}{8}} \right]_{\eta=0} \]

\[ = \frac{P_r}{2\pi} \]  
(30)

III. Results and Conclusion

Numeric calculations are made for temperature distribution, velocity field and heat transfer. Variation of velocity distribution \( f'(\eta) \) and \( s(\eta) \) in the directions of \( x \)-axis and \( y \)-axis against \( \eta \) are shown in table 1 and 2; and shown graphically by curves in figure.
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and Translating Sphere

The heat transfer expression (26) is calculated for Prandtl number Pr (.71, 3.02, 10 and 19.6) and the numerical values are listed in Tables. Temperature distribution (27) is computed for Pr (=.71, 3.02, 10 and 19.6) and results of calculations are entered in Tables and illustrated in figures.

This shows that the thermal boundary layer thickness decreases as Prandtl number Pr increases. The surface heat transfer (28) increases with the Prandtl number Pr. The surface heat transfer (28) at the starting of motion is found to be strongly dependent on the Prandtl number Pr. But it is dependent of magnetic field, buoyancy force Bp and Rotation Parameter Ro.

For non-conduction fluid (M=0) and without boundary force Bp for Steady state (ζ=0) equations (12) and (13) become:

\[ f^{''''}(\eta) + f(\eta)f^{'''}(\eta) + \frac{1}{2}[1 - f^{'}(\eta) + R_0s^2(\eta)] = 0 \]

And

\[ s''(\eta) + f(\eta)s'(\eta) - f'(\eta)s(\eta) = 0 \]

These equations are same as that of Lee et. al. This deduction confirms the correctness of our approach.

Table – 1

Boundary layer flow in the vicinity of the forward stagnation point of the spinning and translating sphere

<table>
<thead>
<tr>
<th>η</th>
<th>f(η)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.03</td>
<td>0.01128</td>
</tr>
<tr>
<td>0.14</td>
<td>0.05637</td>
</tr>
<tr>
<td>0.28</td>
<td>0.11246</td>
</tr>
<tr>
<td>0.42</td>
<td>0.16800</td>
</tr>
<tr>
<td>0.57</td>
<td>0.22270</td>
</tr>
<tr>
<td>0.71</td>
<td>0.27633</td>
</tr>
<tr>
<td>0.85</td>
<td>0.32863</td>
</tr>
<tr>
<td>0.99</td>
<td>0.37938</td>
</tr>
<tr>
<td>1.13</td>
<td>0.42839</td>
</tr>
<tr>
<td>1.27</td>
<td>0.47548</td>
</tr>
<tr>
<td>1.41</td>
<td>0.52050</td>
</tr>
<tr>
<td>1.56</td>
<td>0.56332</td>
</tr>
<tr>
<td>1.70</td>
<td>0.60386</td>
</tr>
<tr>
<td>1.84</td>
<td>0.64203</td>
</tr>
<tr>
<td>1.98</td>
<td>0.67780</td>
</tr>
<tr>
<td>2.12</td>
<td>0.71116</td>
</tr>
<tr>
<td>2.26</td>
<td>0.74210</td>
</tr>
<tr>
<td>2.40</td>
<td>0.77667</td>
</tr>
<tr>
<td>2.55</td>
<td>0.79691</td>
</tr>
<tr>
<td>2.69</td>
<td>0.82089</td>
</tr>
<tr>
<td>2.83</td>
<td>0.84270</td>
</tr>
<tr>
<td>2.97</td>
<td>0.86244</td>
</tr>
<tr>
<td>3.11</td>
<td>0.88021</td>
</tr>
<tr>
<td>3.25</td>
<td>0.89612</td>
</tr>
<tr>
<td>3.39</td>
<td>0.91031</td>
</tr>
<tr>
<td>3.53</td>
<td>0.92318</td>
</tr>
<tr>
<td>3.67</td>
<td>0.93493</td>
</tr>
<tr>
<td>3.81</td>
<td>0.94591</td>
</tr>
<tr>
<td>3.95</td>
<td>0.95641</td>
</tr>
<tr>
<td>4.09</td>
<td>0.96666</td>
</tr>
<tr>
<td>4.23</td>
<td>0.97699</td>
</tr>
<tr>
<td>4.37</td>
<td>0.98737</td>
</tr>
<tr>
<td>4.51</td>
<td>0.99797</td>
</tr>
<tr>
<td>4.65</td>
<td>1.00895</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>η</th>
<th>s(η)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.03</td>
<td>0.99872</td>
</tr>
<tr>
<td>0.14</td>
<td>0.99436</td>
</tr>
<tr>
<td>0.28</td>
<td>0.88575</td>
</tr>
<tr>
<td>0.42</td>
<td>0.83200</td>
</tr>
<tr>
<td>0.57</td>
<td>0.77730</td>
</tr>
<tr>
<td>0.71</td>
<td>0.72367</td>
</tr>
<tr>
<td>0.85</td>
<td>0.67137</td>
</tr>
<tr>
<td>0.99</td>
<td>0.62062</td>
</tr>
<tr>
<td>1.13</td>
<td>0.57164</td>
</tr>
</tbody>
</table>

DOI: 10.9790/5728-11142637 www.iosrjournals.org
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and ....

Table 3

<table>
<thead>
<tr>
<th>η</th>
<th>θ(η)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.50</td>
<td>0.83200</td>
</tr>
<tr>
<td>1.01</td>
<td>0.67137</td>
</tr>
<tr>
<td>1.51</td>
<td>0.52452</td>
</tr>
<tr>
<td>2.00</td>
<td>0.39614</td>
</tr>
<tr>
<td>2.50</td>
<td>0.28884</td>
</tr>
<tr>
<td>3.00</td>
<td>0.20309</td>
</tr>
<tr>
<td>3.50</td>
<td>0.13756</td>
</tr>
<tr>
<td>4.00</td>
<td>0.089669</td>
</tr>
<tr>
<td>4.50</td>
<td>0.05624</td>
</tr>
<tr>
<td>5.00</td>
<td>0.03389</td>
</tr>
<tr>
<td>5.50</td>
<td>0.01962</td>
</tr>
<tr>
<td>6.00</td>
<td>0.01091</td>
</tr>
<tr>
<td>6.50</td>
<td>0.00582</td>
</tr>
<tr>
<td>7.00</td>
<td>0.00298</td>
</tr>
<tr>
<td>7.50</td>
<td>0.00146</td>
</tr>
<tr>
<td>8.00</td>
<td>0.00069</td>
</tr>
</tbody>
</table>

Figure 1 for Table 1
Boundary Layer Flow in the Vicinity of the Forward Stagnation Point of the Spinning and ....

Figure 2 for Table 2

Figure 3 for Table 3

Acknowledgements
I am very much thankful to professor Dr. B.N. Mishra, Red. University prof. and Head, Deptt.of Mathematics, VinobaBhave University, Hazaribag for suggestions and improvement.

References
[1]. Homann, F.: Der Einfluss Grosser Zähigkeit bei der stromung um demzylinder und um deikugel. ZAMM 16, (1936), 153-164
[2]. Hatrikonstentionou, H: Effect of Mixed convection and viscous dissipation on heat transfer about a porous rotating sphere ZAMM, 70, (1990),457-464

DOI: 10.9790/5728-11142637 www.iosrjournals.org