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# Numerical Investigation of Higher Order Nonlinear Problem in the Calculus Of Variations Using Adomian Decomposition Method

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***Abstract:*** *In this paper, a numerical solution based on Adomian Decomposition Method (ADM) is used for finding the solution of higher order nonlinear problem which arise from the problems of calculus of variations. This approximation reduces the problem to an explicit system of algebraic equations. One numerical example is also given to illustrate the accuracy and applicability of the presented method.*
***Keywords:*** *Adomian decomposition method, Calculus of variations, Haar wavelet series, Higher order nonlinear problem, Single-term Haar wavelet series method.*

## I. Introduction

In a large number of problems arising in analysis, mechanics, geometry and so forth, it is necessary to determine the maximum or minimum of a certain functional. Because of the important role of this subject in science and engineering, considerable attention has been received by this kind of problems. Such problems are called variational problems. Some popular methods for solving variational problems are direct methods. J. Gregory and R. S. Wang [2] introduced discrete variable methods for the dependent variable nonlinear extremal problems in the calculus of variations. M. Dehghan and M. Tatari [1] introduced the use of Adomian decomposition method for solving problems in calculus of variations. Solution of problems in calculus of variations via He's variational iteration method presented by M. Tatari and M. Dehghan [8]. A. Saadatmandi and M. Dehghan [4] introduced the numerical solution of problems in calculus of variation using Chebyshev finite difference method. Mohammad Maleki and Mahmoud Mashali-Firouzi [3] introduced a numerical solution of problems in calculus of variation using direct method and nonclassical parameterization.

In this article we developed numerical methods for addressing higher order nonlinear problem in calculus of variations by Adomian decomposition method which was studied by S. Sekar and team of his researchers [6, 7]. In Section 2, the Adomian decomposition method for solving higher order nonlinear problem in calculus of variations is introduced. In Section 3, the Leapfrog and STHWS [5] method for solving higher order nonlinear problem in calculus of variations is solved. We refer [5-7] for the numerical treatment of higher order nonlinear problem in calculus of variations.

## II. Adomian Decomposition Method

The most familiar and elementary method for approximating solutions of an initial value problem is Euler's Method. Euler's Method approximates the derivative in the form of $y' = f(t, y),\ y(t_0) = y_0,\ y \in R^d$ by a finite difference quotient $y'(t) \approx (y(t+h) - y(t))/h$ . We shall usually discretize the independent variable in equal increments:

$$t_{n+1} = t_n + h,\ n = 0,1,\ldots,t_0 .$$

Henceforth we focus on the scalar case, N = 1. Rearranging the difference quotient gives us the corresponding approximate values of the dependent variable:

$$y_{n+1} = y_n + hf(t_n, y_n),\ n = 0,1,\ldots,t_0$$

To obtain the leapfrog method, we discretize $t_n$ as in $t_{n+1} = t_n + h,\ n = 0,1,\ldots,t_0$ , but we double the time interval, h, and write the midpoint approximation $y(t+h) - y(t) \approx hy'\left(t + \frac{h}{2}\right)$ in the form

$$y'(t+h) \approx (y(t+2h) - y(t))/h$$

and then discretize it as follows:
$y_{n+1} = y_{n-1} + 2hf(t_n, y_n),\ n = 0,1,\ldots,t_0$

The leapfrog method is a linear m = 2-step method, with $a_0 = 0, a_1 = 1, b_{-1} = -1, b_0 = 2$ and $b_1 = 0$. It uses slopes evaluated at odd values of n to advance the values at points at even values of n, and vice versa, reminiscent of the children's game of the same name. For the same reason, there are multiple solutions of the leapfrog method with the same initial value $y = y_0$. This situation suggests a potential instability present in multistep methods, which must be addressed when we analyze them—two values, $y_0$ and $y_1$, are required to initialize solutions of $y_{n+1} = y_{n-1} + 2hf(t_n, y_n),\ n = 0,1,\ldots,t_0$ uniquely, but the analytical problem $y' = f(t, y),\ y(t_0) = y_0,\ y \in R^d$ only provides one. Also for this reason, one-step methods are used to initialize multistep methods.

## III. Higher Order Nonlinear Problem

In this section one nonlinear problem of the calculus of variations (examples taken from the real world applications) are considered. Example 3.1 is a higher-order nonlinear problem taken from Gregory and Wang [6].

**Example 3.1**
Consider the functional (Gregory and Wang [1])

$$J(x) = \int_0^1 \left( \frac{1}{3} e^{-t} \dot{x}_1^3(t) + \frac{1}{2} \dot{x}_2^2(t) + \frac{1}{4} \dot{x}_3^4(t) + \frac{1}{2} e^{-2t} \dot{x}_1^4(t) + 48e^{6t} x_3(t) - e^{-t} x_3(t) x_1(t) - t\dot{x}_3(t) - x_2(t) \sin t \right) dt \qquad (1)$$

subject to the following boundary conditions

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, x(1) = \begin{bmatrix} e \\ \sin 1 \\ e^2 \end{bmatrix} \qquad (2)$$

The problem is to find the minimum of equation (1) subject to equation (2), the exact solution to this problem is

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, x(1) = \begin{bmatrix} e^t \\ \sin t \\ e^{2t} \end{bmatrix}$$

**Table 1: Error Calculations at $x_1(t)$**

| t | Example 3.1 | | | | |
|---|---|---|---|---|---|
| | Exact Solution | STHWS Solution | STHWS Error | ADM Solution | ADM Error |
| 0.1 | 1.10517092 | 1.105180318 | 9E-06 | 1.105171012 | 9E-08 |
| 0.2 | 1.22140276 | 1.221412458 | 1E-05 | 1.221402855 | 1E-07 |
| 0.3 | 1.34985881 | 1.349868808 | 1E-05 | 1.349858908 | 1E-07 |
| 0.4 | 1.4918247 | 1.491834998 | 1E-05 | 1.491824801 | 1E-07 |
| 0.5 | 1.64872127 | 1.648731871 | 1E-05 | 1.648721377 | 1E-07 |
| 0.6 | 1.8221188 | 1.8221297 | 1E-05 | 1.822118909 | 1E-07 |
| 0.7 | 2.01375271 | 2.013763907 | 1E-05 | 2.013752819 | 1E-07 |
| 0.8 | 2.22554093 | 2.225552428 | 1E-05 | 2.225541043 | 1E-07 |
| 0.9 | 2.45960311 | 2.459614911 | 1E-05 | 2.459603229 | 1E-07 |
| 1 | 2.71828183 | 2.718293928 | 1E-05 | 2.718281949 | 1E-07 |

**Table 2: Error Calculations at $x_2(t)$**

| t | Example 3.1 | | | | |
|---|---|---|---|---|---|
| | Exact Solution | STHWS Solution | STHWS Error | ADM Solution | ADM Error |
| 0.1 | 0.09983342 | 0.099837917 | 5E-06 | 0.099833462 | 5E-08 |
| 0.2 | 0.19866933 | 0.198674231 | 5E-06 | 0.19866938 | 5E-08 |
| 0.3 | 0.29552021 | 0.295525507 | 5E-06 | 0.29552026 | 5E-08 |
| 0.4 | 0.38941834 | 0.389424042 | 6E-06 | 0.389418399 | 6E-08 |
| 0.5 | 0.479942554 | 0.479431639 | 6E-06 | 0.4794256 | 6E-08 |
| 0.6 | 0.56464247 | 0.564648973 | 7E-06 | 0.564642538 | 7E-08 |
| 0.7 | 0.64421769 | 0.644224587 | 7E-06 | 0.644217756 | 7E-08 |
| 0.8 | 0.71735609 | 0.717363391 | 7E-06 | 0.717356164 | 7E-08 |
| 0.9 | 0.78332691 | 0.78333461 | 8E-06 | 0.783326987 | 8E-08 |
| 1 | 0.84147098 | 0.841479085 | 8E-06 | 0.841471066 | 8E-08 |

**Table 3: Error Calculations at** $x_3(t)$

| t | Example 3.1 | | | | |
|---|---|---|---|---|---|
| | Exact Solution | STHWS Solution | STHWS Error | ADM Solution | ADM Error |
| 0.1 | 1.22140276 | 1.221410858 | 8E-06 | 1.221402839 | 8E-08 |
| 0.2 | 1.4918247 | 1.491833298 | 9E-06 | 1.491824784 | 9E-08 |
| 0.3 | 1.8221188 | 1.8221279 | 9E-06 | 1.822118891 | 9E-08 |
| 0.4 | 2.22554093 | 2.225550528 | 1E-05 | 2.225541024 | 1E-07 |
| 0.5 | 2.71828183 | 2.718291928 | 1E-05 | 2.718281929 | 1E-07 |
| 0.6 | 3.32011692 | 3.320127523 | 1E-05 | 3.320117029 | 1E-07 |
| 0.7 | 4.05519997 | 4.055211067 | 1E-05 | 4.055200078 | 1E-07 |
| 0.8 | 4.95303242 | 4.953044024 | 1E-05 | 4.95303254 | 1E-07 |
| 0.9 | 6.04964746 | 6.049659564 | 1E-05 | 6.049647585 | 1E-07 |
| 1 | 7.3890561 | 7.389068699 | 1E-05 | 7.389056225 | 1E-07 |

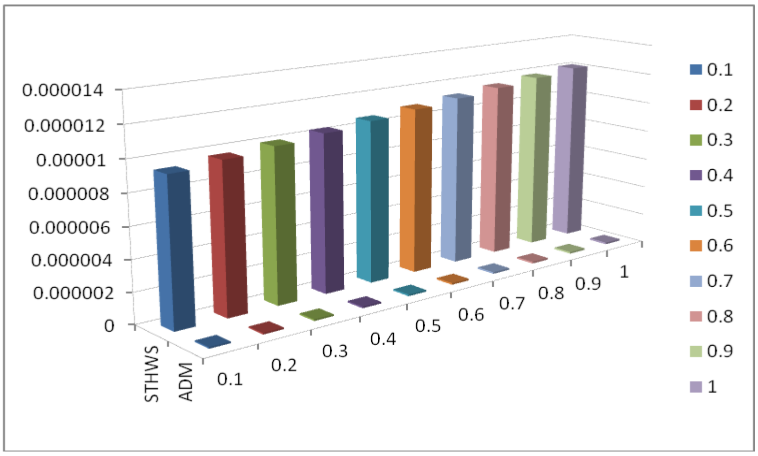


**Fig. 1** Error graph for higher-order nonlinear problem for the various value of "$x_1(t)$".

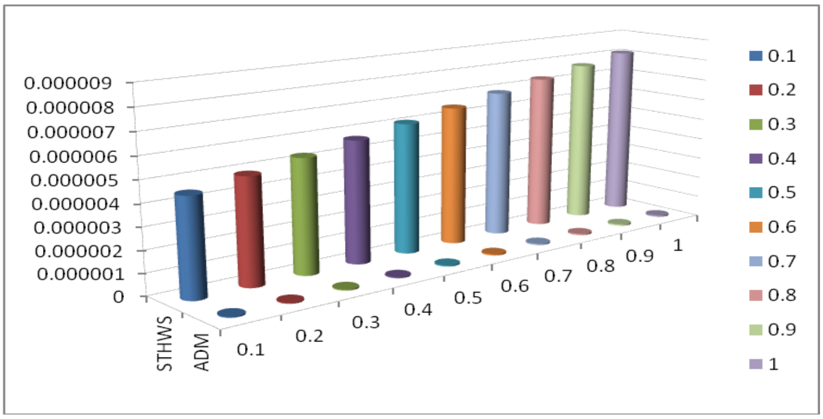


**Fig. 2** Error graph for higher-order nonlinear problem for the various value of "$x_2(t)$".

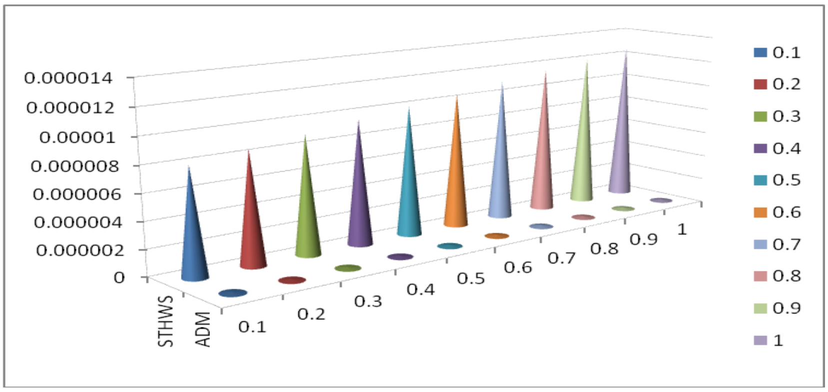


**Fig. 3** Error graph for higher-order nonlinear problem for the various value of "$x_3(t)$".

Eq. (6) has been solved numerically using the STHWS method and ADM method and the obtained results (with step size time = 0.1) along with the exact solutions (from Eq. (8)) are presented in Table 1 - 3 and graphical representation is shown for the higher-order nonlinear problem in Figure 1 - 3, using three-dimensional effects. This result reveals the superiority of the ADM with less complexity in implementation and at the same time the error reduction is less than the STHWS method.

## IV. Conclusion

The aim of this present work is to develop an efficient and accurate method for solving higher order nonlinear problems of the calculus of variations. The problem has been reduced to solving a system of higher order nonlinear algebraic equations. Illustrative examples are included to demonstrate the validity and applicability of the technique. Compare to STHWS method, ADM method gives less error from the Table 1 to Table 3. Also it is clear that from the Fig. 1 to Fig. 3 the ADM method introduced in Section 2 performs better than STHWS method.

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