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Vertex- Edge Dominating Sets and Vertex-Edge Domination Polynomials of Wheels

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Abstract: *Let G = (V, E) be a simple Graph. A set S $\subseteq$ V(G) is a vertex-edge dominating set (or simply ve-dominating set) if for all edges e $\in$ E(G), there exists a vertex v $\in$ S such that v dominates e. In this paper, we study the concept of vertex-edge domination polynomial of wheels, W_n. The vertex-edge domination polynomial of W_n is $D_{ve}(W_n, x) = \sum_{i=1}^{n} d_{ve}(W_n, i)\, x^i$, where $d_{ve}(W_n, i)$ is the number of vertex-edge dominating sets of W_n with cardinality i. We obtain some properties of $D_{ve}(W_n, x)$ and its co-efficients. Also, we calculate the recursive formula to derive the vertex-edge domination polynomials of wheels.*

Keywords: *Star, wheel, vertex-edge dominating sets, vertex-edge domination number $\gamma_{ve}(W_n)$, vertex-edge domination polynomial*

I. Introduction

Let G = (V, E) be a simple graph of order |V| = n. A set S $\subseteq$ V is a dominating set of G, if every vertex in V\S is adjacent to atleast one vertex in S. The domination number of a graph, denoted by $\gamma(G)$, is the minimum cardinality of the dominating sets in G. A set of vertices in a Graph G is said to be a vertex-edge dominating set, if for all edges e $\in$ E(G), there exists a vertex v $\in$ S such that v dominates e. Otherwise, for a graph G = (V, E), a vertex u $\in$ V(G) ve-dominates an edge vw $\in$ E(G) if (i) u = v or u = w (u is incident to vw) or (ii) uv or uw is an edge in G (u is incident to an edge is adjacent to vw).

The minimum cardinality of a ve-dominating set of G is called the vertex-edge domination number of G, and is denoted by $\gamma_{ve}(G)$. Let W_n be the wheel with n vertices. In the next section, we construct the families of the vertex-edge dominating sets of wheels by recursive method. In section 3, we use the results obtained in section 2 to study the vertex-edge domination polynomial of wheels.
We denote the set {1, 2, . . . , n} by [n], throughout this paper.

II. Vertex-edge dominating sets of wheels

Let W_n, n ≥ 3 be the wheel with n vertices $V(W_n)$ = [n] and $E(W_n)$ = {(1, 2), (1, 3), . . . , (1, n), (2, 3), (3, 4), . . . , (n–1, n), (n, 2)}. Let $d_{ve}(W_n, i)$ be the family of vertex-edge dominating sets of W_n with cardinality i.

Lemma 2.1

The following results hold for all Graph G with |V(G)| = n vertices.

(i) $d_{ve}(G,n) = 1$
(ii) $d_{ve}(G,n-1) = n$
(iii) $d_{ve}(G,i) = 0$ if i > n
(iv) $d_{ve}(G,0) = 0$

Lemma 2.2 [3]

For every n ≥ 5, $j \geq \left\lceil \frac{n}{4} \right\rceil$

$d_{ve}(C_n, j) = d_{ve}(C_{n-1}, j-1) + d_{ve}(C_{n-2}, j-1) + d_{ve}(C_{n-3}, j-1) + d_{ve}(C_{n-4}, j-1)$

Theorem 2.3 [4]

Let S_n, n ≥ 3 be a star Graph, then

(i) $d_{ve}(S_n, i) = \binom{n}{i}$, if i ≤ n

(ii) $d_{ve}(S_n, i) = \begin{cases} d_{ve}(S_{n-1}, i) + 1, \text{ if } i = 1 \\ d_{ve}(S_{n-1}, i) + d_{ve}(S_{n-1}, i-1), \text{if } 1 < i \le n \end{cases}$

Theorem 2.4

Let W_n, $n \ge 4$ be the wheel Graph, then $d_{ve}(W_n, i) = d_{ve}(S_n, i) + d_{ve}(C_{n-1}, i) - \binom{n-1}{i}$, $i < n-1$

Proof :

Let S_n be a star and $v \in v(S_n)$ such that v is the center of S_n. Let S_n be a spanning sub graph of W_n and since $W_n - v = C_{n-1}$ and $S_n \cup C_{n-1} = W_n$. The number of vertex-edge dominating sets of the wheel w_n is the sum of the number of vertex edge dominating sets of the star (S_n) and the number of vertex-edge dominating sets of the cycle C_{n-1}, and each time there are $\binom{n-1}{i}$ sets of cardinality i are not vertex-edge dominating sets.

Hence, $d_{ve}(W_n, i) = d_{ve}(S_n, i) + d_{ve}(C_{n-1}, i) - \binom{n-1}{i}$, $i < n - 1$.

Theorem 2.5

Let W_n, $n \ge 4$ be a wheel Graph, then

$$d_{ve}(W_n, i) = d_{ve}(w_{n-1}, i\text{-}1) + d_{ve}(w_{n-2}, i-1) + d_{ve}(w_{n-3}, i-1) + d_{ve}(w_{n-4}, i-1) + \binom{n-5}{i-1} .$$

Proof:

Let W_n, $n \ge 4$ be the wheel Graph. Then by theorem 2.3

$$\begin{aligned} d_{ve}(S_n, i) &= d_{ve}(S_{n-1}, i) + d_{ve}(S_{n-1}, i-1) \\ &= d_{ve}(S_{n-2}, i) + d_{ve}(S_{n-2}, i-1) + d_{ve}(S_{n-1}, i-1) \\ &= d_{ve}(S_{n-3}, i) + d_{ve}(S_{n-3}, i-1) + d_{ve}(S_{n-2}, i-1) + d_{ve}(S_{n-1}, i-1) \\ &= d_{ve}(S_{n-4}, i) + d_{ve}(S_{n-4}, i-1) + d_{ve}(S_{n-3}, i-1) + d_{ve}(S_{n-2}, i-1) + d_{ve}(S_{n-1}, i-1) \end{aligned}$$

by Theorem 2.3, $d_{ve}(S_{n-4}, i) = \binom{n-4}{i}$

Also, by Theorem 2.2,

$$d_{ve}(C_n, i) = d_{ve}(C_{n-1}, i-1) + d_{ve}(C_{n-2}, i-1) + d_{ve}(C_{n-3}, i-1) + d_{ve}(C_{n-4}, i-1)$$

by Theorem 2.4,

$$\begin{aligned} d_{ve}(W_n, i) &= d_{ve}(S_n, i) + d_{ve}(C_{n-1}, i) - \binom{n-1}{i} \\ &= d_{ve}(S_{n-4}, i) + d_{ve}(S_{n-4}, i-1) + d_{ve}(S_{n-3}, i-1) + d_{ve}(S_{n-2}, i-1) + d_{ve}(S_{n-1}, i-1) + d_{ve}(C_{n-2}, i-1) + d_{ve}(C_{n-3}, i-1) + d_{ve}(C_{n-4}, i-1) + d_{ve}(C_{n-5}, i-1) - \binom{n-1}{i} \\ &= d_{ve}(S_{n-1}, i-1) + d_{ve}(C_{n-2}, i-1) - \binom{n-2}{i-1} \\ &\quad + d_{ve}(S_{n-2}, i-1) + d_{ve}(C_{n-3}, i-1) - \binom{n-3}{i-1} \\ &\quad + d_{ve}(S_{n-3}, i-1) + d_{ve}(C_{n-4}, i-1) - \binom{n-4}{i-1} \end{aligned}$$

$$+ \quad d_{ve}(S_{n-4}, i-1) + d_{ve}(C_{n-5}, i-1) - \binom{n-5}{i-1}$$

$$+ \binom{n-2}{i-1} + \binom{n-3}{i-1} + \binom{n-4}{i-1} + \binom{n-5}{i-1}$$

$$+ \binom{n-4}{i} - \binom{n-1}{i}$$

$$d_{ve}(W_n, i) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$$

$$+ \binom{n-2}{i-1} + \binom{n-3}{i-1} + \binom{n-4}{i-1} + \binom{n-5}{i-1}$$

$$+ \binom{n-4}{i} - \binom{n-1}{i}$$

Consider $\binom{n-4}{i-1} + \binom{n-4}{i}$

$$= \frac{(n-4)!}{(i-1)!(n-4-i+1)!} + \frac{(n-4)!}{i!\,(n-4-i)!}$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!} + \frac{(n-4)!}{i!\,(n-i-4)!}$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!} + \frac{(n-4)!(n-i-3)}{i\times(i-1)!(n-i-4)!(n-i-3)}$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!} + \frac{(n-4)!(n-i-3)}{i\times(i-1)!(n-i-3)!}$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!}\left[1 + \frac{n-i-3}{i}\right]$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!}\left[\frac{i+n-i-3}{i}\right]$$

$$= \frac{(n-4)!}{(i-1)!(n-i-3)!} \times \frac{n-3}{i}$$

$$= \frac{(n-3)!}{i!(n-i-3)!} = \binom{n-3}{i}$$

$$\binom{n-3}{i-1} + \binom{n-3}{i} = \binom{n-2}{i}$$

$$\binom{n-2}{i-1} + \binom{n-2}{i} = \binom{n-1}{i}$$

$$\therefore d_{ve}(W_n, i) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$$

$$+ \binom{n-2}{i-1} + \binom{n-3}{i-1} + \binom{n-4}{i-1} + \binom{n-4}{i} + \binom{n-5}{i-1} - \binom{n-1}{i}$$

$$= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$$

$$+ \binom{n-2}{i-1} + \binom{n-3}{i-1} + \binom{n-3}{i} + \binom{n-5}{i-1} - \binom{n-1}{i}$$

$$= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$$

$$+ \binom{n-2}{i-1} + \binom{n-2}{i} + \binom{n-5}{i-1} - \binom{n-1}{i}$$

$$= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$$

$$+ \binom{n-1}{i} + \binom{n-5}{i-1} - \binom{n-1}{i}$$

$$= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1) + \binom{n-5}{i-1}$$

Table 1: $d_{ve}(W_n, i)$, The Number of Vertex-Edge dominating sets of W_n with cardinality i

n \ i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1														
2	2	1													
3	3	3	1												
4	4	6	4	1											
5	5	10	10	5	1										
6	1	15	20	15	6	1									
7	1	15	35	35	21	7	1								
8	1	14	49	70	56	28	8	1							
9	1	12	60	118	126	84	36	9	1						
10	1	9	66	174	243	210	120	45	10	1					
11	1	5	65	230	412	452	330	165	55	11	1				
12	1	5	55	275	627	858	781	495	220	66	12	1			
13	1	5	45	295	867	1464	1632	1275	715	286	78	13	1		
14	1	5	36	291	1092	2275	3068	2899	1989	1001	364	91	14	1	
15	1	5	29	267	1265	3241	5059	5931	4879	2989	1365	455	114	15	1

Theorem 2.6

For every $n \in Z^+$, $n \geq 4$

(i) $d_{ve}(W_n, 1) = 1$, $n > 5$

(ii) $d_{ve}(W_n, 2) = n-1$, n>9

(iii) $d_{ve}(w_n, n-2) = \binom{n}{2}$

(iv) $\gamma_{ve}(W_n) = 1$

(v) $d_{ve}(W_n,i) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)$, $i \geq n-3$, $i \neq n-4$

(vi) $d_{ve}(W_n, i) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1) + 1$, $i = n-4$

Proof:

(i) Let W_n be the wheel and $v \in V(W_n)$ such that v is the center of W_n. then from table, $d_{ve}(W_n,1) = 1$, $n > 5$

(ii) by Theorem 2.4, $d_{ve}(W_n, i) = d_{ve}(S_n, i) + d_{ve}(C_{n-1}, i) - \binom{n-1}{i}$

$\therefore\ d_{ve}(W_n, 2) = d_{ve}(S_n, 2) + d_{ve}(c_{n-1}, 2) - \binom{n-1}{2}$

$= \binom{n}{2} + 0 - \binom{n-1}{2}$ (by theorem 2.3)

($\square$ $d_{ve}(C_{n-1}, 2) = 0$, $n > 9$)

$$= \binom{n}{2} - \binom{n-1}{2}$$

$$= \frac{n!}{2!(n-2)!} - \frac{(n-1)!}{2!(n-3)!}$$

$$= \frac{n!}{2!(n-2)!} - \frac{n\,(n-1)!(n-2)}{n \text{ x } 2! \text{ x } (n-3)!(n-2)}$$

$$= \frac{n!}{2!(n-2)!} - \frac{n!\,(n-2)}{n\text{x}2!\text{x}\,(n-2)!}$$

$$= \frac{n!}{2!(n-2)!}\left[1 - \frac{n-2}{n}\right]$$

$$= \frac{n!}{2!(n-2)!}\left[\frac{n-(n-2)}{n}\right]$$

$$= \frac{n!}{2!(n-2)!} \times \frac{2}{n} = \frac{n\,(n-1)\,(n-2)! \times 2}{2! \times (n-2)! \times n}$$

$$= n-1$$

(iii) by theorem 2.4,

$d_{ve}(W_n, i) = d_{ve}(S_n, i) + d_{ve}(c_{n-1}, i) - \binom{n-1}{i}$

$\therefore\ d_{ve}(W_n, n-2) = d_{ve}(S_n, n-2) + d_{ve}(C_{n-1}, n-2) - \binom{n-1}{n-2}$

$$= \binom{n}{n-2} + (n-1) - \binom{n-1}{n-2} \quad (\therefore\ d_{ve}\ (C_n,$$

n–1) = n)

$$= \frac{n\,!}{(n-2)\,!\,2!} + (n-1) - \frac{(n-1)!}{(n-2)! \times 1}$$

$$= \frac{n\ (n-1)\ (n-2)\,!}{2 \times (n-2)!} + (n-1) - \frac{(n-1)\ (n-2)!}{1! \times (n-2)!}$$

$$= \frac{n\ (n-1)}{2} + (n-1) - (n-1)$$

$$= \frac{n\ (n-1)}{2} = \binom{n}{2}$$

(iv) The center vertex v is enough to cover all the vertices and edges of W_n. therefore, the minimum cardinality of the vertex-edge dominating set of W_n is 1

$\therefore\ \gamma_{ve}(W_n) = 1$

(v) By theorem 2.5,

$$d_{ve}(W_n, i) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-z}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1) + \binom{n-5}{i-1} \quad \text{------} (1)$$

Since $i \geq n - 3$,

$\therefore\ i - 1 \geq n - 4\ \therefore\ i - 1 = n - 4, n - 3, n - 2, n - 1, n, \ldots$

then $\binom{n-5}{i-1} = 0$

and if $i = n - 4\ \therefore \binom{n-5}{i-1} \neq 0\ \therefore\ i \neq n - 4$

substitute in (1) we get

$$\begin{aligned} d_{ve}(W_n, i) &= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) \\ &\quad + d_{ve}(W_{n-3}, i-1) \\ &\quad + d_{ve}(W_{n-4}, i-1) + 0 \\ &= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) \\ &\quad + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1), \qquad i \geq n-3,\ i \neq n-4 \end{aligned}$$

(vi) by theorm 2.5,

$$\begin{aligned} d_{ve}(W_n, i) &= d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) \\ &\quad + d_{ve}(W_{n-3}, i-1) \\ &\quad + d_{ve}(W_{n-4}, i-1) + \binom{n-5}{i-1} \end{aligned}$$

$i = n - 4\ \therefore \binom{n-5}{i-1} = \binom{n-5}{n-5} = 1$

$\therefore\ d_{ve}(W_{n,\,i}) = d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1) + 1,\ i = n - 4.$

III. Vertx – edge domination polynomial of wheel

Definition 3.1

Let $d_{ve}(W_n, i)$ be the family of vertex – edge dominating sets of a wheel W_n with cardinality i, then vertex-edge domination polynomial of W_n is defined as

$$D_{ve}(W_n, x) = \sum_{i=1}^{n} d_{ve}(W_n, i)\, x^i$$

Theorem 3.2

$D_{ve}(W_n, x)$ is the vertex-edge domination polynomial of wheel W_n, $n \geq 5$

(i) $D_{ve}(W_n,x) = D_{ve}(S_n,x) + D_{ve}(C_{n-1}, x) - ((1+x)^{n-1}-1)$

(ii) $D_{ve}(W_n, x) = x\, D_{ve}(W_{n-1}, x) + x\, D_{ve}(W_{n-2}, x) + x\, D_{ve}(W_{n-3}, x) + x\, D_{ve}(W_{n-4}, x) + x(1+x)^{n-5}$

Proof :

(i) From the definition of vertex–edge domination polynomial of wheel, we have

$$D_{ve}(W_n, x) = \sum_{i=1}^{n} d_{ve}(W_n, i)\, x^i$$

$$= \sum_{i=1}^{n} \left[d_{ve}(S_n,i) + d_{ve}(C_{n-1}, i) - \binom{n-1}{i}\right] x^i$$

$$= \sum_{i=1}^{n} d_{ve}(S_n, i)\, x^i + \sum_{i=1}^{n} d_{ve}(C_{n-1}, i) x^i - \sum_{i=1}^{n} \binom{n-1}{i} x^i$$

W have,

$$d_{ve}(C_{n-1}, i) = 0 \text{ if } i < \left\lceil \frac{n-1}{4} \right\rceil \text{ or } i = n$$

$$\therefore \sum_{i=1}^{n} d_{ve}(C_{n-1}, i)\, x^i = \sum_{i=\lceil \frac{n-1}{4} \rceil}^{n-1} d_{ve}(C_{n-1}, i)\, x^i = D_{ve}(C_{n-1}, x)$$

$$\sum_{i=1}^{n} d_{ve}(S_n, i)\, x^i = D_{ve}(S_n, x)$$

$$\sum_{i=1}^{n} \binom{n-1}{i} x^i = \binom{n-1}{1} x^1 + \binom{n-2}{2} x^2 + \ldots\ldots + \binom{n-1}{n-1} x^{n-1}$$

$$= 1 + \binom{n-1}{1} x + \binom{n-1}{2} x^2 + \ldots\ldots \binom{n-1}{n-1} x^{n-1} - 1$$

$$= (1+x)^{n-1} - 1$$

$$\therefore D_{ve}(W_n, x) = D_{ve}(S_n, x) + D_{ve}(C_{n-1}, x) - ((1+x)^{n-1} - 1)$$

(ii)
$$D_{ve}(W_n, x) = \sum_{i=1}^{n} d_{ve}(W_n, i)\, x^i$$

$$= \sum_{i=1}^{n} \left[d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1) + \binom{n-5}{i-1} \right] x^i$$

Since, $d_{ve}(W_n, 1) = 0$ if $i > n$ or $i = 0$

$$D_{ve}(W_n,x) = \sum_{i=2}^{n} [d_{ve}(W_{n-1}, i-1) + d_{ve}(W_{n-2}, i-1) + d_{ve}(W_{n-3}, i-1) + d_{ve}(W_{n-4}, i-1)]x^i + \sum_{i=1}^{n} \binom{n-5}{i-1} x^i$$

$$= x \sum_{i=2}^{n} d_{ve}(W_{n-1}, i-1) x^{i-1} + x \sum_{i=2}^{n-1} d_{ve}(W_{n-2}, i-1) x^{i-1} + x \sum_{i=2}^{n-2} d_{ve}(W_{n-3}, i-1) x^{i-1}$$

$$+ x \sum_{i=2}^{n-3} d_{ve}(W_{n-4}, i-1)x^{i-1}$$

$$+ x \sum_{i=1}^{n-4} \binom{n-5}{i-1} x^{i-1}$$

$$= x D_{ve}(W_{n-1}, x) + x D_{ve}(W_{n-2}, x) + x D_{ve}(W_{n-3}, x) + x D_{ve}(W_{n-4}, x) + x \sum_{i=1}^{n-4} \binom{n-5}{i-1} x^{i-1}$$

Consider

$$\sum_{i=1}^{n-4} \binom{n-5}{i-1} x^{i-1}$$

$$= \binom{n-5}{0} x^{o} + \binom{n-5}{1} x^{1} + \binom{n-5}{2} x^{2} + \ldots\ldots + \binom{n-5}{n-5} x^{n-5}$$

$$= 1 + \binom{n-5}{1} x^{1} + \binom{n-5}{2} x^{2} + \ldots\ldots + \binom{n-5}{n-5} x^{n-5}$$

$$= (1+x)^{n-5}$$

$$\therefore D_{ve}(W_n, x) = x D_{ve}(W_{n-1}, x) + x D_{ve}(W_{n-2}, x) + x D_{ve}(W_{n-3}, x) + x D_{ve}(W_{n-4}, x) + x(1+x)^{n-5}$$

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