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Divisibilty of Selected Primes and the Number Generator Method

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***Abstract**: The research paper aims to determine the divisibility's of a fixed set of common primes namely 7,11,13,17,23,29,31,43,47,43,89& 97 via the compartment method. The method initially during its foundation was extended to 7 but was later successfully extended to the remaining selected numbers. Number Trials (divisibility checker) provide an impetus to check the validation of the compartment method. In spite of the intricacies entailed by the proposed method it successfully aids in determining the divisibilities of other numbers (composites also) as well. An extension to the Number Generator Method has been made to encryption & code generation employing the usage of the former & Euler's Totient function on a primary level.*
***Keywords:** Divisibility, Compartment method, Primes, Number Generator method,Euler-Totient,Greatest Integer Function*

I. The Compartment Method

Statement: A given number X is divisible by the given prime 'N' if that number is divided into compartments starting from the right where each compartment contains 2 digits. The last compartment on the left may contain a single or a double digit depending on the numbers of digits in the number. Each of the compartments are multiplied by consecutively increasing powers of the multiplying factor. The products obtained are summed up. The number obtained has a sum less than the original number. The above stated operations are carried out on the number till the sum is reduced to 2 digit number. If the 2-digit number obtained is divisible by the prime or a multiple of the prime such that it is less than 100, then the original number is divisible by the prime 'N' whose divisibility was to be checked.

Corollary 1: Consider a number X where X is expressed as $a_n\ a_{n-1}\ a_{n-2}a_{n-3}\ldots\ldots\ldots\ldots a_2a_1$ where
a_n represents the nth digit of the number
a_{n-1} represents the n-1 digit of the number ……….. and so on
a_2 represents the 2nd digit of the number
a_1 represents the 1st digit of the number

Let n be even. According to the statement of the compartment method , x is divided into compartments as follows :-

$a_n\ a_{n-1}\ |\ a_{n-2}a_{n-3}\ |\ a_{n-4}a_{n-6}\ |\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.|a_4a_3|\ a_2a_1\ |$

(n/2) th compartment 2nd compartment 1st Compartment

For n digits there are (n/2) compartments.
If n is odd then number of compartments are (n+1)./2. Let Y be an operator performed on X . Symbolically it is denoted as Yk| X where k denotes the multiplying factor. To check the divisibility by 7 , the given number is multiplied by increasing powers of 2. The table below summarises some numbers whose divisibility is to be checked (the value of N) and their corresponding multiplying factors (K).

N	11	7	97	19	47	31
K	1	2	3	5	6	7

Table1(a)

N	23	13	89	29	43	17
K	8	9	11	13	14	15

Table1(b)

PROOF: Consider the number X as:-

$a_n\ a_{n-1}\ |\ a_{n-2}a_{n-3}\ |\ a_{n-4}a_{n-6}\ |\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.|a_4a_3|\ a_2a_1\ |$

Using Greatest integer function , [] the last 2 digits namely $a_n\ a_{n-1}$ and all the digits in the compartment can be represented as:- $a_n\ a_{n-1}$ =[X./10.^(n-2)] the 2nd last digits can be represented as:- [X./10.^(n-4)]-([X./10.^(n-2)]* 10.^2); Similarly

the 3rd last digits can be represented as : [X./10.^(n-6)]-([X./10.^(n-4)]* 10.^2) and so on………….. a_4a_3 can represented as :- [X./10.^2]-([X./10.^4]* 10.^2); and the first compartment a_2a_1 is represented as: - X-([X./10.^2]* 10.^2);
Applying operator Y on the number X ,for checking the divisibility of X, from the table with N= 7 we get k=2 :-

⇒ 2.^(0.5*n-1)* [X./10.^(n-2)] + 2^(0.5*n-2)* ([X./10.^(n-4)]-([X./10.^(n-2)]* 10.^2)) + 2.^(0.5*n-3)*([X./10.^(n-6)]-([X./10.^(n-4)]* 10.^2)) …………………………………………… +…………………………………………………2*([X./10.^2]-([X./10.^4]* 10.^2)) + X-([X./10.^2]* 10.^2)

Now collecting similar powers of 2.^(0.5*n-l) where l=1,2,3,………………..,0.5*n (n is strictly even)

⇒ 2.^(0.5*n-2)* [X./(10.^(n-2))] * (2-10.^2) + 2.^(0.5*n-3)*[X./(10.^(n-4))]*(2-10.^2) + 2.^ (0.5*n-5) * [X./(10.^(n-6))] * (2-10.^2) +………………………………………………………… + [X./100] * (2-10.^2) + X;

Collecting the term (2-10.^2)=-98 and writing 98 as 2*7.^2 outside the brackets we get

⇒ X- 2*7.^2 * (2^(0.5*n-2)* [X./10.^(n-2)] + 2.^(0.5*n-3)* ([X./10.^(n-4)]+…………………………….+ [X./10.^2]) **- (i)**

Now consider 2 cases: Case1: If X is divisible by 7, it can be expressed in the form of x=7*m (m ε N where N is the set of natural numbers), the (1) above becomes:-

⇒ 7 * (m-14*(2^(0.5*n-2)* [X./10.^(n-2)] + 2.^(0.5*n-3)* ([X./10.^(n- 4)]+…………………………….+ [X./10.^2])); **- (ii)**

(2) is clearly divisible by 7.
Case 2: If X is not divisible by 7 then let X be represented as X=7*p + q, then (1) becomes………

⇒ 7 * (p-14*(2^(0.5*n-2)* [X./10.^(n-2)] + 2.^(0.5*n-3)* ([X./10.^(n- 4)]+…………………………….+ [X./10.^2])) + q ;

The summation appearing before q is divisible by 7 but since q is less than 7, the entire sum is not divisible by 7.

Number Trials

Consider the number X = 6284056464, by actual division we can easily find that : (6284056464/7)=897722352 ; By applying the compartment method on this number we get ;
62 |84| 05|64|64 = 5 compartments, which implies multiplication is to be performed till 2.^4=16

62 | 84 | 05 | 64 | 64
↓ ↓ ↓ ↓ ↓
2.^4 2.^3 2.^2 2 1

This gives:- 16*62 +84*8+5*4+64*2 +64= 992+672+20+128+64 = 1876 where the number obtained after performing the Y operator on X is much less than X. This supports the statement validating the compartment method. Again performing the operation on 1876…..

18 | 76
↓ ↓
2 1

This gives 18*2+76=112 again validating the compartment method statement. Performing the operation on 112 gives:-

1 | 12
↓ ↓
2 1

This gives 2*1 +12=14. As 14 is clearly divisible by 7……… from the validation of the compartment method 6284056464 is divisible by 7 which validates the proof….
Consider the X=975433, By applying the compartment method
97|54|33 = 3 compartments, implies multiplication is to be performed till 2.^2=4;

97 | 54 | 33
↓ ↓ ↓
2.^2 2 1

This gives:- 97*4+54*2+33=529 where the number obtained by performing operator Y on X is much less than X thereby supporting the statement validating the compartment method. Again performing the operation on 529 ,

5 | 29

$\downarrow \quad \downarrow$

2.^1 1

This gives:- 5*2+59=39, as the sum finally reduces to a 2 digit number. As 39 is not divisible by 7, so isn't 975433. It can be confirmed that 975433/7 = 139347.57 which again validates the proof. Similarly these trials can be performed for the other set of numbers enlisted in table1(a) & table1(b).

II. Generalised Proof For A Fixed Set Of Primes

In the proof above we had devised the technique to check the divisibility of 7. For checking the divisibility of X with the prime N from the fixed set:-

Consider X= $a_n\, a_{n-1}\, a_{n-2}a_{n-3}$..............$a_2a_1$

Using the Greatest Integer function [] to represent the digits of each compartment:-

$a_n\, a_{n-1}$ = [X./10.^(n-4)]-([X./10.^(n-2)]* 10.^2);

$a_{n-2}a_{n-3}$ = [X./10.^(n-6)]-([X./10.^(n-4)]* 10.^2) and so on…………..

a_2a_1 = [X./10.^2]-([X./10.^4]* 10.^2);

Multiplying X with Y operator with a multiplying factor 'k', we get:-

$a_n\, a_{n-1}$ | $a_{n-2}a_{n-3}$ | $a_{n-4}a_{n-6}$ |…………………………………………………………|a_4a_3| a_2a_1 |

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

k.^(n-1) k.^(n-2) k^1 k.^0

$\Rightarrow$ k.^(0.5*n-1)*[X./10.^(n-2)]+ k.^(0.5*n-2)*([X./10.^(n-4)]-([X./10.^(n-2)]*10.^2) + k.^(0.5*n-3) *([X./10.^(n-6)]-([X./10.^(n-4)]*10.^2))+……………………………………………………….. ……………………………k*([X./10.^2]-([X./10.^4]*10.^2) + X-([X. /10. ^2]*10. ^2); -(iii)

Selecting the corresponding values of K for different values of N from table1(a) & table1(b)

PARTICULAR CASES **-[1],[2]**

Case 1: For N=11, K=1; hence (iii) becomes;

$\Rightarrow$ X-99*([x./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+………………..[X./10.^2])+

Decomposing 99 as 11*9 we get;

$\Rightarrow$ X-11*9*([X./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+………………..[X./10.^2]);

This is clearly divisible by 11 if X can be expressed in the form X=11* where l is a natural number

Case2: For N=7, K=2 ; hence (iii) becomes;

$\Rightarrow$ X+[X./10.^(n-2)]*2.^(0.5*n-2)*(2-10.^2)+[X./10.^(n-4)]*2.^(0.5*n-3)*(2-10.^2)+[X./10.^(n-6)]*2.^(0.5*n-4)*(2-10.^2)+………………………………………………………..+[X./10^4]*4*(2-10.^2)+[X./10.^2]*(2-10.^2)

Taking (2-10.^2=-98=-2*49=-2*7.^2) as common factor & rearranging

$\Rightarrow$ X-2*7.^2*([X./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+………………………………………….+ [X./10.^4]+[X./10.^2])

This is clearly divisible by 7 if X can be expressed in the form X=7*L where L is natural .

Case 3: for K=3,N=97; hence (iii) becomes;

$\Rightarrow$ X+[X./10.^(n-2)]*(3-10.^2)*3.^(0.5*n-2)+ [X./10.^(n-4)]*(3-10.^2)*3.^(0.5*n-3)+ [X./10.^(n-6)]*(3-10.^2)*3.^(0.5*n-4)+…………………………………………………………………………….+ [X./10.^4]*(3-10.^2)*3+ [X./10.^2]*(3-10.^2)

Taking out (3-10.^2=-97) as common & rearranging

$\Rightarrow$ X-97*([x./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+………………..[X./10.^2]);

This is clearly divisible by 97 if X can be expressed of the form X=97*L, where L is a natural number

Case 4: For K=5, N=19; hence (iii) becomes

$\Rightarrow$

X+ [X./10.^(n-2)]*(5-10.^2)*5.^(0.5*n-2) +[X./10.^(n-4)]*(5-10.^2)*5.^(0.5*n-3)+……………………………………………………[X./10.^4]*(5-10.^2)*5 + [X./10.^2]*(5-10.^2);

Decomposing 95=19*5 & substituting in the above expansion

⇨ X-95*([x./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+………………..[X./10.^2]);

This is clearly divisible by 19 if X can be expressed as X=19*L where L is a natural number

Case 5: For K=6,N=47; hence (iii) becomes

⇨ X+ [X./10.^(n-2)]*(6-10.^2)*6.^(0.5*n-2) +[X./10.^(n-4)]*(6-10.^2)*6.^(0.5*n-3)+……………………………………………………[X./10.^4]*(6-10.^2)*6 + [X./10.^2]*(6-10.^2);

Rearrange to get:- X- 47*2*([x./10.^(n-2)]*6.^(0.5*n-2) +[X./10.^(n-4)] *6.^(0.5*n-3) + [X./10.^(n-6)]+………………..[X./10.^2]);

This is clearly divisible by 47 if X can be expressed as X=47*L where L is a natural number

Case 6: For K=7,N=31; hence (iii) becomes

⇨ X + [X./10.^(n-2)]*(7-10.^2)*7.^(0.5*n-2) +[X./10.^(n-4)]*(7-10.^2)*7.^(0.5*n-3)+……………………………………………………[X./10.^4]*(7-10.^2)*7+ [X./10.^2]*(7-10.^2);

Rearrange to get:- X- 31*3*([x./10.^(n-2)]*7.^(0.5*n-2) +[X./10.^(n-4)] *7.^(0.5*n-3) + [X./10.^(n-7)]+………………..[X./10.^2]);

This is clearly divisible by 31 where X can be expressed as X=31*L where L is a natural number

Case 7: For K=8, N=23; hence (iii) becomes

⇨ X+[X./10.^(n-2)]*(8-10.^2)*8.^(0.5*n-2)+ [X./10.^(n-4)]*(8-10.^2)*8.^(0.5*n-3)+……………………………………………[X./10.^4]*(8-10.^2)*8 + [X./10.^2]*(8-10.^2);

Taking out 8-10.^2 =-92=-23*4 as common to get :-

⇨ X-23*4*([X./10.^(n-2)]+[X./10.^(n-4)]
+……………………………………………………………………………………
+ [X./10.^(n-6)]+…………………………………………+[X./10.^4]+[X./10.^2])

This is clearly divisible by 23 if X can be expressed as X=23*L where L is a natural number

Case 8: For K=9, N=13; hence (iii) becomes:-

⇨ X+[X./10.^(n-2)]*(9-10.^2)*9.^(0.5*n-2)+ [X./10.^(n-4)]*(9-10.^2)*9.^(0.5*n-3)+……………………………………………[X./10.^4]*(9-10.^2)*9+[X./10.^2]*(9-10.^2);

Taking out (9-10.^2=-91) as common & rearrange to get:-

⇨ X-13*7*([X./10.^(n-2)]+[X./10.^(n-4)]
+……………………………………………………………………………………
+ [X./10.^(n-6)]+…………………………………………+[X./10.^4]+[X./10.^2])

Clearly this is divisible by 13 if X can be expressed as X=13*L where L is natural number

Case 9: For K=11, N=89; hence (iii) becomes:-

⇨ X+[X./10.^(n-2)]*(11-10.^2)*11.^(0.5*n-2) +[X./10.^(n-4)]*(11-10.^2)*11.^(0.5*n-3)+……………………………………………..[X./10.^4]*(11-10.^2)+ [X./10.^2])

Rearrange to get by taking (11-10.^2) as common factor

X-89*([X./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+……………………………………………………
…………… [X./10.^4]+[X./10.^2])

Clearly this is divisible by 89 if X can be expressed as X=89*L;

Case 10: For K=13, N=29;hence (iii) becomes:-

⇨ X+[X./10.^(n-2)]*(13-10.^2)*13.^(0.5*n-2) +[X./10.^(n-4)]*(13-10.^2)*13.^(0.5*n-3)+……………………………………………..[X./10.^4]*(13-10.^2)+ [X./10.^2])

Taking out (13-10.^2=-87=-29*3) as common factor & rearrange to get

⇨ X-29*3*([X./10.^(n-2)]+[X./10.^(n-3)]+[X./10.^(n-4)]+…………………………………………………
……………………………….+[X./10.^4]+[X./10.^2])

Clearly this is divisible by 13 if X can be expressed as x=13*L where L is a natural number.

Case 11: For K=14, N=43; hence (iii) becomes

⇨ X+[X./10.^(n-2)]*(14-10.^2)*14.^(0.5*n-2) + [X./10.^(n-4)]*(14-10.^2)*14.^(0.5*n-3) + [X./10.^(n-6)]*(14-10.^2)*14.^(0.5*n-4)+………………………………………………………………………………………+
[X./10.^4]*(14-10.^2)*14+ [X./10.^2]*(14-10.^2)

Taking (14-10.^2=-86=-2*43) common and rearranging to get:-

⇨ X-43*2*([X./10.^(n-2)]+[X./10.^(n-3)]+[X./10.^(n-4)]+[X./10.^(n-6)]………………………………………………
……………………………….+[X./10.^4]+[X./10.^2])

Clearly this is divisible by 43 if X can be expressed as X=43*L where L is a natural number
Case 12: For K=15,N=17; hence (iii) becomes
X+[X./10.^(n-2)]*(15-10.^2)*15.^(0.5*n-2)+[X./10.^(n-4)]*(15-10.^2)*15.^(0.5*n-3)+[X./10.^(n-6)]*(15-10.^2)*15.^(0.5*n-4)+……………………………………………………………………………+ [X./10.^4]*15*(15-10.^2)+[X./10.^2]*(15-10.^2);
Taking (15-10.^2=-85=-17*5) common and rearranging to get:-
X-17*5*([X./10.^(n-2)]+[X./10.^(n-4)]+[X./10.^(n-6)]+…………………………………+ [X./10.^4]+[X./10.^2])by 17
Clearly this is divisible if X can be expressed as X=17*L where L is a natural number

III. Number Generator Method

This method is based on the reverse principle of determining the divisibility tests of numbers. It consists of generating solutions from first order Diophantine Equations (n variable linear equations) whose coefficients are determined from the pre-requisite stated rules and the solutions are subjected to certain constraints.

Mathematically it is stated as

a_n* k.^(n-1)+a_{n-1}*k.^(n-2)+ a_{n-2}*k.^(n-3)+…………………………..a_3*k.^2+ a_2*k.^1+ a_1 = S **-(iv)**

Here a_n, a_{n-1},an , ……………… a_3, a_2, a_1 denotes the variables of the n-variable linear equation. K.^(n-1) , K.^(n-2), K.^(n-3)……………………..K denotes the multiplying coefficients of each of the variables where the value of K is determined form for the corresponding values of N from table 1(a) & 1(b). The value of S depends on the value of N while the value of 'n' (number of compartments depends on the number of 2*n or (2*n-1) digits in the number & is determined as S=N*L where L is a natural number while N is the number for which the divisibility is to be checked.

Tables 1(a) & 1(b) impose the following constraints on the Number Generator equation:-

- For all values of k , a fixed value of N & L, the value of an must satisfy:-
  a_n <S./(K.^(n-1)) where n denotes the number of compartments
- To generate solutions containing at most 2*n digits, a_n ~=0 for all n
- If 'h' is a solution to the number generator equation such that a_n=h, then max (a_n)=99 & min(a_n)=0.
- If the value of a_n is a single digit number, a zero is prefixed before it . For instance if a_n =9 ,it is represented as 09 and not simply 9.

  Consider that for n=5; if solutions are: - a5=A ,a4=B ,a3=C ,a4=D ,a5=E, the entire solution is represented as $a_5a_4a_3a_2a_1$=ABCDE

AN EXTENSIVE APPLICATION -[4],[6]

Code Encryption: - An extension of the Number Generator Method has been made to encrypt codes transferred from the sender to the receiver where the receiver is aware of the parameters used by the sender. The code generation takes the following into consideration

- The sender generates a code using the equation:-
  a_n* k.^(n-1)+a_{n-1}*k.^(n-2)+ a_{n-2}*k.^(n-3)+……………….a_3*k.^2+ a_2*k.^1+ a_1 = S **-(v)**
  Where S=L*N as defined earlier in (3),

  If m_1 m_2 m_3 m_4……………………….. m_{n-3} m_{n-2} m_{n-1} m_n are the roots of the above equation, then the generated solution is the code: - m_n m_{n-1} m_{n-2} m_{n-3} …………………………………… m_4 m_3 m_2 m_1 ;
- The input equation is known only to the sender.
- The receiver however must be apprised of the parameter used by the sender.
- The sender hints the receiver regarding the values of N, the multiplying factor & the set information which is denoted by the notation Set(X,a[n][m]) . This contains all the codes sent by the sender to the receiver.
- The receiver generates the input equation used by the sender.

COROLLARY 2:- The term a[n][m] does not denote an array, the following are denoted:- **-[5]**
N=Coefficient of L
K=Multiplying Factor corresponding to N
x=Number of terms in the equation a=set containing cryptic code for the number of codes sent by the sender to the receiver n=product of composite factors of the number of solutions m=number of prime factors of the solutions

BASIS :- The set function (Ψ(a) [n][m]) derives its foundation from the famous Euler's Totient Theorem where the value Ψ(a) is calculated using Euler's Totient Formulae

Implementation Technique -[7]

The implementation is based on the known parameters.
The receiver receives this information and verifies the following:- Whether the codes sent in by the sender verify the equations and verifies these from the information. The number of codes received by the receiver is in accordance with the number of codes sent in by the sender. From the information set(x, a[n][m]), the receiver generates the number of codes that he has received. The receiver employs the Euler Totient function in order to encrypt the given codes.

Illustration
Putting N=47 & n=5 gives k=6 and setting l=1296, (4) becomes:-

$1296a_5+219a_4+36a_3+6a_2+a_1=60912$
We have the following solution table generated using MATLAB:-

a_5	a_4	a_3	a_2	a_1
45	12	0	0	0
45	11	6	0	0
45	11	3	12	36
42	25	27	14	24
42	25	27	12	36
36	66	0	0	0

If it is assumed that only 6 out of the total codes are selected, the using the validation of the number generator method we get the solutions as:-
x1=4511031236 x2=4225271424
x3=4225271236
x4=4512000000
x5= 4511060000

The encoder (the sender) sends these generated codes to the receiver along with the set information:- N=47, l=1296, Set(x,ψ(a)[n][m]) which in this case is denoted as :-
(47,1296 ,2[Φ][2])

The value of ψ(a) is calculated as:-
$\Psi(a) = a*(1-1./j_1)*(1-1./j_2)*(1-1./j_3)\ldots\ldots\ldots\ldots*(1-1./j_n)$

Where j_1, j_2,j_3………………………& j_n denotes the prime factors of a. If no prime actors exist it is denoted as Φ.
Consider that the sender send 120 codes to the receiver, using Euler's Totient function
Ψ(120)=32,
If there are 5 terms in the equation & 120 has 3 prime factors with 4 as the least composite factor, the set information becomes:- Set(5,32[4][3]).

Merits/Demerits:-

- It will enable the sender to check the authenticity of the data sent by him/her to the receiver.
- The receiver can check that:-
- ➢ Any of the codes that have been either inserted or deleted .
- ➢ Any interchange of digits in the codes will be detected after verification in the required equation.

However the codes suffer some serious drawbacks.

- The codes eliminated or tampered by an intruder cannot be restored.
- Data Protection & non –repudiation is not ensured.

IV. Conclusion

The compartment method provides an extensive but a tedious methodology to determine the divisibility of selected primes. Infact this method can be extended to selected composites as well but the same has been omitted . Via programming in C/C++, the compartment method has aimed to find its vitality. An extensive extension to the Number Generator method and in cryptography has enabled the

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