An Alternate Formula in terms of Pi to find the Area of a Triangle and a Test to decide the True Pi value (Atomic Energy Commission Method*)

R.D. Sarva Jagannadha Reddy

Abstract: Circle, square and triangle are basic geometrical constructions. \( \pi \) constant is associated with the circle. In this paper, circle-triangle interlationship chooses the real value of \( \pi \) and calculating the area of the triangle involving \( \pi \) of the inscribed circle. The alternate formula to find the area of the triangle is

\[
\frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \cdot d^2. 
\]

This formula has a geometrical backing.

Keywords: Altitude, base, circle, diameter, perimeter, triangle

I. Introduction

The official \( \pi \) value is 3.14159265358… It is an approximation, inspite of having trillions of its decimals. A new value to \( \pi \) was discovered in March 1998. The value is \( \frac{14 - \sqrt{2}}{4} = 3.14644660942… \) Both the \( \pi \) values have their own supporting arguments. Triangle is another geometrical entity. Its area is calculated using the formula \( \frac{1}{2} ab \), where \( a \) = altitude and \( b \) = base. In this paper, a different formula has been derived to find out the area of the equilateral triangle based on the \( \pi \) of its inscribed circle. The formula \( \frac{1}{2} ab \) gives the area of the triangle. No other formula is necessary for the area of triangle. The main purpose of derivation of new area formula of triangle is, to test the correctness of \( \pi \) value involved in the new formula.

One advantage in using the new formula for area of the triangle is, the resulting value tally’s exactly with the value of \( \frac{1}{2} ab \) only, when the chosen \( \pi \) value is correct one. If the wrong/ approximate \( \pi \) value is involved in the new formula, it does not give exact value of the triangle. In other words, out of the two numbers

3.14159265358… and \( \frac{14 - \sqrt{2}}{4} = 3.14644660942… \) only one number, gives the exact area of the triangle.

The number which fails in giving exact value to the area of the triangle is decided as the wrong \( \pi \) value.

Procedure

Draw a circle with centre ‘O’ and radius \( \frac{d}{2} \). Draw three equidistant tangents on the circle. The tangents intersect at A, B and C, creating an equilateral triangle ABC. DE is the hypotenuse of DOE triangle or the chord of the circle.

Calculations:

Centre = O
Radius = OD = OE = \( \frac{d}{2} \)
Diameter = DF = d
Triangle = ABC
Side = AB = BC = AC = a = \( \sqrt{3} \) d

* This author was awarded Merit Scholarship by Atomic Energy Commission, Trombay, Bombay for his M.Sc., (Zoology) study in S.V. University College, Tirupatt, Chittoor Dt. A.P.,India, during 1966-68. This author is highly indebted to the AEC and hence this paper is named in the AECs’ Honour.
An Alternate Formula in terms of Pi to find the Area of a Triangle and a Test to decide the ......

Altitude = DC = \( \frac{3d}{2} \)

Radius = OE = OD = \( \frac{d}{2} \)

Hypotenuse = Chord = DE = \( \frac{d}{2} \times \sqrt{2} = \frac{\sqrt{2}d}{2} \)

Area of the triangle by Conventional formula
\[
\frac{1}{2} \times DC \times AB = \frac{1}{2} \times \frac{3d}{2} \times \sqrt{3}d = \left( \frac{3\sqrt{3}}{4} \right) d^2
\]

New formula to find the area of ABC triangle

Perimeter of the ABC triangle \( 3 \times AB = 3 \times \sqrt{3}d = 3\sqrt{3}d \)

\[
\text{Area of the ABC triangle} = \frac{4}{14} \times \text{diameter DF of circle} - 2 \times \text{chord DE}
\]

\[
= \frac{4 \left( 3\sqrt{3}d \right)}{14d - 2 \left( \frac{\sqrt{2}d}{2} \right)} = \frac{12\sqrt{3}}{14 - \sqrt{2}}
\]

To find out the area of the ABC triangle, multiply the area of the circle with \( \frac{12\sqrt{3}}{14 - \sqrt{2}} \)

Area of the circle = \( \frac{\pi d^2}{4} \)

Area of the ABC triangle = Area of the circle \times \( \frac{12\sqrt{3}}{14 - \sqrt{2}} \)

\[
= \frac{\pi d^2}{4} \times \frac{12\sqrt{3}}{14 - \sqrt{2}} = \left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2
\]

where \( d = \text{diameter of the inscribed circle} \)

Thus, \( \left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2 \) is the new formula to find out the area of the superscribed triangle about a circle.

Where \( \pi \), may be 3.14159265358 or 3.14644660942 = \( \frac{14 - \sqrt{2}}{4} \)

Area of the ABC equilateral triangle besides from the \( \frac{1}{2} ab \) formula is = \( \left( \frac{\sqrt{3}}{4} \right) a^2 \)

where \( a = \sqrt{3}d \)

\[
= \frac{1}{2} \times \frac{3d}{2} \times \sqrt{3}d = \left( \frac{3\sqrt{3}}{4} \right) d^2 = \left( \frac{\sqrt{3}}{4} \right) (\sqrt{3}d)^2 = \left( \frac{3\sqrt{3}}{4} \right) d^2
\]

So, the value \( \left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2 \) as the area of the ABC triangle, should also be obtained, using the above new formula, derived in terms of \( \pi \)

\[
\left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2 \text{ should be equal to } \left( \frac{3\sqrt{3}}{4} \right) d^2
\]
An Alternate Formula in terms of Pi to find the Area of a Triangle and a Test to decide the .....

The unknown one in the above is $\pi$. Let us write formula with two different $\pi$ values.

$$\left( \frac{3\sqrt{3} \times 3.14159265358}{14 - \sqrt{2}} \right) d^2$$

**OR**

$$\left( \frac{3\sqrt{3} \times 4}{14 - \sqrt{2}} \right) d^2$$

$$\left( \frac{3\sqrt{3} \times 3.14159265358}{14 - \sqrt{2}} \right) \times 1 \times 1$$

where $d = 1$

= 1.29703410738, we have obtained this value instead of $\frac{3\sqrt{3}}{4} = 1.2990381057$

So, official $\pi$ value 3.14159265358 has failed to give $\frac{3\sqrt{3}}{4}$ as the value of the area of ABC triangle. On the otherhand, new $\pi$ value $\frac{14 - \sqrt{2}}{4}$ has given $\frac{3\sqrt{3}}{4}$ as the area of ABC triangle. This shows, that the new $\pi$ value $\frac{14 - \sqrt{2}}{4}$ is the real $\pi$ value.

Equating conventional formulas $\frac{1}{2} ab = \left( \frac{\sqrt{3}}{4} \right) a^2$ to new formula $\left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2$ is itself a justification and a naked truth of latter’s correctness.

II. Conclusion

$\frac{1}{2} ab$ is the formula to find out the area of a triangle. In this paper, a new formula $\left( \frac{3\sqrt{3}\pi}{14 - \sqrt{2}} \right) d^2$ is derived. This formula by giving the exact area of ABC triangle shows, that circle and the equilateral triangle are clearly interrelated and are not very different as we have been believing. The benefit we derive by using this formula is, this formula chooses the real $\pi$ value, discarding other approximate values attributed to $\pi$. This method alone, for the first time in the history of mathematics, acts as a testing method of $\pi$ and boldly says Archiemedes’ upper limit of $3 \frac{1}{7}$ or $\frac{22}{7}$ is a lower value compared to the real $\pi$ number.

Post script

As it is proved in this paper that the real $\pi$ value is $\frac{14 - \sqrt{2}}{4}$ based on the area of the triangle ABC, it has made possible to demarcate the length of the circumference of the inscribed circle in the straight-lined perimeter of the ABC triangle. How?

Let us know first the length of the circumference of the inscribed circle with the known $\pi$ value $\frac{14 - \sqrt{2}}{4}$

Diameter = $d$

Circumference = $\pi d$
An Alternate Formula in terms of \(\pi\) to find the Area of a Triangle and a Test to decide the ...

\[
\pi = \frac{14 - \sqrt{2}}{4}
\]

When the diameter is equal to 1, the circumference = \(\pi\) value

Let us search for the line-segments equal to \(\frac{14 - \sqrt{2}}{4}\).

Diameter = \(DF = d\)

Altitude of ABC triangle = \(DC = \frac{3d}{2}\)

Radius = \(OH = \frac{d}{2}\)

\(DE = \text{hypotenuse} = \text{Chord} = \frac{\sqrt{2}d}{2}\)

\(G\) is the mid point of \(DE\)

So, \(DG = GE = OG = \frac{\sqrt{2}d}{4}\)

\(OG = \frac{\sqrt{2}d}{4}\)

\(GH = \text{Radius} - OG = \frac{d}{2} - \frac{\sqrt{2}d}{4} = \left(\frac{2 - \sqrt{2}}{4}\right)d\)

\(CI = DC = \frac{3d}{2}\)

\(IJ = GH = \left(\frac{2 - \sqrt{2}}{4}\right)d\)

Circumference of the circle \(\pi d = \left(\frac{14 - \sqrt{2}}{4}\right)d\), So, the demarcated length \(DCJ\) in the perimeter of the ABC triangle is equal to the circumference of the inscribed circle.

\(DC + CI + IJ = \frac{3d}{2} + \frac{3d}{2} + \left(\frac{2 - \sqrt{2}}{4}\right)d = \left(\frac{14 - \sqrt{2}}{4}\right)d\)

Thus the straight lined length equal to length of the curvature of the circumference of the inscribed circle is called rectification of circumference of a circle. It was done never before. People tried earlier in the perimeter of the square but never in the perimeter of the triangle.

**References**


An Alternate Formula in terms of Pi to find the Area of a Triangle and a Test to decide the ....