Weibull Deterioration, Quadratic Demand Under Inflation

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Abstract: Deterministic inventory model is developed for deteriorating products when the demand rate is considered as quadratic function of time, further incorporating two parameter Weibull rate of deterioration with inflation. The model is solved for when shortages are not allowed. A numerical example and sensitivity of the models is also studied at the end.

Key words: Weibull deterioration, Quadratic demand, salvage value, Inflation

I. Introduction


Ajanta Roy (2008) developed an inventory model for deteriorating items with price dependent demand and time varying holding cost with shortages and without shortages. Mishra and Singh (2010) studied an inventory model for deteriorating items with time dependent demand and partial backlogging. Mishra (2012) proposed an inventory model with Weibull rate of deterioration and constant demand. He incorporated variable holding cost with shortages, salvage value also considered for deteriorated items. Vikas Sharma and Rekha (2013) proposed an inventory model for time dependant demand for deteriorating items with Weibull rate of deterioration. In this model shortages are taken consideration. Mohan and Venkateswarlu (2013a) developed an inventory model with variable holding cost and salvage value. Mohan and Venkateswarlu (2013b) proposed an inventory model for quadratic demand with respect to time with salvage considering deterioration items. Mohan and Venkateswarlu (2013c) developed an inventory model with Quadratic Demand, Weibull distribution deterioration rate with Variable Holding Cost and Salvage value. Shital S. Patel, Raman Patel studied an inventory model for deteriorating items with linear demand under permissible delay in payments and inflation in incorporating shortages.

In this paper, we consider an inventory model with weibull deterioration rate and demand rate is quadratic function of time. Shortages are not allowed in this case and the time horizon is infinite. The optimal total cost is obtained considering the salvage value for deteriorated items. The numerical example and sensitivity analysis is done at the end.
II. Mathematical Assumptions and Notations

This model is developed using the following assumptions and notations:

i) The demand rate $D(t)$ at time $t$ is assumed to be $D(t) = at^2 + bt + c$, $a \neq 0$, $b \neq 0$, $c \geq 0$.

Here $c$ is the initial rate of demand, $b$ is the rate with which the demand rate increases and $a$ is the rate with which the change in the rate demand rate itself increases.

ii) The deterioration rate follows two parameter Weibull distribution and is given by

$$\theta(t) = t^{\beta-1} \alpha \beta ; 0 \leq a \leq 1, \beta \geq 1.$$

iii) $D$, the number of deteriorated units

iv) Replenishment rate is infinite

v) Lead time is zero.

vi) $C$, the cost per unit

vii) $Q(t)$ is the inventory level at time $t$.

viii) $A$ is the order cost per unit order.

ix) The salvage value $\gamma C$, $0 \leq \gamma < 1$ is associated with deteriorated units during a cycle time.

x) $R$, inflation rate

III. Formulation and solution of the model

It is assumed that the inventory level depletes as the time passes due to demand rate and deterioration. The differential equation which describes the inventory level at time $t$ can be written as

$$\frac{d(Q(t))}{dt} + \theta(t)Q(t) = -D(t) ; \theta(t) = t^{\beta-1} \alpha \beta ; 0 \leq t \leq T$$

where $D(t) = (at^2 + bt + c)$, with $Q(t) = 0$ at $t = T$  

(1)

$$Q(t)e^{\alpha t} = \left( \frac{at^3}{3} + \frac{bt^2}{2} + ct \right) - \alpha \left( \frac{at^{\beta+3}}{\beta + 3} + \frac{bt^{\beta+2}}{\beta + 2} + \frac{ct^{\beta+1}}{\beta + 1} \right) + k_1$$

where $k_1$ is an integral constant. Here we have expanded $e^{\alpha t}$ and ignored higher order terms as $\alpha$ is small.

The solution of the above differential equation using the boundary conditions is given by
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\[
Q(t) = \left[ \frac{\alpha}{3} \left( T^3 - t^3 \right) + \frac{b}{2} \left( T^2 - t^2 \right) + c \left( T - t \right) \right] e^{-\alpha t^\beta} + \alpha \left[ \frac{a}{\beta + 3} \left( T^\beta + 3 - t^\beta + 3 \right) + \frac{b}{\beta + 2} \left( T^\beta + 2 - t^\beta + 2 \right) \right] \left( T^\beta + 1 - t^\beta + 1 \right)
\]

Since \( Q(0) = Q \), we get

\[
Q = \left( \frac{aT^3}{3} + \frac{bT^2}{2} + cT \right) + \alpha \left[ \frac{aT^\beta + 3}{\beta + 3} + \frac{bT^\beta + 2}{\beta + 2} + \frac{cT^\beta + 1}{\beta + 1} \right]
\]

(2)

IV. Inventory Model with Salvage value:

The following costs are calculated to find the total cost of the system when shortages are not allowed:

Ordering cost, \( (OC) = A \)

Inventory Holding cost, \( (IHC) = \frac{T}{b} \int_0^T Q(t) e^{-Rt} dt \)

\[
IHC = \frac{\alpha^2}{4} \left( \frac{aT^4}{3} + \frac{bT^3}{2} + cT^2 \right) + \alpha \left[ \frac{aT^\beta + 3}{\beta + 3} + \frac{bT^\beta + 2}{\beta + 2} + \frac{cT^\beta + 1}{\beta + 1} \right]
\]

(3)

The number of units that deteriorated during this cycle time is

\[
D = \int_0^T D(t) e^{-Rt} dt
\]

where \( D(t) = (at^2 + bt + c) \) is the rate of Demand.

Cost due to deterioration (CD) =

\[
= \alpha \left[ \frac{aT^\beta + 4}{\beta + 4} + \frac{bT^\beta + 3}{\beta + 3} + \frac{cT^\beta + 2}{\beta + 2} \right] - R \left[ \frac{cT^2}{2} + \frac{bT^3}{3} + \frac{at^4}{4} \right] - \frac{R^2}{2} \left[ \frac{cT^3}{3} + \frac{bT^4}{4} + \frac{at^5}{5} \right]
\]

Thus the total cost is obtained as

Total Cost (TC) = Ordering cost + Holding cost + Cost due to deterioration-Salvage value

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The necessary condition for a minimize total cost per unit time is

\[ \left( \frac{\alpha + \beta}{4}, \frac{\alpha^3 + \beta^3}{3}, \frac{\alpha^2 + \beta^2}{2} \right) = \left( \frac{\alpha^4 + \beta^4}{8}, \frac{\alpha^6 + \beta^6}{12}, \frac{\alpha^8 + \beta^8}{24} \right) \]
The optimum value of \( T \) is obtained by solving Equation (5) using MATHCAD and classified into four models. The following tables show the accelerated growth/decline models or retarded growth/decline models depending on the signs of \( a \) and \( b \):

- Depending on the signs of \( a' \) and \( b' \), one may have the following different types of relative demand patterns:
  - \( a > 0 \) and \( b > 0 \) gives accelerated growth in demand.
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- $a > 0$ and $b < 0$ gives retarded growth in demand
- $a < 0$ and $b > 0$, gives retarded decline in demand
- $a < 0$ and $b < 0$ gives accelerated decline in demand.

- The above four types of demand curves are given below:

- **Accelerated Growth Demand Curve**
  Fig. 2

- **Retarded Growth Demand Curve**
  Fig. 3

- **Retarded Decline Demand Curve**
  Fig. 4
4.1 Numerical Example:

To illustrate the models developed, we assume the following data:

\[ \begin{align*}
    a &= 5 \\
    b &= 10 \\
    c &= 100 \\
    A &= 100 \\
    C &= 8 \\
    \alpha &= 0.1 \\
    \beta &= 1.5 \\
    \gamma &= 0.1 \\
    h &= 3 \\
    R &= 0.01
\end{align*} \]

<table>
<thead>
<tr>
<th>Model Type</th>
<th>T</th>
<th>Q</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Demand</td>
<td>1.635</td>
<td>200.435</td>
<td>408.664</td>
</tr>
<tr>
<td>Linear Demand</td>
<td>1.729</td>
<td>205.512</td>
<td>397.956</td>
</tr>
</tbody>
</table>

Table 1: MODEL-I: \((a > 0, b > 0 \text{ and } c > 0)\)

<table>
<thead>
<tr>
<th>Model Type</th>
<th>T</th>
<th>Q</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Demand</td>
<td>1.882</td>
<td>214.936</td>
<td>384.52</td>
</tr>
<tr>
<td>Linear Demand</td>
<td>1.729</td>
<td>205.512</td>
<td>397.956</td>
</tr>
</tbody>
</table>

Table 2: MODEL-II: \((a < 0, b > 0 \text{ and } c > 0)\)

<table>
<thead>
<tr>
<th>Model Type</th>
<th>T</th>
<th>Q</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Demand</td>
<td>1.87</td>
<td>198.845</td>
<td>372.434</td>
</tr>
<tr>
<td>Linear Demand</td>
<td>2.13</td>
<td>212.771</td>
<td>353.865</td>
</tr>
</tbody>
</table>

Table 3: MODEL-III: \((a > 0, b < 0 \text{ and } c > 0)\)

<table>
<thead>
<tr>
<th>Model Type</th>
<th>T</th>
<th>Q</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Demand</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Linear Demand</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: MODEL-IV: \((a < 0, b < 0 \text{ and } c > 0)\)

It is observed that the Models II has shown marginal improvement in Total cost (TC) of the inventory system when comparing the quadratic time dependent demand models with linear dependent demand models when the deterioration rate follows Weibull deterioration rate. Hence we present the sensitivity analysis for the model II which is significant for further analysis.
4.2 Sensitivity analysis

MODEL-II: \((a < 0, b > 0 \text{ and } c > 0)\)

Table 5: Sensitivity of the scale parameter \(\alpha\)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(T)</th>
<th>(Q)</th>
<th>(TC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.882</td>
<td>214.936</td>
<td>384.52</td>
</tr>
<tr>
<td>0.15</td>
<td>1.675</td>
<td>196.391</td>
<td>322.282</td>
</tr>
<tr>
<td>0.2</td>
<td>1.536</td>
<td>183.782</td>
<td>354.46</td>
</tr>
<tr>
<td>0.25</td>
<td>1.433</td>
<td>174.359</td>
<td>482.863</td>
</tr>
<tr>
<td>0.3</td>
<td>1.353</td>
<td>167.047</td>
<td>508.493</td>
</tr>
</tbody>
</table>

Table 6: Sensitivity of the shape parameter \(\beta\)

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(T)</th>
<th>(Q)</th>
<th>(TC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.882</td>
<td>214.936</td>
<td>384.52</td>
</tr>
<tr>
<td>2</td>
<td>1.686</td>
<td>191.458</td>
<td>394.243</td>
</tr>
<tr>
<td>2.5</td>
<td>1.559</td>
<td>175.848</td>
<td>401.533</td>
</tr>
<tr>
<td>3</td>
<td>1.47</td>
<td>164.716</td>
<td>407.143</td>
</tr>
<tr>
<td>3.5</td>
<td>1.406</td>
<td>156.629</td>
<td>411.568</td>
</tr>
</tbody>
</table>

Table 7: Sensitivity of the salvage parameter \(\gamma\)

<table>
<thead>
<tr>
<th>(\gamma)</th>
<th>(T)</th>
<th>(Q)</th>
<th>(TC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.15</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

From Tables 5 to 7, the following points are noticed:

The total cost (TC) is more sensitive to the changes made in scale parameter and shape parameter but it does’t satisfy when the salvage value greater than 0.1

V. Discussions:

Neglecting \(\alpha^2\) and higher powers of \(\alpha\) and putting \(R = 0\) the Total cost function of (5) will be reduced to ‘R Venkateswarlu and R Mohan, An Inventory Model for time dependent Quadratic Demand Weibull Rate of Deterioration Rate and Salvage value Tenth AIMS International Conference on Management (Proceedings) 184-189’. When we compared this model (with inflation) and as mentioned above model the following variations are studied.

| Cycle time (T) is less | In this model cycle time is more |
| The number of order quantity (Q) is less | The number of order quantity (Q) is significantly more |
| Total cost is less | Total cost is significantly more |
| Two models are existing | One model is existing |

VI. Conclusions:

We have developed inventory management models for deteriorating items when the demand rate is assumed to be quadratic function of time. It is assumed that the deterioration rate is two parameter Weibull distribution. We have solved the model without shortages under inflation.

References

Weibull Deterioration, Quadratic Demand Under Inflation