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Acts Freely on Prime and semi prime Γ-near Rings

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Abstract: *Let M be a Γ-near ring. An element $a\in M$ is called a dependent element on a mapping f if $f(x)\alpha a= a\alpha x$ for all $x\in M$, $\alpha\in\Gamma$. In this paper we study and investigate concerning dependent elements of M by certain mappings on prime and semi prime Γ-near rings using certain assumption (A), and also we study the generalized Γ-derivation F of Γ-near ring M and Γ-derivations D which are free action.*
KeyWords: *Γ-near ring, generalized Γ-derivation, reverse centralizer, dependent element, free action.*

I. Introduction And Preminiries

Some researchers have studied the notion of free action. Laradji and Thaheem in [1] initiated the study of dependent elements of endemorphism of semiprime ring. F.J.Murray and J.Von Neuman in [2], introduced the notions of dependent elements and free action. In [8],Vukman and Ireana investigate some properties of dependent elements of derivations, generalized derivations and automorphisms of prime and semiprime rings. M.S.Samman and M. Anwar in [3] have studied some properties of dependent elements of left centralizers. Vukman and Kosi-UIbI in [7] and Vukman in [4] and [5] on dependant elements of mappings of semiprime rings, Vukman and Kusi-UIbI in [6] have studied centralizers in general from work of semiprime rings.

In this paper we investigate some mappings related to centralizer, reverse-centralizer, Γ-derivations and generalized Γ-derivations are free actions on prime and semi prime Γ-near rings.

Throughout M will represent an associative Γ-near ring with center Z(M) the commutator $[x,y]_\alpha$ will denoted by $x\alpha y-y\alpha x$ for all $x,y\in M$ and $\alpha\in\Gamma$, and we use the identities below
$[x\beta y,z]_\alpha = x\beta[y,z]_\alpha+[x,z]_\alpha\beta y+ x\beta z\alpha y- x\alpha z\beta y$ and
$[x,y\beta z]_\alpha = y\beta[x,z]_\alpha +[x,y]_\alpha\beta z + y\beta x\alpha z -y\alpha x\beta z$, for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$.
We shall take an assumption (A)… $x\alpha y\beta z = x\beta y\alpha z$, for all $x,y,z\in M$ and $\alpha,\beta\in\Gamma$.

A Γ-near ring M is a triple (M, +,Γ) where
(i) (M, +) is a group (not necessarily abelian).
(ii) Γ is a non empty set of binary operations on M-such that for each $\alpha \in \Gamma$, $(M,+,\alpha)$ is a near ring.
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all x, y, $z \in M$ and $\alpha,\beta\in \Gamma$.

M is said to be prime Γ-near ring if $x\Gamma M\Gamma y = (0)$ for all x, $y\in M$ implies x = 0 or y = 0 and semi prime Γ-near ring if $x\Gamma M\Gamma x = (0)$ for all $x \in M$ implies x = 0. A Γ-near ring M is said to be 2-torsion free whenever 2x=0 , for all $x\in M$, then x=0.A Γ-derivation on M is defined to be an additive endomorphism D of M satisfying the product rule $D(x\alpha y)=D(x)\alpha y+x\alpha D(y)$ or equivalently $D(x\alpha y)=x\alpha D(y)+D(x)\alpha y$, for all $x,y\in M$ and $\alpha\in\Gamma$.An additive mapping F: $M \to M$ is called generalized Γ-derivation if there exists Γ-derivation D of M such that $F(x\alpha y) = F(x)\alpha y + x\alpha D(y)$.An additive mapping T: $M \to M$ is called left (resp.right) centralizer if $T(x\alpha y) = T(x)\alpha y$, $(T(x\alpha y) = x\alpha T(y))$, for all $x,y\in M$ and $\alpha\in \Gamma$. If T is a both left as well right centralizer then T is centralizer for all $x,y\in M$ and $\alpha\in\Gamma$,and T is called left (resp. right) reverse centralizer of M if $T(x\alpha y) = T(y)\alpha x$, $(T(x\alpha y) = y\alpha T(x))$ hold for all $x,y\in M$ and $\alpha\in\Gamma$. If T is both left as well right reverse-centralizer, then T is a reverse-centralizer. An element $a\in M$ is called a dependent element of mapping f: $M \to M$ if $f(x)\alpha a = a\alpha x$ holds for all $x\in M$ and $\alpha\in\Gamma$.A mapping f: $M \to M$ is said to be free action if the only depend element of f is zero. The symbol $D^*(f)$ is denoted to the collection of all dependent elements.

II. Results

We consider M in all our results satisfying the assumption (A).

Theorem (2.1):
Let M be a prime Γ-near ring with a non zero Γ-derivation D, then D is a free action.

Theorem (2.2):
Let M be a semi prime Γ-near ring ,and F be a non zero generalized Γ-derivation associated with a Γ-derivation D, then F is a free action.

Proof:
For all $x \in M$,$a\in M$ and $\alpha\in\Gamma$, we have the relation
$F(x)\alpha a = a\alpha x$ …(1)
Putting $x\beta y$ for x ,and using (1), we get
$F(x)\beta y\alpha a + (x\alpha a - a\alpha x)\beta y = 0$,for all $x,y\in M$ and $\alpha,\beta\in\Gamma$ …(2) Putting $y\delta z$ for y in (2), and other hand right multiplication of (2) by z, and subtraction two equations , we obtain
$F(x)\beta y\delta(z\alpha a - a\alpha z) = 0$,for all $x,y,z\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(3)
Replacing y by $a\lambda y$ in (3),and using (1),we obtain
$a\beta x\lambda y\delta(z\alpha a - a\alpha z) = 0$ …(4)
Replacing x by $z\alpha x$ in (4),and other hand left multiplication of (4) by z ,and subtracting two equations, we obtain:
$(z\alpha a-a\alpha z)\beta x\lambda y\delta(z\alpha a-a\alpha z)=0$,for all $x,y,z\in M$ and $\alpha,\beta,\delta,\lambda\in\Gamma$ …(5)
Replacing $x\lambda y$ by r in (5) ,we obtain
$(z\alpha a - a\alpha z)\beta r\delta(z\alpha a - a\alpha z) = 0$,for all $z,r\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(6)
Since M is semi prime, we get
$(z\alpha a - a\alpha z) = 0$ … (7)
Substitution (7) in (2)for all $z\in M$, and replacing y by $a\delta y$ using (1), we get
$a\beta x\delta y\alpha a = 0$ … (8)
Putting z for $x\delta y$ in (8), we get:
$a\beta z\alpha a = 0$, for all $z \in M$, and $\alpha,\beta\in\Gamma$.
Since M is semi prime, we get a = 0.This completes the proof.

Theorem (2.3):
Let M be a prime Γ-near ring and F be a generalized Γ-derivation on M associated with a non zero Γ-derivation D. If $a\in M$, a is dependent element of F, then $a\in Z(M)$.
Proof:
Since a an element dependent on F, therefore
$F(x)\alpha a = a\alpha x$,for all $x\in M$ and $\alpha\in\Gamma$ …(1)
Replacing x by $x\beta y$ in (1) and using (1), we get
$(F(x)\alpha a-x\alpha a)\beta y = D(x)\beta y\alpha a$,for all $x,y\in M$ and $\alpha,\beta\in\Gamma$ …(2)
Right multiplication (2) by z, other hand replacing y by $y\delta z$ in (2),and subtracting two equations , we get
$D(x)\beta y\delta[a,z]_\alpha = 0$,for all $x,y,z\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(3)
Since M is a prime and $D \neq 0$, we obtain
$[a,z]_\alpha = 0$ that means $a\in Z(M)$, for all $z\in M$, $\alpha\in\Gamma$.This completes the proof.

Corollary (2.4):
Let M be a prime Γ-near ring, and let $a,b \in M$ be a fixed elements. Suppose that $c\in M$ is dependent element of $F(x)= a\alpha x+x\alpha b$. Then $c\in Z(M)$.
Proof:
For all $x\in M$ and $\alpha\in\Gamma$, we obtain:
$F(x) = a\alpha x$ …(1)
Replacing x by $x\beta y$ in (1), we get:
$F(x\beta y)= (a\alpha x+x\alpha b)\beta y + x\beta[y,b]_\alpha$,for all $x,y\in M$ and $\alpha,\beta\in\Gamma$ …(2)
Replacing $[y,b]_\alpha$ by $D(y)$ in (2) and using (1), we get:
$F(x\beta y) = F(x)\beta y + x\beta D(y)$,for all $x,y\in M$ and $\beta\in\Gamma$ …(3)
This mean F is a generalized Γ-derivation where it follows according the theorem (2.3), $c\in Z(M)$.

Theorem (2.5):
Let M be a prime Γ-near ring, and let $a, b \in M$ be fixed elements. Suppose that $c\in M$ is dependent element on the mapping $\psi(x)=a\alpha x\beta b$, then $a\alpha c\in Z(M)$ or $b\alpha c\in Z(M)$.
Proof:
For all $x\in M$ and $\delta\in\Gamma$, we have
$\psi(x)\delta c = c\delta x$, …(1)
Then, $(a\alpha x\beta b)\delta c = c\delta x$ …(2)
Replacing x by $x\lambda y$ in (2),and using (2) , we get
$a\alpha x\beta[b\delta c,y]_\lambda = 0$, for all $x,y\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(3)

Replacing x by $c\delta x\lambda y$ in (3),other hand replacing x by $c\delta x$ in (3) with left multiplying by y,and subtracting two equations, we obtain

$[a\delta c,y]_\lambda \alpha x\beta[b\delta c,y]_\lambda = 0$, for all $x,y\in M$ and $\alpha,\beta,\delta,\lambda\in\Gamma$ …(4)

Since M is a prime, for all $x\in M$,we get

$[a\delta c,y]_\lambda = 0$ or $[b\delta c,y]_\lambda = 0$.

That means either $a\alpha c\in Z(M)$ or $b\alpha c\in Z(M)$.

Theorem (2.6):

Let M be a 2-torsion free prime Γ-near ring ,and F_1 and F_2 are two generalized Γ- derivations on M associated with Γ-derivations D_1 and D_2 respectively, then the mapping $\psi(x)$ is free action, for all $x\in M$, when:

1) $\psi(x) = D_1(x) + D_2(x)$

2) $\psi(x) = D_1^2(x) + D_2(x)$

3) $\psi(x) = D_1(x) + D_2^2(x)$

4) $\psi(x) = F_i(x) + D_i^2(x)$, for all i= 1,2.

5) $\psi(x) = F_1^2(x) + F_2(x)$

6) $\psi(x) = F_1(x) + F_2^2(x)$.

Proof:

We will prove (4). The proof of other results is by the same way.

Let i = 2, for all $x\in M$, we have

$\psi(x) = F_2(x) + D_2^2(x)$.

Let $a \in D^*(\psi)$, then $\psi(x)\alpha a = a\alpha x$, for all $x \in M$ and $\alpha \in \Gamma$.

That is

$a\alpha x = F_2(x)\alpha a + D_2^2(x)\alpha a$, for all $x\in M$ and $\alpha\in\Gamma$ …(1)

Putting $x\beta a$ for x in (1), and using (1), we get

$a\alpha x\beta a = 2D_2(x)\beta D_2(a)\alpha a + x\beta D_2(a)\alpha a + x\beta D_2^2(a)\alpha a$ ...(2)

Replacing x by $y\delta x$ in (2), and using (2), we get:

$2(D(y)\delta x\beta D_2(a)\alpha a = 0$,

Since M is 2-torsion free, we have

$D_2(y)\delta x\beta D_2(a)\alpha a=0$, for all $x,y\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(3)

Putting z for $D_2(a)$ in (3), we get

$D_2(y)\delta x\beta z\alpha a = 0$, for all $x,y,z\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(4)

Replacying $x\beta z$ by r, we get

$D_2(y)\delta r\alpha a = 0$, for all $y,r\in M$ and $\alpha,\delta\in \Gamma$

Since M is prime and $D \neq 0$, we get a = 0, that's mean our mapping is a free action.

Theorem (2.7):

Let M be a semi prime Γ-near ring, and let T be a centralizer, F be a generalized Γ-derivation with associated Γ-derivation D. If a is dependent element of D, $a\in M$, then $\psi = (F\circ T)$ is a free action.

Proof:

Let $\psi= (F\circ T)$, and $a\in D^*(\psi)$, then $\psi(x)\alpha a = a\alpha x$,for all $x\in M$ and $\alpha\in\Gamma$, That is

$(F\circ T)(x)\alpha a = a\alpha x$, …(1)

Replacing x by $x\beta y$ in (1), and using (1) ,we get

$(F\circ T)(x)[a,y]_\alpha =T(x)\beta D(y)\alpha a$, for all $x,y\in M$ and $\alpha,\beta\in\Gamma$ …(2)

Replacing y by $a\delta y$ in (2), and using (1) ,we get

$a\beta x\delta[a,y]_\alpha =T(x)\beta D(a)\delta y\alpha a + T(x)\beta a\delta D(y)\alpha a$, for all $x,y\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(3)

Multiplying (3) on the left by z, other hand replacing x by $z\alpha x$ in (3) and Subtracting two equations ,we get

$[a,z]_\alpha\beta x\delta[a,z]_\alpha=0$,for all $x,z\in M$ and $\alpha,\beta,\delta\in\Gamma$ …(4)

Replacing z by y in (4), we get:

$[a,y]_\alpha\beta x\delta[a,y]_\alpha = 0$, which by semiprimeness of M, implies

$[a,y]_\alpha = 0$,for all $y \in M$, $\alpha \in \Gamma$ …(5)
Substitute (5) in (2), and by the hypothesis ,we get
$T(x)\beta a\alpha y = 0$, for all $x,y \in M$ and $\alpha,\beta \in \Gamma$ ...(6)
Replacying y by $y\delta T(x)\beta a$ in (6), and using semiprimeness of M ,we get
$T(x)\beta a = 0$, for all $x \in M$, $\beta \in \Gamma$ …(7)
Taking F of (7), and using (3) and (1),we get
$a\alpha x\beta a = 0$, for all $x \in M$ and $\alpha,\beta \in \Gamma$
Since M is semi prime, we get $a = 0$, which implies that $(F \circ T)$ is a free action.

Theorem (2.8):
Let T be a reverse centralizer of Γ-near ring M. Then $\psi: M \to M$ which defined by $\psi(x) = [T(x),x]_\alpha$, for all $x \in M$ and $\alpha \in \Gamma$ is a free action.
Proof:
Let $a \in D^*(\psi)$, then $\psi(x)\beta a = a\beta x$, for all $x \in M$ and $\alpha,\beta \in \Gamma$
That is
$[T(x),x]_\alpha\beta a = a\beta x$, for all $x \in M$ and $\alpha,\beta \in \Gamma$ …(1)
Linearizing (1) ,and using (1) , we get:
$[T(x),y]_\alpha\beta a + [T(y),x]_\alpha\beta a = 0$, for all $x,y \in M$ and $\alpha,\beta \in \Gamma$ …(2)
Replacing y by $a\delta y$ in (2), and using (2),we get
$-a\delta[T(y),x]_\alpha\beta a + [T(x),a]_\alpha\delta y\beta a + T(y)\delta[a,x]_\alpha\beta a + [T(y),x]_\alpha\delta a\beta a = 0$, for all $x,y \in M$ and $\alpha,\beta,\delta \in \Gamma$ …(3)
Replacing y and x by a in (3),and other hand replacing x by a in (3),and using them ,we get
$- a\delta a\beta a + a\delta a\beta a + a\delta a\beta a = 0$
That is, $a\delta a\beta a = 0$, we get $a = 0$.Hence ψ is free action.

References

[1]. A.Laradji and A.B.Thaheem, On Dependent Elements in Semiprime Ring, Math.Japonica, 47 (1998), No.1, 576-584. F. J. Murray and von Neumann J., On Rings of Operators, , 37, 116-229, Ann. Math.,(1936).
[2]. M. S. Samman and Anwar M., Dependent Element of Left Centralizer of Semiprime Rings, 33, 313-319, The Arabian Journal for Science and Engineering (2008).
[3]. J.Vakman, On Dependent Elements and Related Problems In Ring, Internationsl Math. Journal, 8 (2005), No.2, 93-112.
[4]. J.Vakman, Free Actions of Semiprime Rings with Involution Induced by Aderivation, Demenstratio Mathematica, 38 (2006), No.4, 811-817.
[5]. J.Vakman and I.Kosi-UIbI, On Dependent Elements in Rings, International Mathematical Journal, 6 (2005), No.2, 93-112.
[6]. J.Vakman, I.Kosi-UIbI, Centralizer on Ring and Algebras, Bull, Austral.Math.Soc, 71, (2005), 234-255.
[7]. J.Vukman and Ireana, On Dependent Elements in Rings, 54, 2895-2906, IJMMS,(2004).