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Newsboy problem with lost sales recapture as function of $\log_m\left(1+\frac{r}{p}\right)$ and normally distributed demand error

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Abstract: *We consider an extension to the lost sale recapture model in a newsvendor framework developed earlier by the authors. As in real practice, we have considered that there may be an opportunity to backlog the lost sales, by offering some incentive for waiting. The back log fill rate is modelled as a log function of adding one to the proportion of rebate relative to the price. The retailer's decision includes selling price, order quantity and the rebate that will maximize its expected profit. Sensitivities of the demand errors in the form of normal distribution rather than the uniform distribution serve as an extension to the previous work by the authors.*
***Keywords*:** *newsvendor problem, lost sales, rebates, price dependent demand*

I. Introduction And Literature Review

This paper considers the buying and ordering policies of a newsvendor-type retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation. The backordering occurs through an emergency purchase of the items in question at some premium over the regular purchasing cost. In turn, the retailer offers to the end-customers left out of the initial sale a rebate incentive upon purchase of each item backordered.

The problem of backordering shortage items has been considered by Weng (2004) and Zhou and Wang (2009). A different model of lost sales recapture was discussed by Arcelus, Gor and Srinivasan (2012). This paper is similar in lines of and Patel and Gor (2013, 2014(a),(b)). Here, we use an entirely different fill rate function than Arcelus, Gor and Srinivasan (2012) and Patel and Gor (2013) and include sensitivities to the normal distribution over and above the one for uniform distribution discussed in Patel and Gor (2014(b)). We describe the characteristics of the model, develop the objective function and derive the profit-maximizing optimality conditions that are shown to be unique. We present a numerical example. In addition to illustrating the main features of the model and discussing some comparative statics of interest, this section attempts to conjecture the behavioural relationship between various parameters and variables. A conclusions section completes the paper. Table 1 lists the notations used throughout the paper.

Table 1: Notation

p	The selling price per unit (*decision variable*)
v	The salvage value per unsold unit
q	The order quantity (*decision variable*)
r	The rebate per backordered item (*decision variable*)
c	The acquisition cost per unit
s	The shortage penalty per unsold unit
D	The total demand rate per unit of time
g, ε	The deterministic and stochastic components, respectively, of D
a,b	The upper and lower values, respectively, of ε
μ, σ	The mean and standard deviation, respectively, of ε
f, F	The density function and the cumulative distribution function, respectively, of ε
δ_0,δ_1	The intercept and slope, respectively, of the deterministic linear demand function
γ_0, γ_1	The intercept and the demand elasticity, respectively, of the iso-elastic deterministic demand function
Ω	The fill rate of backlogged demand
d	The premium on the purchase price of each backlogged unit acquired
z	The stocking factor
Λ, Φ	The expected number of leftovers and shortages, respectively
e	The price elasticity of demand
I_ε	The generalized failure rate function
$\pi(p,q,r)$	The retailer's profit function

E(p,q,r)	The retailer's expected profit function
U.D	The uniform Distribution
N.D	The Normal Distibution

II. Model Formulation

In this section, we describe the key characteristics of the model, formulate the retailer's profit-maximizing objective function and derive the optimality conditions. Observe that, in the development of the models, the arguments of the functions are omitted whenever possible, to simplify notation.

Characteristics of the model

Characteristic 1: Key properties of the demand function.

The random single-period total demand, D(p,ε), is of the form:

$$D(p,\varepsilon) = g(p) + \varepsilon, \quad \text{if additive error} \qquad g(p)\varepsilon, \quad \text{if multiplicative error} \tag{1}$$

g(p) has an IPE or increasing price elasticity, e, which satisfies the following condition:

$$e_p^{'} = \frac{\partial e}{\partial p} \geq 0, \quad where \quad e = \frac{\partial g}{\partial p}\frac{p}{g}$$

ε has a GSIFR or generalized strictly increasing failure rate, I_ε*, since*

$$I_\varepsilon^{'} = \partial I_\varepsilon / \partial\varepsilon \geq 0, \quad where \quad I_\varepsilon = \varepsilon f/(1-F)$$

Characteristic 2: A fill rate, Ω, given by the following expression:

$$\Omega = \log_m\left(1+\frac{r}{p}\right), \quad where \quad 0 < r < p, \quad 0 < \Omega < 1, \ 2 < m < \infty, m > \left(1+\frac{r}{p}\right) \tag{2}$$

Characteristic 3: The stocking factor, z

$$z = q - g, \quad \text{if additive}$$
$$= q/g, \quad \text{if multiplicative}$$
$$\Phi = \int_z^B (\varepsilon - z) f(\varepsilon) d\varepsilon \tag{3}$$
$$\Lambda = \int_A^z (z-\varepsilon) f(\varepsilon) d\varepsilon = \Phi + z - \mu$$

Detailed discussion on the above three characteristics can be found in Patel and Gor (2013), Patel and Gor (2014 (a)) and Patel and Gor (2014 (b)).

The retailer's profit-maximizing objective

The retailer profit function is decomposable into two parts, depending upon whether the retailer order quantity exceeds or understates the demand for the product. If the first, then *q* exceeds *D* and the retailer sells *D* units at *p* per unit, disposes of the rest at a salvage value of *v* per unit and incurs an acquisition cost of *c* for each of the *q* units ordered. If the second, *q* is below *D*, in which case the retailer buys and sells the *q* units at a profit margin of (*p-c*) per unit, acquires a fraction *Ω* of the shortage demand at a premium *d* per unit, sells it at (*p-r*), the regular selling price, *p*, net of the per unit rebate offered, *r*, and pays a shortage penalty of *s* per unit on the rest of the merchandise. Formally, the functional form of the retailer's profit function, *π(p,q,r)*, is as follows:

$$\pi(p,q,r) = pD - cq + v(q-D), \quad if \ q \geq D$$
$$= (p-c)q + \left[(p-r)-(c+d)\right]\Omega(D-q) - s(1-\Omega)(D-q), \quad if \ q \leq D \tag{4}$$

The objective is to find the levels of *p, q* and *r* that maximizes *E(p,q,r),* the retailer's expected profit. Using (3) and (4), it can be readily seen that *E* may be written as follows:

$$E(p,q,r) = (p-c)(g+\mu) - (c-v)\Lambda - \left[(p-c+s)(1-\Omega) + \Omega(r+d)\right]\Phi, \ \text{if additive}$$
$$= (p-c)g\mu - g(c-v)\Lambda - g\left[(p-c+s)(1-\Omega) + \Omega(r+d)\right]\Phi, \quad \text{if multiplicative} \tag{5}$$

First-order optimality conditions:

To simplify the explanation, only the additive-error/linear-demand case will be discussed. The multiplicative case can be developed along the same lines. Let $E_i^{'} = \partial E/\partial i, i = p,r,Q$ be the first derivative of the expected profit with respect to each of the decision variables. Setting these derivatives to zero, we obtain the following first-order optimality conditions.

$$\begin{aligned} E_p^{'} &= 0 = (g+\mu)+g_p^{'}(p-c)-(1-\Omega)\Phi+(p-c+s-r-d)\Phi\Omega_p^{'} \\ E_r^{'} &= 0 = \Phi\Omega_r^{'}(p-c+s-r-d)-\Phi\Omega \\ E_z^{'} &= 0 = -(c-v)-\Phi_z^{'}\left[(p-v+s)-\Omega(p-c+s-r-d)\right] \end{aligned} \qquad (6)$$

where $\Omega_p^{'}$ and $\Omega_r^{'}$ are defined in (3). The detailed economic interpretations of the optimality conditions above can be found in Patel and Gor (2014(b)).

III. Numerical Analysis

Given the central objective of the paper, our numerical analysis centers on the impact of fluctuations in base *m* of the fill rate function, upon the fill rate, *Ω*, and through it, upon the retailer's profit-maximizing pricing, ordering, rebate policies. All computations were carried out with MAPLE's Optimization toolbox.

Base-case numerical structure

The starting point consists of two sets of examples that serve as the base-case for the analysis of this section. The first (second) set, denoted by *AL (MI)*, assumes the deterministic demand, *g*, to be linear (iso-elastic) and its error, additive (multiplicative), i.e.

$$D(p) = \begin{cases} \delta_0 - \delta_1 p + \varepsilon, \quad \delta_0 > 0, \quad \delta_1 > 0, & \text{for AL total demand} \\ \gamma_0 p^{-\gamma_1}\varepsilon, \quad \gamma_0 > 0, \quad 0 > \gamma_1 > 1, & \text{for MI total demand} \end{cases} \qquad (7)$$

For comparability purposes, this section operates with the parameter values of Patel and Gor(2014(b)), to which suitable values for the remaining parameters have been added. These values appear in Table 2 (N. D.). In this way, any sensitivity analysis can be carried out by adroit manipulation of the appropriate parameter values for any of the components of the base-case.

Further for maximum comparability among probability distributions, all cases are related to a random variable uniformly distributed and normal distributed over the interval (-3,500, 1,500), for the AL demand model and (0.7, 1.1), for its MI counterpart. Either support interval describes the normal distribution completely.

Base-case numerical results

Having described the nature of the numerical structure that gives rise to the parameter values of the *AL* and *MI* components of the base case, we now discuss the numerical results. Unless otherwise stated, we concentrate our remarks on the *AL* demand case. As mentioned latter on in this section, the results for the *MI* case can be interpreted in similar fashion.

Table 2.Numerical Analysis: Base Case Optimal Policies(N. D.)

DISTRIBUTION		Support, mean and Standard deviation		
NORMAL DISTRIBUTION Additive Error and Linear Demand. **A > -a** Multiplicative Error and Iso-elastic demand.A>0		support [A,B] [-3500, 1500] , Mean = -1000, SD = 1440 [0.7, 1.1], Mean = 0.9 , SD = 0.07		
Additive Error Linear Demand Parameter values: γ_0=100000 ; γ_1 =1500 ; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	***Λ***	***Φ***
346866	50.36	23295	245	399
Multiplicative Error Iso-Elastic Demand Parameter values: γ_0 = 500000000; γ_1 = 2.5; c = 35; d = 3; v = 10; s = 3				
Profit	***p***	***q***	***Λ***	***Φ***
377413	59.90	16290	538	452

IV. Numerical Example And Interpretations

The optimal results using MAPLE for the fill rate model are shown in Table 3(N. D.). The reader can refer to Patel and Gor (2014(b)) for comparability purposes with the uniform distribution case. Both the cases Additive Error Linear Demand and Multiplicative Error Iso-elastic Demand are showcased to highlight the variations in the optimal solutions too. The following observations and interpretations are made:

(a) The optimal policy for the fill rate model with m=2, as shown in row 1 of Table 3(N. D.) in Additive Error Linear Demand case, consists of the retailer acquiring q*=23,228 units at a unit cost of c=$35 and selling them at a unit price of p*=$50.38. With respect to the fill rate, approximately Ω^*= 19% of the shortages are recaptured at an extra purchasing cost of d=$3.00 to the retailer, who allows a rebate of r*=$7.42 per unit backlogged. Afterwards, all unsold units, i.e. $[(1-\Omega^*)(D-q^*)]$, will be assigned a unit shortage penalty of s=$3.
On the other hand, when demand falls below the q*=23,228 units ordered and all purchased at the cost of c=$35 per unit, D units are sold at the regular unit price of p*=$50.38 and the remaining, at the salvage value of v=$10.00 per unit.
The resulting optimal policy is π*[p*, q*, r*]=$347516 [50.38, 23228, 7.42].
As shown in Table 2, these results contrast with the optimal solution for the *AL* certainty case of π*[p*; q*] =$346866 [$50.36; 23,295]

(b) Similar interpretation follows for the other models in the Additive Error Linear Demand case, where the power on $\log_m\left(1+\frac{r}{p}\right)$ increases Table 3(N. D.). The increase in the base of the fill rate function tends to increase the optimal order quantity and the rebate, whereas decreases selling price as well as profits.

(c) Table 3(N. D.) also gives results for the *MI* case. Observe though that unlike its Additive Error Linear Demand counterpart, in this case, increase in the base of the fill rate function, tends to increase the order quantity and the rebate and also the selling price. Profits decrease with the increase in the base of the fill rate function.

Table 3. Optimal Policies for lost sale recapture model with fill rate Ω= $\log_m\left(1+\frac{r}{p}\right)$ **(N.D)**

Additive Error Linear Demand							
m	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
2	347516	50.38	23228	7.42	0.19	227	425
3	347271	50.37	23254	7.36	0.12	234	415
4	347186	50.37	23262	7.42	0.09	236	412
5	347141	50.37	23267	7.42	0.08	237	410
Multiplicative Error Iso-Elastic Demand							
m	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
2	379033	59.86	16208	11.88	0.26	486	503
3	378415	59.87	16241	11.88	0.16	506	483
4	378202	59.88	16252	11.88	0.13	513	476
5	378090	59.88	16257	11.88	0.11	516	473

V. Sensitivity Analysis

Table 4(N. D.) describes the sensitivities of the optimal policies to the change in the salvage and shortage costs in the Additive Error and Linear Demand case. Corresponding results for the Iso-elastic demand and multiplicative error case can be easily computed. The primary objective to carry out the sensitivity analysis is to observe the directional change in the short ages and the leftover values. Observe that, through Table 6 and 7, we have tried to construct examples where the relationship between shortages and leftovers is $\Lambda^* > \Phi^*$ as well as $\Lambda^* < \Phi^*$.

Table 4: Sensitivities to the salvage and shortage costs in Additive Error Linear Demand Case for *m*=2(N. D.)

Linear Demand Additive Error Case for *m*=2								
v	**s**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
16	**3**	349048	50.40	23333	7.43	0.19	219	348
17	**3**	349341	50.40	23354	7.44	0.19	300	333
18	**3**	349648	50.41	23377	7.44	0.19	314	319
19	**3**	349970	50.41	23401	7.44	0.19	329	304
20	**3**	350308	50.42	23427	7.44	0.19	346	288
21	**3**	350663	50.42	23455	7.44	0.19	364	272
Linear Demand Additive Error Case for *m*=2								
10	**12**	344985	50.41	23348	11.58	0.26	302	331
10	**13**	344757	50.41	23359	12.04	0.30	309	324
10	**14**	344537	50.41	23369	12.49	0.31	316	317
10	**15**	344325	50.42	23379	12.94	0.32	323	310
10	**16**	344121	50.42	23389	13.39	0.33	329	304
10	**17**	343923	50.42	23399	13.83	0.34	336	298

Next, as shown in Table 5, we perform sensitivity analysis to the change in the support values [A,B] for the Normal distribution for the fill rate model with base *m*=2. Similar sensitivities can be performed for various other values of m, as well as support structures.

Table 5: Sensitivities to the Normal Distribution Support Changes: CASE *m*=2

Linear Demand and Additive Error								
SUPPORT	**Mean**	π^*	p^*	q^*	r^*	Ω^*	Λ^*	Φ^*
-3500,1500	-1000	347516	50.38	23228	7.42	0.19	227	425
1500,3500	2500	411161	51.61	24999	8.01	0.20	99	169
1500,5500	3500	421907	51.90	25502	8.14	0.20	99	163
-5500,1500	-2000	326077	49.98	22706	7.24	0.19	332	643
-1500,3500	1000	378950	51.05	24239	7.74	0.20	234	415
Iso-elastic Demand and Multiplicative Error								
.7,1.1	0.9	379033	59.86	16208	11.88	0.26	486	503
.8,1.2	1.0	379033	59.86	18011	11.88	0.26	486	503
.6,1.0	0.8	379033	59.86	14405	11.88	0.26	486	503
.6,1.2	0.9	368431	60.59	15745	12.21	0.26	691	689
.8,1.4	1.1	386128	59.40	20187	11.66	0.25	710	739

Table 6, shows the percentage change in the optimal policies when for capturing the demand errors, the normal distribution is used instead of the uniform distribution (Patel and Gor, 2014(b)).

Table 6. % change Optimal Policies for lost sale recapture model with m=2: N. D. used instead of U. D.

Additive Error Linear Demand							
Dist	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
U. D.	335356	50.25	23125	7.36	0.04	321	1027
N. D.	347516	50.38	23228	7.42	0.19	227	425
% CHANGE	3.62↑	0.25↑	0.44↑	0.81↑	3.75↑	0.29↓	0.58↓
Multiplicative Error Iso-Elastic Demand							
Dist	**Profit**	***p***	***q***	***r***	***Ω***	***Λ***	***Φ***
U. D.	359274	61.27	15351	12.53	0.12	726	1017
N. D.	379033	59.86	16208	11.88	0.26	486	503
% CHANGE	5.49↑	2.30↓	5.58↑	5.18↓	1.16↑	0.33↓	0.50↓

VI. Some Concluding Comments

The primary contribution of this paper has been to consider the impact upon the ordering and pricing policies of a newsvendor-type, profit-maximizing retailer, faced with the possibility of backordering at least some of the shortages incurred from demand underestimation, by offering some rebate incentives for waiting.

The backlog fill rate, representing the probability of the end-customers returning to satisfy their unfilled demand, is modelled as a function of the size of the rebate offered relative to the selling price. The decision variables are the selling price, the order size and the rebate offered as an incentive to satisfy at least a portion of the unfulfilled demand. Sensitivities of the demand errors in the form of normal distribution rather than the uniform distribution serve as an extension to the previous work by the authors.

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