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Steady State Analysis of Repairable M/G/1 Retrial Queuing Model with Modified Server Vacation Balking and Optional Reservice

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Abstract: *A retrial queue with general retrial times, modified vacation policy and Bernoulli feedback is analyzed in this paper. If the server is busy or on vacation, an arriving customer either enters on orbit with probability b, or balks with probability 1-b. Otherwise the service of the arriving customer commences immediately. At the service completion epoch, the test customer may either enter the orbit for another service with probability p or leave the system with probability q (=1-p). If the orbit is empty, the server takes at most J vacations until at least one customer is recorded in the orbit when the server returns from a vacation. By applying supplementary variable technique some analytical results for the system are derived. The effects of various parameters on the system performance are analyzed numerically.*

Key words: *Retrial queues, balking, Bernoulli feedback, supplementary variable technique, Laplace Stieljes transform.*

I. Introduction

Retrial queuing systems are characterized by the fact that an arriving customer who finds the server busy upon arrival is obliged to leave the service area and repeats his request for service after some random time. Such queues play very important roles in the analysis of telephone switching systems, computer and communication systems [6,2]. Review of retrial queue literature could be found in Yang and Templeton, Falin and Templeton, and Artalejo [11,3,1]. A comprehensive study on the vacation models can be found in Takagi [9]. Jan-Chnan ke et al [5] first introduced the modified vacation policy in retrial queues. Queuing system with server breakdowns are very common in communication systems and manufacturing systems. Single server queuing systems with server breakdowns and Bernoulli vacation have been studied by many researchers including Li et al , Wang et al, Peishu Chen et al [7,10,8]. Many queuing situations have the feature that the customers may be served repeatedly for a certain reason. When the service of a customer is unsatisfied, it may be tried again and again until a successful re service is completed. These queuing models arise in the stochastic modeling of many real life situations. Artalejo and Lopez Herrero [1] studied the M/G/1 retrial queue with balking probabilities depending on the number of customers in the system upon arrival, where the limiting distribution of the number of customers in the system is determined with the help of a recursive approach based on the theory of regenerative approach.

In this paper an M/G/1 retrial system with modified vacation, server breakdown, balking and feedback is considered. This paper is organized as follows: The mathematical model for the M/G/1 retrial queue with balking and feedback where the server takes at most J vacations utilizing the idle time subject to random server breakdown is constructed. After that the steady state distribution of the server state and the number of customers in the system/orbit are obtained. Some important performance measures of the system are derived. Numerical example is carried out.

II. Model Description

In this paper, a repairable M/G/1 retrial queue with balking and feedback, where the server applies a modified vacation policy when no customer is recorded in the orbit is considered. New customers arrive from outside according to Poisson process with rate λ. There is a single server to all arriving customers. Once an arriving customer finds the server free it begins his service, if the server is busy or down or on vacation, the arrivals either leave the service area and join the pool of blocked customers called the orbit with nonzero probability b or balk the system with probability 1-b. The customers from the orbit try to request his service later and inter retrial times have an arbitrary distribution A(x) with corresponding Laplace Stieljes transform (LST) Stieljes transform (LST) $\tilde{A}(\theta)$. There is a single server who provides service to all arriving customers. The service time follows a general distribution S(x) with corresponding Laplace Stieljes transform $\tilde{S}(\theta)$ and nth factorial moment s_n. As soon as a customer served completely, he will decide either to join the orbit for another service with probability p or to leave the system with probability q. Whenever the orbit is empty, the server leaves for a vacation of random length V if no customers appears in the orbit when the server returns from a vacation it leaves again for another vacation with the same length. Such pattern continues until it returns from a vacation to find at least one customer is recorded in the orbit or it has already taken J vacations. If the orbit is empty at the end of the J^{th} vacation, the server remains idle for the new arrivals in the system. At the vacation completion epoch the orbit is non empty, the server waits for the customer in the orbit, or new customers to arrive. The vacation time V has distribution function V(x) with corresponding Laplace Stieljes transform $V(\theta)$ and the nth factorial moment V_n. It is also assumed that when the server is busy it fails at an exponential rate β. When the server fails it repaired immediately and the customer first being served before the server breakdown wait for the server until repair completion in order to accomplish its remaining service. The repair time is a random variable with probability distribution function R(x), Laplace Stieljes transform $R(\theta)$ and the nth factorial moment r_n. Various stochastic process involved in the system are independent of each other.

III. System Analysis

The state of the system at time t can be described by the {C(t) , N(t) , $\xi_0(t)$, $\xi_1(t)$, $\xi_2(t)$, $\xi_3(t)$ },where C(t) denote the steady state at time t.

C(t) = {0, if the server is free at time t
1, if the server is busy t time t
2, if the server is on repair at time t
ω_1, if the server is on vacation with first vacation at time t
ω_2, if the server is on vacation with second vacation at time t
.
.
.
ω_j, if the server is on vacation with jth vacation at time t
.
.
.
ω_J, if the server is on vacation with Jth vacation at time t

where N(t) represents the number of customers in the retrial group at time t. When C(t) =1, then $\xi_1(t)$ represents the corresponding elapsed service time at time t; when C(t) = 2, then $\xi_2(t)$ represents the corresponding elapsed repair time at time t. When C(t) = ω_j then $\xi_j(t)$ represents the elapsed vacation time in jth ($1 \le j \le J$) vacation at time t.

In steady state it is assumed that, A(0) = 0 , A(∞) = 1 , S(0) = 0 , S(∞) = 1 , R(0) = 0 , R(∞) = 1 , V(0) = 0, V(∞) = 1 and α(x), s(x), γ(x), ω(x) are the conditional completion rate for repeated attempts, for service , for repair and for vacation respectively. i. e.

$$\alpha(x)dx = \frac{dA(x)}{1-A(x)} \quad s(x)dx = \frac{dS(x)}{1-S(x)} \quad \gamma(x)dx = \frac{dB(x)}{1-B(x)} \quad \omega(x)dx = \frac{dV(x)}{1-V(x)}$$

Let $\{t_n ; n = 1,2……\}$ be the sequence of epochs at which either service period completion occurs or a repair achievement or a vacation time ends occurs. The sequence of random vectors $Z_n = \{ C(t_n^+) , N(t_n^+) \}$ form a Markov chain which is embedded in the retrial queuing system. By similar argument of [3 , 4] it can be shown that $\{Z_n, n = 1,2,….\}$ is ergodic if and only if $\xi_1[1 - A(\lambda) + p + \lambda b s_1(1+\beta r_1)] < 1$

For the process $Z_n = \{ C(t_n^+) , N(t_n^+) \}$, in steady state the following probabilities can be defined

$$P_0 = \lim_{t \to \infty} P\{C(t) = 0, N(t) = 0\}$$

$$P_n(x,t)dx = \lim_{t \to \infty} P\{N(t) = n , \xi_0(t) = A(t) ; x < A(t) \le x+dx\} \quad x > 0 , n \ge 1$$

$$\pi_n(x, t)\, dx = \lim_{t \to \infty} P\{N(t) = n , \xi_1(t) = S(t) ; x < S(t) \le x+dx\} \quad x > 0 , n \ge 0$$

$B_n(x,t)\,dx = \lim_{t\to\infty} P\{N(t)=n,\ \xi_2(t) = R(t);\ y < R(t) \le y+dy\}\quad x>0,\ n\ge 0$

$\Omega_{j,n}(x,t)\,dx = \lim_{t\to\infty} P\{N(t)=n,\ \xi_3(t) = V(t);\ x < V(t) \le x+dx\}\quad x>0,\ n\ge 0$

By the method of supplementary variable technique, the Kolmogrov forward equations which govern the system under the steady state conditions can be written as follows:

$$\lambda P_0 = \int_0^\infty \Omega_{J,0}(x)\omega(x)dx \qquad \text{…………….. (1)}$$

$$\frac{dP_n(x)}{dx} + (\lambda + \alpha(x))P_n(x) = 0,\ x>0,\ n\ge 1 \qquad \text{………………. (2)}$$

$$\frac{d\pi_0(x)}{dx} + (\lambda + \beta + s(x))\pi_0(x) = \lambda(1-b)\pi_0(x),\ x>0 \qquad \text{……………. (3)}$$

$$\frac{d\pi_n(x)}{dx} + (\lambda + \beta + s(x))\pi_n(x) = \lambda(1-b)\pi_n(x) + \lambda b\pi_{n-1} + \int_0^\infty B_n(x,y)\gamma(y)dy\quad y>0,\ n\ge 0 \qquad \text{……...(4)}$$

$$\frac{dB_0(x,y)}{dy} + (\lambda + \gamma(y))B_0(x,y) = \lambda(1-b)R_0(x,y)\quad x>0,\ y>0 \qquad \text{………….(5)}$$

$$\frac{dB_n(x,y)}{dy} + (\lambda + \gamma(y))B_n(x,y) = \lambda(1-b)R_n(x,y) + \lambda bR_{n-1}(x,y)\quad x>0,\ y>0,\ n>0 \qquad \text{………….. (6)}$$

$$\frac{d\Omega_{j,0}(x)}{dx} + (\lambda + \omega(x))\Omega_{j,0}(x) = \lambda(1-b)\Omega_{j,0}(x)\quad x>0 \qquad \text{……………..(7)}$$

$$\frac{d\Omega_{j,n}(x)}{dx} + (\lambda + \omega(x))\Omega_{j,n}(x) = \lambda(1-b)\Omega_{j,n}(x) + \lambda b\,\Omega_{j,n-1}(x)\quad x>0,\ n>0 \qquad \text{…………….(8)}$$

The boundary conditions are

$$P_n(0) = \int_0^\infty \Omega_{1,n}(x)\omega(x)dx + \int_0^\infty \Omega_{2,n}(x)\omega(x)dx + \ldots + \int_0^\infty \Omega_{J,n}(x)\omega(x)dx + q\int_0^\infty \pi_n(x)s(x)dx + p\int_0^\infty \pi_{n-1}(x)s(x)dx\quad x>0 \qquad \text{……………….(9)}$$

$$\pi_0(0) = \int_0^\infty P_1(x)\alpha(x)dx + \lambda P_0 \qquad \text{……………..(10)}$$

$$\pi_n(0) = \int_0^\infty P_{n+1}(x)\alpha(x)dx + \lambda\int_0^\infty P_n(x)dx,\ n\ge 1 \qquad \text{……………(11)}$$

$$B_n(x,0) = \beta\pi_n(x)\quad n\ge 0 \qquad \text{………….(12)}$$

$$\Omega_{1,n}(0) = \begin{cases} \int_0^\infty \pi_0(x)s(x)dx & n=0 \\ 0 & n\ge 1 \end{cases} \qquad \text{……… (13)}$$

$$\Omega_{j,n}(0) = \begin{cases} \int_0^\infty \Omega_{j-1,0}(x)\omega(x)dx, & n=0,\ j=2,3\ldots J \\ 0 & n\ge 1,\ j=2,3\ldots J \end{cases} \qquad \text{………… (14)}$$

and the normalizing condition is

$$P_0 + \sum_{n=1}^\infty \int_0^\infty P_n(x)dx + \sum_{n=0}^\infty \int_0^\infty \pi_n(x)dx + \sum_{n=0}^\infty \int_0^\infty\int_0^\infty B_n(x,y)dxdy + \sum_{j=i}^J \left(\sum_{n=0}^\infty \int_0^\infty \Omega_{j,n}(x)dx\right) = 1 \qquad \text{…….. (15)}$$

To solve the equations (1) – (15) we define the probability generating functions

$P(x,z) = \sum_{n=1}^\infty P_n(x)z^n$, $\pi(x,z) = \sum_{n=1}^\infty \pi_n(x)z^n$, $B(x,z) = \sum_{n=1}^\infty B_n(x)z^n$, $\Omega_j(x,z) = \sum_{n=1}^\infty \Omega_{j,n}(x)z^n$

Now multiplying equation (2) by z^n and summing over n (n = 1, 2 ,3…..) finally yields

$$\frac{\partial P(x,z)}{\partial x} + (\lambda + \alpha(x))P(x,z) = 0 \,,\ x > 0 \qquad \text{……………… (16)}$$

Similarly proceeding (3)-(14),

$$\frac{\partial \pi(x,z)}{\partial x} + (\lambda b(1-z) + \beta + s(x))\pi(x,z) = \int_0^\infty B(x,y,z)\gamma(y)dy \qquad \text{…………………..(17)}$$

$$\frac{\partial B(x,z)}{\partial y} + (\lambda b(1-z) + \gamma(y))B(x,y,z) = 0 \qquad \text{……………………(18)}$$

$$\frac{\partial \Omega_j(x,z)}{\partial x} + (\lambda b(1-z)) + \omega(x))\,\Omega_j(x,z) = 0 \qquad \text{……………………(19)}$$

$$P(0,z) = \sum_{j=1}^{J}\int_0^\infty \Omega_j(x,z)\omega(x)dx + (q+pz)\int_0^\infty \pi(x,z)s(x)dx - \sum_{j=1}^{J}\Omega_{j,0}(0) - \lambda P_0 \qquad \text{……………………(20)}$$

$$\Pi(0,z) = \frac{1}{z}\int_0^\infty P(x,z)\alpha(x)dx + \lambda[\int_0^\infty P(x,z)dx + P_0] \qquad \text{……………………(21)}$$

$$B(x,0,z) = \beta\pi(x,z) \qquad \text{…………………….(22)}$$

Solving the partial differential equations (16), (18) and (19) we have

$$P_0(x,z) = P_0(0,z)[1-A(x)]e^{-\lambda x} \qquad \text{…………………...(23)}$$

$$B(x,y,z) = \beta\pi(x,z)[1-R(y)]e^{-\lambda b(1-z)y} \qquad \text{…………………...(24)}$$

$$\Omega_j(x,z) = \Omega_j(0,z)\,[1-V(x)]e^{-\lambda b(1-z)x}\,,\ j = 1,2,\ldots.J \qquad \text{…………………..(25)}$$

From (24) and (17)

$$\Pi(x,z) = P(0,z)[1-S(x)]\, e^{-\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z))} \qquad \text{……………………(26)}$$

From equation (7)

$$\Omega_{j,0}(x) = \Omega_{j,0}(0)\,[1-V(x)]e^{-\lambda bx}\,,\ j = 1,2,\ \ldots\ldots\ldots J \qquad \text{…………………….(27)}$$

Putting j = J into (27), and integrating with respect to x from 0 to ∞, then from (1), we have

$$\Omega_{j,0}(0) = \frac{\lambda P_0}{\tilde{V}(\lambda)} \qquad \text{…………………..(28)}$$

Using (28) in (19) and combining (27), we obtain

$$\Omega_{J-1,0}(0) = \frac{\lambda P_0}{[\tilde{V}(\lambda)]^2}$$

Recursively over the range j = J-1, J-2……..1. On simplification

$$\Omega_{j,0}(0) = \frac{\lambda P_0}{[\tilde{V}(\lambda)]^{J-j+1}} \quad j=1,2,\ldots.J \qquad \text{…………………(29)}$$

Note that $\Omega_{j,0}$ represents the steady state probability the steady state probability that no customers appear while the server is on the jth vacation. Integrating equation (27) with respect to x from 0 to ∞ and using (29) to get

$$\Omega_{j,0} = \frac{P_0[1-\tilde{V}(\lambda)]}{[\tilde{V}(\lambda)]^{J-j+1}} \quad j = 1,2,\ldots..J \qquad \text{………………(30)}$$

From (13) and (14)

$$\Omega_j(0,z) = \Omega_{j,0}(0) = \frac{\lambda P_0}{[\tilde{V}(\lambda)]^{J-j+1}} \quad j = 1,2,\ldots.J \qquad \text{………………(31)}$$

Using (23) in (21) to obtain

$$\pi(0,z) = P(0,z)\,\frac{\tilde{A}(\lambda) + z(1-\tilde{A}(\lambda))}{z} + \lambda P_0 \qquad \text{……………………(32)}$$

Using (25), (26), (29) in (20) to get

$$P(0,z) = \lambda P_0\{N(z) + \pi(0,z)(q+pz)\tilde{S}(\lambda b(1-z) + \beta - \beta\tilde{R}(\lambda b(1-z)) \qquad \text{………………...(33)}$$

Where $N(z) = \dfrac{(1-\tilde{V}(\lambda))^{J}(\tilde{V}(\lambda b(1-z))}{(1-\tilde{V}(\lambda))(\tilde{V}(\lambda))^{J}}$

Using (32) in (33) after simplification, we get

$$P(0,z) = \frac{\lambda P_0\{(N(z)-1)z + z(q+pz)S(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z))}{z-(q+pz)(\tilde{A}(\lambda)+z(1-\tilde{A}(\lambda))\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))} \quad \ldots\ldots\ldots\ldots\ldots\ldots(34)$$

Thus

$$P(z) = \int_0^\infty P(x,z)dx = \frac{zP_0[1-\tilde{A}(\lambda)]\{N(z)-1+(q+pz)\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))\}}{z-(q+pz)(\tilde{A}(\lambda)+(1-\tilde{A}(\lambda))\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))} \quad \ldots\ldots(35)$$

Using (35) in (24)

$$\pi(z) = \frac{\lambda P_0\{(N(z)-1)(\tilde{A}(\lambda)+z(1-\tilde{A}(\lambda))+z\}\{1-\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))\}}{z-(\tilde{A}(\lambda)+z(1-\tilde{A}(\lambda)\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))} \quad \ldots\ldots\ldots(36)$$

using (36) in (24) we get

$$B(z) = \beta\pi(z)\frac{[1-\tilde{R}(\lambda b(1-z))]}{\lambda b(1-z)} \quad \ldots\ldots\ldots\ldots(37)$$

and

$$\Omega_j(z) = \frac{P_0[1-\tilde{V}(\lambda b(1-z))]}{b(1-z)\tilde{V}(\lambda b)^{J-j+1}} \quad \ldots\ldots\ldots\ldots\ldots(38)$$

The unknown constant P_0 can be determined by using the normalization condition (15)

$$P_0 + P(1) + \pi(1) + B(1) + \sum_{j=1}^{J}\Omega_j(1) = 1$$

Thus

$$P_0 = \frac{\tilde{A}(\lambda) - p - \lambda b s_1(1+\beta r_1)}{(N^1(1)+\tilde{A}(\lambda))(1-p+s_1\lambda(1-b)(1+\beta r_1))}$$

where

$$N^1(1) = \frac{\lambda b v_1(1-\tilde{V}(\lambda b)^J)}{(1-\tilde{V}(\lambda b))\tilde{V}(\lambda b)^J}$$

IV. Performance Measures

Now the system performance measures of the M/G/1 retrial queue with balking, feedback, modified server vacations, and random server breakdown is derived. Note that $p + \lambda b s_1(1+\beta r_1) < \tilde{A}(\lambda)$ is the stability condition.

1. The system is idle with probability

$$P_0 = \frac{\tilde{A}(\lambda) - p - \lambda b s_1(1+\beta r_1)}{\tilde{A}(\lambda) + N^1(1)(1-p+\lambda s_1(1-b)(1+\beta r_1))}$$

2. The server is idle with probability

$$\text{Pr(server is idle)} = \frac{\{N^1(1)+\tilde{A}(\lambda)[1-N^1(1)-\lambda b s_1(1+\beta r_1)]\}}{(N^1(1)+A(\lambda))(1-p+s_1\lambda(1-b)(1+\beta r_1))}$$

3. The system is busy with probability

$$\pi(1) = \frac{\lambda s_1}{1-p+\lambda b s_1(1-b)(1+\beta r_1)}$$

4. The system is under repair with probability

$$B(1) = \frac{\beta\lambda s_1 r_1}{1 - p + \lambda b s_1 (1 - b)(1 + \beta r_1)}$$

5. The server is on Jth vacation with probability

$$V = \sum_{j=1}^{J} \Omega_j(1) = P_0 N^1(1)$$

6. The probability generating function of the number of customers in the orbit is

$$\Phi_q(z) = P_0 + P(z) + \pi(z) + B(z) + \sum_{j=1}^{J} \Omega_j(z)$$

$$\frac{\begin{aligned}&-(((z^2 bp\tilde{A}(\lambda) - bpz\tilde{A}(\lambda) - N(z)pz\tilde{A}(\lambda) - N(z)qz - N(z)z\tilde{A}(\lambda) - bq\tilde{A}(\lambda) - N(z)q\tilde{A}(\lambda) + \\ &N(z)pz^2\tilde{A}(\lambda) - N(z)pz^2 - N(z)\tilde{A}(\lambda) + N(z)q\tilde{A}(\lambda)z - N(z)z + zbq\tilde{A}(\lambda) + \tilde{A}(\lambda) - z\tilde{A}(\lambda)) \\ &\tilde{S}(\lambda b1 - z) + \beta - \beta\tilde{R}(\lambda b(1 - z))) + z\tilde{A}(\lambda) + zb\tilde{A}(\lambda) - z^2 b\tilde{A}(\lambda) - \tilde{A}(\lambda) + zbN(z) - z^2 bN(z) - N(z)z\tilde{A}(\lambda) + \\ &N(z)\tilde{A}(\lambda) - zbN(z)\tilde{A}(\lambda) + 2N(z)z + z^2 bN(z)\tilde{A}(\lambda))P_0)\end{aligned}}{\begin{aligned}&((-1 + z)b((-q\tilde{A}(\lambda) - qz + qz\tilde{A}(\lambda) - pz\tilde{A}(\lambda) - pz^2 + \\ &pz^2\tilde{A}(\lambda)\tilde{S}(\lambda b(1 - z) + \beta - \beta\tilde{R}(\lambda b(1 - z))) + z)\end{aligned}}$$

7. The mean number of customers in the orbit is given by

$$L_q = \Phi_q^1(1)$$

$$= \frac{1}{8}\,\frac{1}{(b^2(\tilde{A}(\lambda) - p - s_1\lambda b(1 + \beta r_1))^2}$$

$$\begin{aligned}&(-2b(\tilde{A}(\lambda) - p - s_1\lambda b(1 + \beta r_1))(-3\tilde{A}(\lambda)s_1 r_2\beta\lambda^2 b^2 + 6b^2 s_1 r_1\tilde{A}(\lambda)\beta\lambda p - 6\tilde{A}(\lambda)s_2 r_1\beta\lambda^2 b^2 - \\ &3\tilde{A}(\lambda)s_2 r_1^2\beta^2\lambda^2 b^2 + 6b^3\tilde{A}(\lambda)s_2 r_1\beta\lambda^2 + 3b^3\tilde{A}(\lambda)s_2 r_1^2\lambda^2\beta^2 + 3b^3\tilde{A}(\lambda)s_1 r_2\lambda^2\beta - 3\tilde{A}(\lambda)s_2\lambda^2 b^2 + \\ &3b^3\tilde{A}(\lambda)s_2\lambda^2 + 6b^2 s_1 p\tilde{A}(\lambda)\lambda + N^1(1)(-6s_1 r_2\beta\lambda^2 b^2 - 12s_1 r_1\lambda b\beta - 12s_1\lambda b + 12\tilde{A}(\lambda)s_1 r_1\beta\lambda b - \\ &6s_1 r_1\lambda b\beta p - 12s_2 r_1\beta\lambda^2 b^2 - 6s_2 r_1^2\beta^2\lambda^2 b^2 + 6p\tilde{A}(\lambda) + 6b\tilde{A}(\lambda) - 6p - 6b - 6s_2\lambda^2 b^2 + 12\tilde{A}(\lambda)s_1\lambda b \\ &-6ps_1\lambda b) + N^{11}(1)(3\tilde{A}(\lambda) - 3b + 3b\tilde{A}(\lambda) - 3p - 6s_1 r_1\lambda b\beta - 6s_1\lambda b))P_0 + 6b(-s_2\lambda^2 b^2 - 2ps_1\lambda b \\ &-2p + 2p\tilde{A}(\lambda) - 2s_1\lambda b - 2s_1 r_1\lambda b\beta p + 2\tilde{A}(\lambda)s_1\lambda b - 2s_2 r_1\beta\lambda^2 b^2 - s_2 r_1^2\beta^2\lambda^2 b^2 - 2s_1 r_1\lambda b\beta + \\ &2\tilde{A}(\lambda)s_1 r_1\beta\lambda b - s_1\beta r_2\lambda^2 b^2))((-p + \tilde{A}(\lambda) - 2s_1\lambda b - 2s_1 r_1\lambda b\beta + b\tilde{A}(\lambda) - b)N^1(1) - b\tilde{A}(\lambda) - \tilde{A}(\lambda)s_1\lambda b + \\ &b^2 s_1\tilde{A}(\lambda)\lambda + b^2 s_1 r_1\tilde{A}(\lambda)\beta\lambda + bp\tilde{A}(\lambda) - \tilde{A}(\lambda)s_1\beta r_1\lambda b)P_0)\end{aligned}$$

8. The Probability generating function of the number of customers in the system is

$$\Phi_s(z) = P_0 + zP(z) + z\pi(z) + zB(z) + \sum_{j=1}^{J} \Omega_j(z)$$

$$\frac{\begin{array}{l}-(((-N(z)z\tilde{A}(\lambda)+N(z)z^2\tilde{A}(\lambda)+z\tilde{A}(\lambda)-N(z)z^2-z^2\tilde{A}(\lambda)-N(z)pz^2-\\ N(z)q\tilde{A}(\lambda)-N(z)qz-bq\tilde{A}(\lambda)+N(z)qz\tilde{A}(\lambda)-N(z)pz\tilde{A}(\lambda)+N(z)pz^2\tilde{A}(\lambda)+\\ bqz\tilde{A}(\lambda)-bpz\tilde{A}(\lambda)+bpz^2\tilde{A}(\lambda))\tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))+N(z)z\tilde{A}(\lambda)-\\ z\tilde{A}(\lambda)+zbN(z)-z^2bN(z)-z^2b\tilde{A}(\lambda)+N(z)z+N(z)z^2+z^2\tilde{A}(\lambda)-N(z)z^2\tilde{A}(\lambda)-\\ zbN(z)\tilde{A}(\lambda)+z^2bN(z)\tilde{A}(\lambda)+zb\tilde{A}(\lambda))P_0)\end{array}}{\begin{array}{l}((-1+z)b((-q\tilde{A}(\lambda)-qz+qz\tilde{A}(\lambda)-pz\tilde{A}(\lambda)-pz^2+pz^2\tilde{A}(\lambda))\\ \tilde{S}(\lambda b(1-z)+\beta-\beta\tilde{R}(\lambda b(1-z)))+z)\end{array}}$$

9. The mean number of customers in the system is given by

$$L_s = \Phi_S^1(1)$$

$$=\frac{1}{12}\frac{1}{b^2(\tilde{A}(\lambda)-p-s_1\lambda b(1+\beta r_1))^2}$$

$$\begin{array}{l}(-2b((\tilde{A}(\lambda)-p-s_1\lambda b(1+\beta r_1)((-6s_2\lambda^2b^2-12s_2r_1\beta\lambda^2b^2-6s_2\lambda^2b^2\beta^2r_1^2-6b+6p\tilde{A}(\lambda)-6ps_1r_1\beta\lambda b+\\ 12\tilde{A}(\lambda)s_1r_1\beta\lambda b-18s_1r_1\beta\lambda b-6s_1r_2\beta\lambda^2b^2-18s_1r_1\beta\lambda b-6s_1r_2\beta\lambda^2b^2-18s_1\lambda b+6b\tilde{A}(\lambda)-6p-6s_1\lambda bp+\\ 12\tilde{A}(\lambda)s_1\lambda b)N^1(1)+N^{11}(1)(-6s_1\lambda b-3b-6s_1r_1\beta\lambda b+3\tilde{A}(\lambda)-3p+3b\tilde{A}(\lambda))-3\tilde{A}(\lambda)s_2\lambda^2b^2+\\ 3b^3s_2\tilde{A}(\lambda)\lambda^2-3\tilde{A}(\lambda)s_1r_2\lambda^2b^2\beta+6b^3s_2r_1\tilde{A}(\lambda)\beta\lambda^2+3b^3s_2\tilde{A}(\lambda)\lambda^2\beta^2r_1^2-6\tilde{A}(\lambda)s_2\lambda^2b^2\beta r_1-\\ 3\tilde{A}(\lambda)s_2r_1^2\lambda^2\beta^2b^2+3b^3s_1\lambda^2\beta r_2\tilde{A}(\lambda)+6b^2s_1p\tilde{A}(\lambda)\beta\lambda r_1-6\tilde{A}(\lambda)s_1r_1\beta\lambda b-6\tilde{A}(\lambda)s_1\lambda b+\\ 6b^2s_1p\tilde{A}(\lambda)\lambda)P_0+6b(-s_2\lambda^2b^2-2s_1\lambda pb-2p+2p\tilde{A}(\lambda)-2s_1\lambda b-2ps_1r_1\beta\lambda b+2\tilde{A}(\lambda)s_1\lambda b-\\ 2s_2\lambda^2b^2\beta r_1-s_2\lambda^2b^2\beta^2r_1^2-2s_1r_1\beta\lambda b+2\tilde{A}(\lambda)s_1r_1\beta\lambda b-s_1\lambda^2b^2\beta r_2)(b^2\tilde{A}(\lambda)s_1r_1\beta\lambda-\tilde{A}(\lambda)s_1r_1\beta\lambda b+\\ bp\tilde{A}(\lambda)+b^2\tilde{A}(\lambda)s_1\lambda-b\tilde{A}(\lambda)-\tilde{A}(\lambda)s_1\lambda b+N^1(1)(\tilde{A}(\lambda)-p-b+b\tilde{A}(\lambda)-2s_1\lambda b-2s_1r_1\beta\lambda b))P_0)\end{array}$$

10. The average time a customer spends in the system in steady state is

$$W_s = L_s / \lambda$$

V. Numerical example

In this section a numerical example is presented in order to illustrate the effect of various parameters in the system performance measures of our system, where all retrial times, service time, vacation time and repair time are exponentially distributed. Arbitrary values are assumed to the parameters such as steady state condition is satisfied.

Let $A(x) = ve^{-vx}$, $S(x) = e^{-x}$, $B(x) = e^{-x/2}$, $V(x) = e^{-4x}$, $J = 10$, $\lambda = 0.3$

The following table presented here gives the values of P_0, L_s, L_q for the various retrial rates, breakdown rates balking probabilities and feed back probabilities.

B	B	P	V	Po	Lq	Ls
0.2	0.3	0.6	2	0.1820	1.3376	2.9082
			4	0.2479	0.7092	1.9710
			6	0.2702	0.5632	1.7545
			8	0.2811	0.4999	1.6606
			10	0.2879	0.4625	1.6050
0.4			2	0.1361	2.4857	4.8880
			4	0.2041	1.2750	3.0795
			6	0.2271	1.0255	2.7079
			8	0.2384	0.9207	2.5519
			10	0.2454	0.8596	2.4610
0.6			2	0.0903	4.9668	8.8673
			4	0.1604	2.2418	4.7890
			6	0.1841	1.7837	4.1051
			8	0.1957	1.5997	3.8306
			10	0.2029	1.4945	3.6738
			2	0.2904	0.7254	2.1017
			4	0.3411	0.4457	1.6861

0.4	0.2	0.5	6	0.3583	0.3677	1.5704
			8	0.3666	0.3320	1.5175
			10	0.3718	0.3104	1.4855
	0.3		2	0.2372	1.1782	2.8112
			4	0.2926	0.7352	2.1513
			6	0.3113	0.6193	1.9790
			8	0.3204	0.5673	1.9018
			10	0.3261	0.5361	1.8555
	0.4		2	0.1807	1.9724	1.0347
			4	0.2409	1.1836	2.8569
			6	0.2613	0.9967	2.5784
			8	0.2712	0.9154	2.4572
			10	0.2775	0.8671	2.3854
0.2	0.3	0.3	2	0.4233	0.3218	1.1597
			4	0.4573	0.2163	1.0023
			6	0.4688	0.1836	0.9536
			8	0.4744	0.1682	0.9306
			10	0.4779	0.1587	0.9164
		0.4	2	0.3574	0.4624	1.4321
			4	0.4004	0.2979	1.1873
			6	0.4149	0.2493	1.1149
			8	0.4219	0.2266	1.0813
			10	0.4263	0.2127	1.0606
		0.5	2	0.2784	0.7195	1.8913
			4	0.3319	0.4349	1.4679
			6	0.3499	0.3569	1.3521
			8	0.3587	0.3215	1.2996
			10	0.3642	0.3001	1.2678

Fig.1: The effect of different values of β and V on Lq.

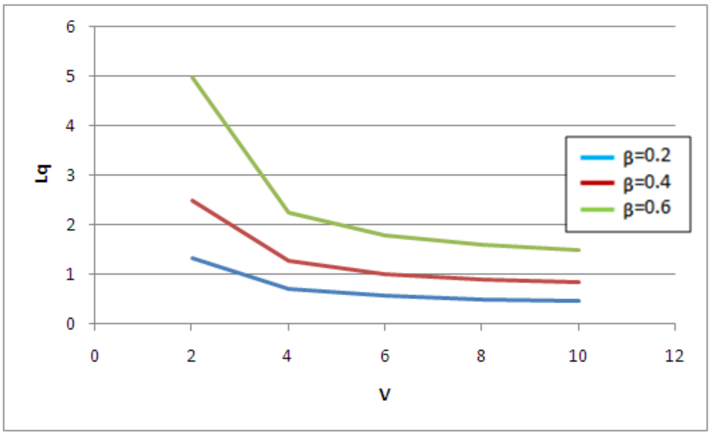


Fig.2: The effect of different values of b and V on Ls.

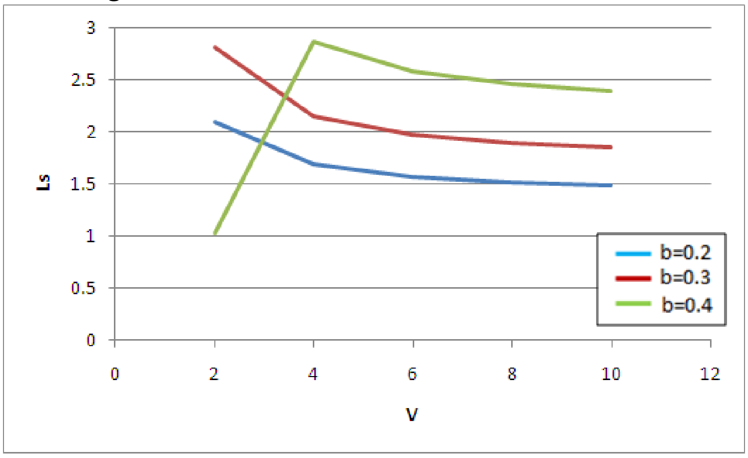
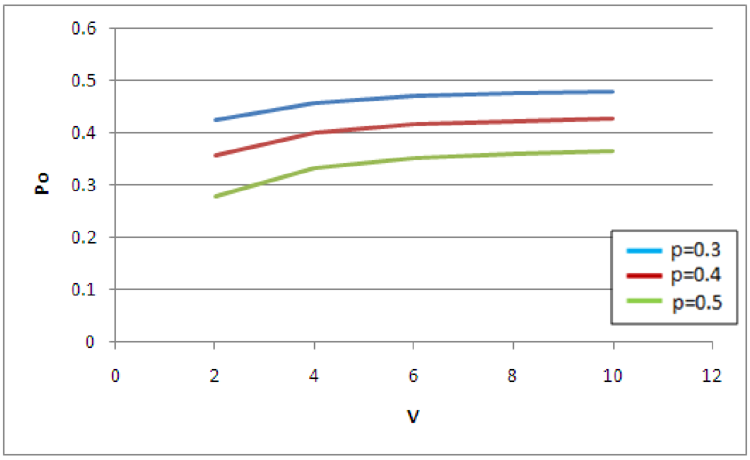


Fig.3: The effect of different values of p and V on Po.



VI. Conclusion

In this paper a single server retrial queuing system with modified vacation, random server breakdown, balking and optional re service is studied. The probability generating functions of the number of customers in the system when it is idle, busy, on vacation and under repair is found using supplementary technique. The performance measures like, the mean number of customers in the system/orbit were obtained. The effect of various parameters on the performance measure has been illustrated numerically. The model discussed in this paper can be widely used in telecommunication systems.

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