A numerical analysis of a steady equation of a power-law fluid flow on a moving wall.

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Abstract: We present a paper on the numerical analysis of a power-law fluid flow on a moving wall. We investigate the steady boundary-layer flow of a non-Newtonian fluid, represented by a power-law model over a moving flat plate. The governing partial differential equation is transformed into ordinary differential equation through a similarity variable. The finite difference method is employed to obtain solution of the non-linear problem. We investigate the effect of the power-law viscosity index $n$, and the results obtained are discussed.

Keywords: Steady boundary-layer flow; Moving plate; Non-Newtonian power-law fluid and Finite difference method.

I. Introduction

The forced convection flow over a moving fluid has many practical engineering applications such as the cooling of polymer films or sheets and metallic plates on conveyers. Accurate and comprehensive computational techniques such as finite difference method can be applied to solve partial differential equations that model the flow of a power-law fluid.


In this paper we present a numerical analysis of the flow behavior when $n=1$ (Newtonian fluids) using finite difference method.

II. Mathematical Formulation

Consider a steady, two-dimensional laminar flow of a power-law fluid passing a moving flat plate with constant velocity $U_w$, in the same or opposite direction to the free stream $U_\infty$. The $x-\text{axis}$ extends parallel to the plate, while the $y-\text{axis}$ extends upwards, normal to it. The boundary layer equations governing the flow in a power-law are [8],[2] and [9].

The steady equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} \tag{2.2}
\]

Where $u$ and $v$ are velocity components along the $x$ and $y$ axes, respectively. $\tau_{xy}$ is the shear stress, and $\rho$ is the fluid density. The boundary conditions are

$u = U_w, v = 0$

$y = 0, u \rightarrow U_\infty$

$y \rightarrow \infty \tag{2.3}$

As the stress tensor is defined as [7] and [4]

Where
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\[ \tau_{ij} = 2k \left| 2D_{kl} D_{kl} \right|^{(n-\frac{1}{2})} D_{ij} \]  

(2.4)

\[ D_{ij} = \frac{1}{2} \left| \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right| \]  

(2.5)

Denotes the stretching tensor is called the coefficient and \( n \) is the power-law index. The index \( n \) is non-dimensional, and the dimension of \( k \) depends on the value of \( n \). The two parameter rheological Eq.(2.4) is known as the Ostwald-de-Waele model or the power-law model.

The parameter \( n \) is an important index to subdivide fluids into pseudo plastic fluids (\( n < 1 \)), dilatants fluid (\( n > 1 \)) and for \( n = 1 \), the fluids is Newtonian. Therefore, the deviation of \( n \) from unity indicates the degree of deviation from Newtonian behavior[9]. Eq.(2.4) represents shear-thinning (\( n < 1 \)) and shear-thickening (\( n > 1 \)) fluids.

Using Eqs.(2.4) and (2.5), the shear stress appearing in Eq.(2.2) can be written as:

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{k}{\rho} \left( \frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial u}{\partial y} \]  

(2.6)

The continuity equation (2.1) is satisfied by introducing a stream function \( \psi \) such that:

\[ u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \]  

(2.7)

The momentum equation can be transformed into the corresponding ordinary differential equation by introducing the following new variable [8]

\[ \eta = \left( \frac{\text{Re}}{x/l} \right)^{\frac{n}{l}} \frac{y}{l}, \psi = l U_\infty \left( \frac{x/l}{\text{Re}} \right)^{\frac{n}{l}} f(\eta), \]  

(2.8)

\[ \tau_{xy} = k \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \]  

(2.9)

differentiating Eqs.(2.8) and (2.9) to get

\[ u = U_\infty f' \]  

(2.10)

\[ \frac{\partial u}{\partial y} = U_\infty f'' \]  

(2.11)

\[ \left| \frac{\partial u}{\partial y} \right|^{n-1} = U_\infty^{n-1} \left| f'' \right|^{n-1} \]  

(2.12)

\[ \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} = U_\infty^n \left| f'' \right|^{n-1} f'' \]  

(2.13)
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Substitute Eqs.(2.3)&(2.11)-(2.13) into (2.7) to get
\[
\left(\frac{d^2 f}{d\eta^2}\right)^{n-1} f'' + \frac{1}{n+1} f = 0
\]
(2.14)

Where prime denote differentiation with respect to \( \eta \)
The transformed boundary conditions are
\[
f(0) = 0, \quad f'(0) = \varepsilon, \quad f' (\eta) \rightarrow 1, \quad \text{as} \ \eta \rightarrow \infty, \quad f'(\infty) = 1
\]
(2.15)

Where \( \varepsilon = \frac{U_w}{U} \) is the velocity ratio parameter.

III. Method of Solution

In order to solve the problem and keep it tractable, the set of non-linear ordinary differential equations (2.14) subject to boundary conditions (2.15) have been solved numerically using finite difference method.

We need some other conditions
\[
f'' = f'' = -1
\]
(3.1)

Integrating Eq.(2.11) together with the boundary conditions (3.1) to get
\[
\left(\frac{d^2 f}{d\eta^2}\right)^{n-1} f'' + \frac{1}{n+1} f = -1
\]
(3.2)

Let \( n = \frac{1}{2} \) in Eq.(3.2) to get
\[
(f'') + \frac{1}{2} f' + 1 = 0
\]

IV. Result

Fig4.1: Graph of the velocity function \( f \) against the similarity variable \( \eta \)
when \( n = 1 \).

V. Conclusion

We obtained a suitable expression for the steady boundary- layer flow of a power-law fluid on a moving wall. The power-law viscosity index \( n \) subdivide the fluid into Newtonian fluid, the upper branch solutions are physically stable, while the lower branch solutions are not. We plot the graph of velocity function \( f \) against the similarity variable \( \eta \). It is seen from the graph that parameter \( n \) affects the flow characteristics.
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Reference


