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An Application Of Queuing Theory To The TV Show Kaun Banega Crorepati (KBC)

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Abstract: *The present paper explains the application of queuing theory to one episode of the TV show Kaun Banega Crorepati during a week. By analysis of one episode, we can understand the general scenario of the KBC. In one episode of this show,* 10 *contestants or participants wait for their turn to go on the hot seat. At one time only one contestant can seat on the hot seat and other contestants are waiting in a queue. So, we can apply queuing theory to the TV show Kaun Banega Crorepati. We also seek to find the probability of one contestant to be selected for the hot seat and the expectation of contestants on each day of a week.*
Keywords: *Queue; Queuing Theory; Kaun Banega Crorepati (KBC); Hot seat.*

I. Introduction

There are three episodes of KBC during each week from Friday to Sunday. In one episode of the TV show Kaun Banega Crorepati, 10 contestants or participants are allowed to participate in the show. Depending on the Fastest Finger First (FFF) contest, one participant can occupy the hot seat in front of Amitabh Bachchan. From the durarion of one participant on the hot seat, we can derive the service rate of the hot seat. If we assume that, the time span of FFF contest is negligible, the arrival rate is same as the service rate.

The first contestant is considered as a customer being served by a single server i.e. the hot seat. The other contestants are considered as the waiting customers in a queue. So, we can apply queuing theory to KBC. We assume that, the arrival rate is same as the service rate for the hot seat. For this reason, we consider the utilization factor for the hot seat as 1. Because of the single server, that is, the single hot seat, $\mathrm{M/M/1}$ queuing model can be useful for analysing the TV show KBC. M stands for Markovian arrival time and Markovian service time distributions. The service time for one customer is random and therefore, the arrival time is also random.

$\mathbf{M/M/1}$ Queuing Model: In this model, the customers are served by the single server i.e. the hot seat. The maximum number of customers in the system is 10. This model is the single server model with finite limit N. N is the maximum number of customers in the system (maximum queue length $= \mathrm{N} - 1$). In the context of KBC, $\mathrm{N} = 10$ during each week. That means, there are 10 customers to be served by the hot seat in the 3 episodes of one week. We assume that, the arrival rate is λ customers per unit time and the service rate is μ customers per unit time.

During three episodes of KBC in one week, if 10 contestants are served, then no more contestants are allowed. That is, when the number of customers reaches N, no more customers are allowed in the system. Thus, we have

$$\lambda_n = \begin{cases} \lambda, & n = 0,1,2,\dots, N-1 \\ 0, & n = N, N+1, \dots\dots \end{cases}.$$

$$\mu_n = \mu, n = 0,1,2,\dots\dots$$

Setting $\rho = \frac{\lambda}{\mu}$, the probability of n customers in the system is

$$p_n = \begin{cases} \rho^n p_0, & n \le N \\ 0, & n > N \end{cases}.$$

The value of p_0, the probability of 0 customers in the system, is determined by the equation,

$\sum_{n=0}^{\infty} p_n = 1$, which gives

$$p_0(1 + \rho + \rho^2 + \cdots. + \rho^N) = 1.$$

$$\therefore\ p_0 = \begin{cases} \frac{1-\rho}{1-\rho^{N+1}}, & \rho \ne 1 \\ \frac{1}{N+1}, & \rho = 1 \end{cases}.$$

Hence, the probability of n customers in the system is

$$p_n = \begin{cases} \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \ne 1 \\ \frac{1}{N+1}, & \rho = 1 \end{cases} \quad n = 0,1,\dots, N.$$

The value of $\rho = \frac{\lambda}{\mu}$ need not be less than 1 in this case, as the number of customers has limit N. In the context of KBC, we take $\rho = 1$.

The expected number of customers in the system is determined by

$$L_s = \sum_{n=0}^{N} np_n$$
$$= p_0(\rho + 2\rho^2 + 3\rho^3 + \cdots + N\rho^N)$$
$$= p_0(1 + 2 + 3 + \cdots + N)$$
$$= \frac{1}{N+1}\left[\frac{N(N+1)}{2}\right]$$
$$\therefore\ L_s = \frac{N}{2}.$$

This means that, the expected number of customers (contestants) in the three episodes of KBC during each week is,

$$L_s = \frac{10}{2} = 5.$$

The relationship between L_s and W_s (also$L_q$and W_q) is known as Little's formula and is given as

$$L_s = \lambda_{eff} W_s \qquad \text{and} \qquad L_q = \lambda_{eff} W_q.$$

The parameter λ_{eff} is the effective arrival rate at the system. When all arriving customers are allowed in the system, it is equal to the arrival rate λ.If some customers can not be allowed in the system, then $\lambda_{eff} < \lambda$. In this case, $\lambda_{lost} = \lambda - \lambda_{eff}$ is the rate at which, the customers are lost i.e. can not be allowed in the system. So, an arriving customer may enter the system or will be lost with rates λ_{eff} or λ_{lost}, which means that,

$$\lambda = \lambda_{eff} + \lambda_{lost}.$$

A customer will be lost from the system, if N customers are already in the system. This means that, the proportion of customers that will be lost from the system is p_N. Thus,

$$\lambda_{lost} = \lambda p_N.$$
$$\therefore\ \lambda_{eff} = \lambda - \lambda_{lost} = \lambda - \lambda p_N = \lambda(1 - p_N).$$

In the context of KBC, $\lambda_{eff} = \lambda\left(1 - \frac{1}{11}\right) = \left(\frac{10}{11}\right)\lambda = \left(\frac{10}{11}\right)\mu$.

$$\therefore\ \lambda_{eff} < \mu.$$

Obviously, $\lambda_{eff} < \lambda$, since $\lambda = \mu$.

By definition, $\begin{pmatrix}\text{Expected waiting}\\ \text{time in system}\end{pmatrix} = \begin{pmatrix}\text{Expected waiting}\\ \text{time in queue}\end{pmatrix} + \begin{pmatrix}\text{Expected service}\\ \text{time}\end{pmatrix}$.

$$\therefore\ W_s = W_q + \frac{1}{\mu}.$$

Multiplying this formula by λ_{eff}, we can determine relationship between L_s and L_q, using Little's formula, that is

$$\lambda_{eff} W_s = \lambda_{eff} W_q + \frac{\lambda_{eff}}{\mu}$$
$$\therefore\ L_s = L_q + \frac{\lambda_{eff}}{\mu}.$$

In the context of KBC, $L_q = L_s - \frac{\lambda_{eff}}{\mu}$

$$= L_s - \frac{\left(\frac{10}{11}\right)\mu}{\mu}$$
$$= 5 - \frac{10}{11}$$
$$\therefore\ L_q = 4.09.$$

That is, the expected number of customers (or contestants) in the queue are 4.09. That means that, 4 contestants are waiting in the queue actually, in a week.

Sample Space: During three episodes of KBC, everytime onecontestant is chosen from the group of 10 contestants. We name these 10 contestants as $C_1, C_2, \ldots\ldots, C_{10}$. Any one contestant among these is chosen through the contest of FFF. In this event, the sample space is $S = \{C_1, C_2, \ldots.., C_{10}\}$. In this event, the probability of each contestant to be selected is same and is

$$P(C_i) = \frac{1}{11}, \ i = 1,2, \ldots\ldots,10.$$

Therefore, the expectation of contestants or the expected number of contestants in each week is

$$E(\text{contestants}) = \sum_{i=1}^{10} iP(C_i) = 1\left(\frac{1}{11}\right) + 2\left(\frac{1}{11}\right) + \cdots.. + 10\left(\frac{1}{11}\right)$$
$$= \frac{1}{11}(1 + 2 + \cdots.. + 10)$$
$$= \frac{1}{11}\left(\frac{10 \times 11}{2}\right) = \frac{10}{2} = 5.$$

II. Calculation of probabilities:

On a first day of a week i.e. on Friday, the probability of each contestant to be selected is $\frac{1}{N+1} = \frac{1}{11}$ and the expected number of contestants to be served by the hot seat is 5 out of 10 contestants during a week. We assume that, on average 2 contestants are served by the hot seat on day one.

On a second day of the week i.e. on Saturday, the probability of each contestant to be selected is $\frac{1}{N+1} = \frac{1}{9}$ because there are 8 contestants remaining on day two. Therefore, the expected number of contestants to be served on second day of the week is

$$\begin{aligned} \mathrm{E(contestants)} &= \textstyle\sum_{i=1}^{8} iP(C_i) = 1\left(\frac{1}{9}\right) + 2\left(\frac{1}{9}\right) + \cdots..+8\left(\frac{1}{9}\right) \\ &= \frac{1}{9}(1 + 2 + \cdots..+8) \\ &= \frac{1}{9}\left(\frac{8\times 9}{2}\right) = \frac{8}{2} = 4. \end{aligned}$$

Again, we assume that, on average 2 contestants are served by the hot seat on day two. On a third day of the week i.e. on Sunday, the probability of each contestant to be selected is $\frac{1}{N+1} = \frac{1}{7}$ because there are 6 contestants remaining on day three. Therefore, the expected number of contestants to be served on third day of the week is

$$\begin{aligned} \mathrm{E(contestants)} &= \textstyle\sum_{i=1}^{6} iP(C_i) = 1\left(\frac{1}{7}\right) + 2\left(\frac{1}{7}\right) + \cdots..+6\left(\frac{1}{7}\right) \\ &= \frac{1}{7}(1 + 2 + \cdots..+6) \\ &= \frac{1}{7}\left(\frac{6\times 7}{2}\right) = \frac{6}{2} = 3. \end{aligned}$$

III. Conclusion

We conclude that, theprobability of each contestant on first day is $\frac{1}{11}$ and the expectation is 5 contestants for a week of three days. The probability of each contestant on second day is $\frac{1}{9}$ and the expectation is 4 contestants for two days of a week. The probability of each contestant on third day is $\frac{1}{7}$ and the expectation is 3 contestants for one day of a week. So, we can say that, as the probability on each day increases, the expectation also increases.

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