

Higher Order Vagueness(Corrigendum¹)

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Abstract A discussion on this so debatable and debated concept, Higher Order Vagueness. After going through the issues that make Higher Order Vagueness an object of scientific attention, we conclude that vagueness is undesirable in logical systems but desirable in language, logical systems possess three levels of reasoning or existence, and Higher Order Vagueness could as well be called Lower Order Vagueness.

Keywords Vagueness, Higher order, Logic, Language, Sorites

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I. Introduction

What is a concept?

A concept is a set of two elements: sigmatoid (Pinheiro 2015, p. 19), and world reference. A sigmatoid is a string of unspaced symbols.

Xxxooo is a sigmatoid.

Xx is a sigmatoid. *Xooo* is a sigmatoid as well. Therefore *xx xooo* is one sigmatoid followed by another. We use sigmatoids to point at world references, and those can be material or immaterial.

Material world references are things such as what we identify in the world as being a bottle when we say *bottle of water*.

Immaterial world references are things such as what we identify in our imagination as being sound when we say *the sound of silence*.

Higher Order Vagueness, three sigmatoids that are found together, like this, in discussions involving Paradoxes of Language, such as The Sorites (Pinheiro 2016a), is what we call concept when used in that context, of the paradoxes of language, because both the expression, which is the three sigmatoids together, and the immaterial world reference, which is the idea of blurred line and talk about the own talk, must be in our Inner Reality (Pinheiro 2016b, p. 68) as a whole for us to understand the content of discourse.

While *vagueness* (Pinheiro 2017b, pp. 1-4) came about as an ideal token to explain the shock we get when confronted with paradoxes of language, such as The Sorites, *higher order* just seems wrong, out of place.

In this paper, we will prove that it is indeed wrong and out of place: lower order could also do the trick.

It is probably better saying *indiscernibility of boundaries in the application of the logical system due to indeterminacy inherited from its metalanguage axioms*.

We could also say *logical systems with identified gap* (Pinheiro 2013, para. 1) *and/or glut* (Pinheiro 2012, para. 1) *area*.

The point that made us follow a different path from that of Priest and Hyde in 2000 was exactly this: we thought that progress was a machine becoming a human being, but they thought that progress was a human being becoming a machine (Pinheiro 2016c, para. 12). If a human being becomes a machine, everything they say is computable, and therefore can be perfectly translated into a logical system, its entries or results. If a machine becomes a human being, it can do things we have not programmed it to do, and some of them might be irrational or illogical or impossible to be translated into a logical system. If there are never gaps or gluts in the application of the metalanguage, a human being has been translated into a logical system, its entries or results with perfection, and therefore has become a machine.

We will first assume that is doable, and go with that to an extreme.

What are the components of the metalanguage of a logical system? How can we fix Higher Order Vagueness (HOV)?

We try to have the lowest number of axioms as possible, so that we are after the *smallest generator* or the *smallest generation set*.

The metalanguage could tell the computer that an atom (Yamamoto 1991, p. 73), in the logical system we have, is considered to be true if and only if its attribution is 3, and it is considered to be false otherwise.

Now we are given an atom that has attribution 2.999. Regardless of what the metalanguage is saying, we know that we round 2.999, and it becomes 3.

Since the metalanguage does not mention what to do in this case, we are left *in the gap*: can we round 2.999 up, and tag the atom as true or it is false? That is when we say that we are facing The Problem of Higher Order Vagueness.

If we think about the concept of god, Catholic god, and therefore the world reference *creator of all*, we can think that god is the ² *generator of our logical system*, so that, if there is a mistake with the *thinking of god*, there is a mistake of the *higher order* or the order above us. Would the generator of a logical system be inside of the compiler (Englopedia 2022)? The text editor and the programming language we type in it is more human, and therefore is considered to belong to a higher level when compared to the compiler (Venkat 2022). The logical system itself is described in the compiler, since there the computer puts all text inputs as entries, interprets them to the machine level, and applies the axioms, and rules of the system at hand. In this paper, we will discuss this idea further.

II. Development

The reason behind the creation of a logical system has to be making the machines interact with us in a different way, so that the logical system is their *persona*, thinking here that we could define a Mechanical Persona (in analogy with the Human Persona (Pinheiro 2017, pp. 21-24), which could be regarded as part of the set of the Inhuman Personae).

A mechanism created by us has to be inferior to us because it is created according to our thinking, and because of our actions, similar to the way things are between humans and god.

In this sense, writing the metalanguage is explaining how things work to the machine, but the closer we go to the human language, the higher we go in orders in Computer Science (Computer Science GCSE GURU 2018, para. 1-11).

Since Logic is either philosophical or computational and every logical system in the sense of Graham Priest (NCL) bears no completeness or soundness issues (Priest 2000), and therefore is fully computational, we are going closer to the machine when translating our thinking into what they can understand or metalanguage, and therefore we are going lower in Computer Science. Another way to see this is acknowledging that the metalanguage is passed to the computer via machine language, say C++ (cplusplus.com 2022), but the system goes in more human language, say Windows.

Because it cannot be the case that we refer to the purely philosophical logic when putting logical systems together, the issues we find when analysing The Sorites Paradox have to do with lower order vagueness, not higher.

Maybe our thinking processes go higher in levels of complexity (Pinheiro 2015b, p. 136) when trying to explain our logical system to others or to a machine, but we ourselves then suffer from higher order problems, not the logical system under analysis. In this case, when we say HOV, we could be right only if referring to Philosophy of Mind or Psychology or something of the type.

In order to correctly decide on how to refer to the issues we here mention, it is worth assessing some extracts of texts found in scientific journals:

1) In (Hu 2017, p. 1)

“all vague predicates are higher-order vague.”

>>If we talk about higher-order vagueness of predicates, then we talk about language, and that means that we are inside of Philosophy of Language or Logic, but not Philosophy of Mind or Psychology or the alike.

2) In (Burgess 1990, p. 427):

The notion of *absolute* definiteness I have just introduced cannot always be thought of as a concept expressible in the semantic theory in which we discuss a vague language. Rather, it is a concept which belongs in the metalanguage in which we are discussing that semantic theory; but one which includes the language in which that theory is couched. If higher-order vagueness terminates at some finite or infinite order, then ‘absolutely definitely A’ is equivalent to ‘A’ preceded by the minimum number of definitely-operators needed to reach the point beyond which further iteration yields nothing new. (If the order of termination is infinite, then, obviously some of those definitely-operators will need to be ‘strong’ operators.³³)

>>Here we see order as level of operation in a string of operations or level of the function in a string of functions: it is the same sense we have when we talk about the fourth derivative of a function. That is the

derivative of the third order derivative, which is the derivative of the second, which is the derivative of the first (Houghton Mifflin Harcourt 2018, para. 1-2).

We could say that we have the first order definition of truth, then the second order, which would be the definition of the truth of that definition of truth, and so on.

The problem here is that the metalanguage stops all on the second order: all else seems to be misunderstanding in the mind of the thinker.

For each logical system, we only have three levels of reasoning or existence (Cambiucci 2008, para. 1-25): the *metalevel* (where we tell the machine what to do to cope with our logical system), the *system level* (where we describe the system itself, our particular view of things), and the *application level* (where we try to use the logical system to handle the issues we have at hand, so say solving a problem in Mathematics, the program MAPLE).

If we are not applying the system, we only have two universes where we could be stepping: metalevel or system itself. We could then think, as Bobzien (Bobzien & Keefe 2015) points out, that we talk about vagueness of **vagueness of bald**, so second hierarchical order of vagueness, or unmentioned vagueness of the elements that form the basis of the metalanguage. The problem is that, in this case, the metalanguage would be either one level above or one level below the own language of the logical system, and that is a mistake in the definition, which makes the system unacceptable, and therefore that is a mistake that makes it be a non-system.

We could never have third order vagueness in this case, since we should have realized the mistake and simply deleted the system, or fixed it/its metalanguage axioms, on the second level of vagueness.

We do not mathematicize language: whatever is vague should remain outside of the scope of Computer Science. Vagueness is not regression in linguistic knowledge; vagueness is a progression instead, since it allows us to do more with the sigmatoids than we have done before (mouse from the past -> an animal versus mouse from nowadays -> an animal or a computer item).

We always prefer, if we have only 10 dollars, buying something that can do lots of things (Harper College 2018, para. 1-40) instead of something that does just one thing: say we have a machine that is fax, photocopier, printer, and scanner (Nashuatec Global 2018, para. 1-2) for 10 dollars and another that is only fax (Officeworks 2018, para. 1-8).

Both do the same jobs in the same way when it comes to the fax option. Which one are we going to buy?

We are lazy: we prefer learning one sigmatoid that does the job in ten situations to learning ten sigmatoids that do different jobs in different situations.

Optimisation is a universal tool: we all prefer things that have been through optimisation processes, so that everyone prefers the 2i (Drive.com.au 2018) to 2 in cars or the four wheel traction to two or one.

Vagueness is evolution because we seek comfort, pleasure, and efficiency (Pinheiro 2016c, p. 2).

Whatever is machine-like, we decided a long time ago, is lower-level of reasoning or utility, and whatever is vague is closer to what we, as purely human creatures, wish for.

In this case, *heap* is good for us, for our daily lives, but is bad for computers, and therefore should be avoided in the same sense that driving is good for responsible adults but can be a hazard for kids.

Let's keep vague vague and therefore agree that whatever is vague has got multiple world references, and we could then even talk about Worlds of Discourse.

Sigmatoids that bear non-null vagueness score are definitely not suitable for use with logical systems because there will always be confusion that the computer cannot deal with originating from that inclusion.

Consider The World of Mathematics. Select the sigmatoid *function*.

We have at least real and complex functions.

If we are not aware of the vagueness involved in the match (function; its world reference), we may find only the real intersects of our function, and deal with those as if they were all the zeroes available.

That may lead to decrease in our marks at school. That may be something we cannot deal with.

The user might think that their machine can deal with complex numbers. After entering the function in the computer, the computer crashes.

Only what is perfectly defined, so only what has got vagueness zero, so say 2, could be part of a logical system (Pinheiro 2015, pp. 19-22).

3) In (Hyde 1994, p. 37):

So, if the presence of a penumbral region is taken as definitive of vagueness then it is not itself characterised merely by the existence of border cases; there must be border cases of border cases. But why stop here? There appears to be no more reason to suppose that there is a sharp boundary between the determinately red and the vaguely determinately red than there was to suppose a sharp boundary between the determinately red and the vaguely red. “At no point does it seem natural to call a halt to the increasing orders of vagueness” (Fine 1975, p. 297), so the iteration seems endless: border cases echo up through the hierarchy.

The real lesson of higher-order vagueness is that vague predicates *draw no apparent sharp boundaries*, not merely that they apparently fail to draw a sharp boundary at the first level, or the first and second levels, or ...

After having said all of the above one might think, like Sainsbury, that the iterative conception of vagueness is both inescapable on the paradigmatic approach to vagueness and misguided, thus motivating a search for an entirely new approach which avoids the so-called “problem of higher-order vagueness” (Sainsbury 1991, p. 179). Alternatively, one might think that the iterative conception, with qualifications *ad infinitum*, constitutes an adequate reply. These two responses presuppose there to be a real problem to be addressed.

I want to argue that the iterative conception captures a feature of vagueness that is real enough—the phenomenon of higher orders of vagueness—but that this phenomenon is ultimately an echo of a more basic feature of border cases. There is no real problem. Recourse to an infinite string of qualifications, like those above, betrays an ignorance of the ambiguity of “border case”.

To make it easier, say that

$P(\text{yellow}) = 1$ if hue = 0.15

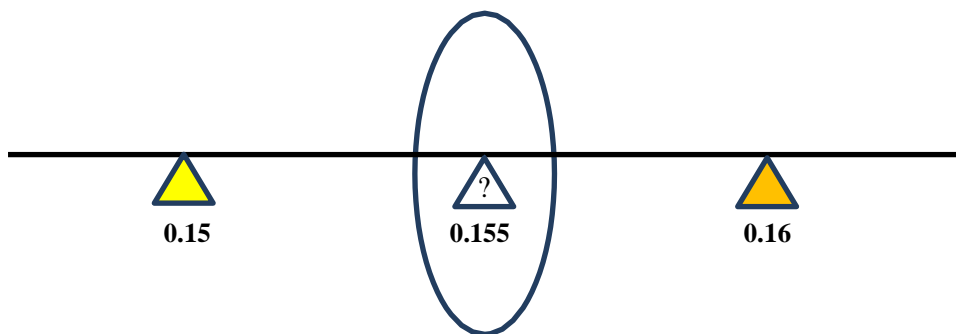
$P(\text{orange}) = 1$ if hue = 0.16

$P(\text{yellow}) = 0$ if hue = 0.16

$P(\text{orange}) = 0$ if hue = 0.15

The yellow set contains a borderline case: 0.155 is the same as 0.15 if we can truncate (Third Space Learning 2022) instead of rounding

The orange set contains a borderline case: 0.155 is the same as 0.16 if we can round (Varsity Tutors 2022) instead of truncating



Detailing the figures we have further would be going down on the interval of detail: we go 0.005 up from 0.15, and 0.005 down from 0.16, but we go from 0.01 to 0.005 of distance between components, and therefore we go lower on this distance. That is to solve the problem with this particular vagueness: we then say, in the metalanguage, that we round up whenever we have a 5 followed by something that is 5 or more. In this case, ?, which is hue = 0.155, is orange.

We had 10^{-2} , and now we have 10^{-3} in the detail, and therefore we solved the problem of vagueness by lowering our level of detail. We can then say we had a problem of vagueness with the higher order of detail, but now we don't have the problem anymore because we used a lower order for the detail. Yet the problem is also with the lower order of detail not being mentioned in the metalanguage, so that we can say it is a lower order vagueness problem instead. As seen in (Lutkevich 2022), we are decreasing the order of magnitude of our detail to solve the problem in the metalanguage, so that we had a lower magnitude problem or a lower order problem, thus a problem of lower order vagueness to solve.

In derivatives, a second order derivative measures the variation of the first order derivative. A third order derivative measures the variation of the second order derivative. We could see 0.155 as a new operation over the previous numbers, 0.15, and 0.16, say average, and see 0.15 as the average between 0.1, and 0.2. Operation over operation, then including 0.155 in the metalanguage solved the problem, and therefore the problem existed when we had a higher order of average, and it got solved when we introduced a lower order of average to our metalanguage. It seems that the problem existed because there was vagueness in the lower order while the lower order was not introduced in the metalanguage, though.

We can then say that vagueness is the quality that means border cases between the positive and the negative applications of a sigmatoid to world objects, and there is a lower order vagueness problem in The Sorites.

III. Conclusions

The phenomenon of Higher Order Vagueness could as well be called Lower Order Vagueness instead: it happens in the lower levels of the conversations between computers and human beings, and also in the lower levels of the possible differences between positive and negative applications of a predicate.

Vagueness is not an unwanted occurrence when it comes to purely human language, since it provides us with more freedom and fulfillment, but it is an unwanted occurrence when it comes to logical systems, since we really do not want computers or human beings who use those to face the consequences of the confusion caused by those terms: impossible computations, misleading results, and so on.

The Linguistic Universe is divided into Worlds of Discourse, so that we have the World of Mathematics, the World of Religion, and therefore what could be called Big Worlds, and the World of Arithmetic, the World of Algebra, the World of Catholicism, and therefore what could be called Small Worlds.

Some sigmatoids might have a single reference in a particular world, say the World of Arithmetic, and more than one reference in another world, say the Linguistic World.

Vagueness does come by degrees, and we'd better have the meter on zero when we talk about sigmatoids that are part of logical systems.

We should now establish a better way to describe the relationship between signs and our world: (sigmatoid; world reference; world of pertinence) seems to be a first step in that direction.

Vagueness is not a regression in human knowledge: it is a progression instead, since we are increasing utility/application, which is something seen as progress in all other areas (multifunction machines are preferred everywhere on earth to single-function ones).

For each logical system, we have only three levels of reasoning or existence: the metalevel (where we tell the machine what to do to cope with our logical system), the system level (where we describe the system itself, our particular view of things), and the application level (where we try to use the logical system to handle the issues we have at hand, so say solving a problem in Mathematics, the program MAPLE).

We could never have third order vagueness in this case, since we must realize the mistake, and simply delete the system, or fix it/its metalanguage axioms by at most the second level of vagueness.

We do not mathematicize language: whatever is vague should remain outside of the scope of Computer Science.

The metalanguage is closer than the logical system to the machine language. In this case, it has to be more objective than the language of the logical system. If we use MAPLE to operate in the World of Mathematics, then we cannot have *definitely true* in the metalanguage of MAPLE, since that is vaguer than *belongs to*. We would at most have *definitely true* means *belongs to*, which finishes with the vagueness contained in *definitely true*.

A question of the type *how true is your definition of truth* is a question about the metalanguage, not the logical system, so that it must be a question of *lower order*, not higher.

1 This article corrects the article Higher Order Vagueness, published by the International Journal of Advances in Philosophy in 2018, 2(2): 38-43 DOI: 10.5923/j.ap.20180202.02

REFERENCES

- [1]. Pinheiro, M. R. 2015, 'Words for Science', Indian Journal of Applied Research, vol. 5, no. 5, pp. 19-22, <https://www.researchgate.net/publication/287206279_Words_for_Science>
- [2]. Pinheiro, M. R. 2016a, The Sorites was upon a Time and it is now Over, CreateSpace.
- [3]. Pinheiro, M. R. 2016b, 'Possible Worlds x Psychiatry', International Journal of Humanities and Social Science Invention, vol. 5, no. 10, pp. 68-76, viewed 04 August 2018, <https://www.researchgate.net/publication/308968450_Possible_Worlds_x_Psychiatry>
- [4]. Pinheiro, M. R. 2013, The Words from the Gap, viewed 04 August 2018, <<https://www.proz.com/translation-articles/articles/3947/1/The-words-from-The-Gap>>

- [5]. Pinheiro, M. R. 2012, What about the Gluts?, viewed 04 August 2018, <<https://www.proz.com/translation-articles/articles/3610/1/WHAT-ABOUT-THE-GLUTS%3F>>
- [6]. Pinheiro, M. 2017, 'Human Persona', *International Journal of Advances in Philosophy*, vol. 1, no. 2, pp. 21-24, viewed 03 June 2018, <https://www.researchgate.net/publication/318038905_Human_Persona>
- [7]. Priest, G. 2000, *Introduction to non-classical logic: moving about in worlds not realized*, Cambridge University Press, Cambridge.
- [8]. Pinheiro, M. R. 2015b, 'Translation Techniques', vol. May, no. 4, pp. 121-144, viewed 04 August 2018, <https://www.researchgate.net/publication/286165517_Translation_Techniques>
- [9]. Pinheiro, M. R. 2016a, *The Sorites was upon a Time and it is now Over*, CreateSpace, New York.
- [10]. Pinheiro, M. R. 2016c, *The Intrinsic Vagueness of Being*, viewed 04 August 2018, <<https://drmarciapineiro.wordpress.com/2016/09/19/the-intrinsic-vagueness-of-being/>>
- [11]. Computer Science GCSE GURU 2018, *High and Low Level Languages*, viewed 04 August 2018, <<https://www.computerscience.gcse.guru/theory/high-low-level-languages>>
- [12]. Clarke, E. M. 2018, *Lecture 1: Propositional Logic, Lecture Slides and Readings, Bug Catching 15-414*, Carnegie Mellon, delivered 28 July 2018, viewed 28 July 2018, <https://www.cs.cmu.edu/~emc/15414-f12/lecture/propositional_logic.pdf>
- [13]. Hu, I. 2017, 'Vague' at Higher Orders', *Mind*, vol. 126, no. 504, pp. 1189-1216, <<https://doi.org/10.1093/mind/fzw046>>
- [14]. Burgess, J. A. 1990, 'The Sorites Paradox and Higher-Order Vagueness', *Synthese*, vol. 85, no. 3, pp. 417-474, <<https://doi-org.simsrad.net.ocs.mq.edu.au/10.1007/BF00484836>>
- [15]. Houghton Mifflin Harcourt 2018, *Higher Order Derivative*, viewed 04 August 2018, <<https://www.cliffsnotes.com/study-guides/calculus/calculus/the-derivative/higher-order-derivatives>>
- [16]. Cambiucci, W. 2008, *Arquitetura de Soluções*, viewed 05 August 2018, <<https://blogs.msdn.microsoft.com/wcamb/2008/01/13/gramticas-linguagens-e-compiladores-um-retorno-s-bases/>>
- [17]. Bobzien, S. & Keefe, R. 2015, 'Vagueness', *Proceedings of the Aristotelian Society Supplementary Volume lxxxix*, doi:10.1111/j.1467-8349.2015:00244:x
- [18]. Harper College 2018, 6a – Customer Decisions: Utility Maximization, *Lectures*, viewed 05 August 2018, <<http://www2.harpercollege.edu/mhealy/eco211/lectures/utilmax/util.htm>>
- [19]. Nashuatec Global 2018, *Product Comparison Results, Product Compare Result*, viewed 05 August 2018, <<http://www.nashuatec.com/products/product-compare-result.aspx?product1=41634&product2=44200>>
- [20]. Officeworks 2018, *Brother Mono Laser Fax Machine FAX-2840*, Officeworks, viewed 05 August 2018, <<https://www.officeworks.com.au/shop/officeworks/p/brother-mono-laser-fax-machine-fax-2840-brfax2840>>
- [21]. Drive.com.au 2018, *2017 Subaru XV 2.0i-P new car review*, Drive, viewed 04 August 2018, <<https://www.drive.com.au/new-car-reviews/2017-subaru-xv-2-0i-p-road-test-review-65015>>
- [22]. Pinheiro, M. R. 2017b, 'The Intrinsic Vagueness of Language', *International Journal of Advances in Philosophy*, vol. 1, no. 1, pp. 1-4, viewed 04 August 2018, <https://www.researchgate.net/publication/308356437_The_Intrinsic_Vagueness_of_Being>
- [23]. Hyde, D. 1994, 'Why High-Order Vagueness is a Pseudo-Problem', *Mind*, vol. 103, no. 409, pp. 35-41, <<https://www.jstor.org/stable/pdf/2253957.pdf?refreqid=excelsior%3A773c85e70119b86102d655f92c22d56f>>
- [24]. Yamamoto, A. 1991, 'Elementary Formal System as a Logic Programming Language', in H. Tanaka, T. Fujisaki & K. Furukawa, *Logic Programming '89: Proceedings of the 8th Conference, 12-14 Jul, 1989, Volume 8, Part 1989*, Springer Science & Business Media, Tokyo, pp. 73-86.
- [25]. Lutkevich, B. 2022, *order of magnitude*, WhatIs.com, viewed 13 August 2022, <<https://www.techtarget.com/whatis/definition/order-of-magnitude>>
- [26]. cplusplus.com 2022, *Compilers*, viewed 13 August 2022, <<https://cplusplus.com/doc/tutorial/introduction/>>
- [27]. Englopedia 2022, *Metalanguage definition characteristics type usage*, viewed 14 August 2022, <<https://englopedia.com/metalanguage-definition/>>
- [28]. Venkat 2022, *Low Level Programming Languages*, viewed 14 August 2022, <<https://sillycodes.com/low-level-programming-languages/>>
- [29]. Varsity Tutors 2022, *Rounding Numbers*, viewed 20 August 2022, <https://www.varsitytutors.com/hotmath/hotmath_help/topics/rounding-numbers>
- [30]. Third Space Learning 2022, *Truncation*, viewed 20 August 2022, <<https://thirdspacelearning.com/gcse-maths/number/truncation/>>