Abstract: Due to diversification, the value at risk (VaR) of a portfolio will be less than or equal to the sum of the VaRs of the positions in the portfolio. Diversified VaR is simply the VaR of the portfolio where the calculation takes into account the diversification effects. Paper under study includes three asset class i.e. Equity (BSE Sensex), Gold (Goldman Sachs Gold Exchange Traded Scheme) and Agri Commodity Index (Agri Index Dhaanya) and calculates Individual VaR and shows the effects of diversification in reducing portfolio VaR. Based on Secondary data. Five year publicly available data in terms of daily closing prices (1-07-2008 to 30-06-2013) of the above mentioned assets classes are taken for the study. There are in all 1240 data points and it conforms to normal distribution (bell shaped curve). Each asset class standard deviation, Individual VaR at 95% and 99% confidence level, Correlation among (Sensex & Gold) (Sensex & Agri), Portfolio standard deviation (Sensex & Gold) (Sensex & Agri), Portfolio VaR (Sensex & Gold) (Sensex & Agri) at 95% and 99% are calculated. When considered each asset class in isolation Individual VaR tends to be higher than compared with portfolio VaR consisting of (Sensex & Gold) or (Sensex & Agri). The main reason is from the benefits of negative correlation between Sensex return & Gold return and Sensex return & Agri commodity Index return for the period under study. Zero or a negative correlation among securities/assets class can help in reducing the existing VaR of the portfolio. This advantage of VaR can help in the portfolio allocation process in selecting asset class/securities.

INTRODUCTION
Managing risk is an essential activity for all financial institution focuses on commercial or investment division. In order to manage risk, financial institutions must first measure their exposure to risk. Often a financial institution’s portfolio depends on hundreds or even thousands of market variables. A huge number of these variables for measuring risk are produced each day. While very useful to traders, these risk measure does not provide senior management and individuals that regulate financial institutions with a measure of the total risk to which a financial institution is exposed. Value at risk (VaR) is an attempt to provide a single number that summarizes the total risk in a portfolio. It was pioneered by JP Morgan and has become widely used by corporate treasurers and fund managers as well as financial institutions.

DEFINITION OF VaR
Value at risk (VaR) is a probabilistic method of measuring the potential loss in portfolio value over a given time period and for a given distribution of historical returns. VaR is the dollar or percentage loss in portfolio (asset) that will be equaled or exceeded only X percent of the time. In other words, there is an X percent probability that the loss in portfolio value will be equal to or greater than the VaR measure. Calculating VaR requires assumption that the asset returns conform to a standard normal distribution. Standard normal distribution is defined by two parameters, its mean (µ = 0) and standard deviation = 1, and is perfectly normal.
 VaR mathematically is defined as: \( \text{VaR (X \%)} = (Z_{x\%} \times \sigma) \times \text{asset value in rupee terms} \) 
Where \( Z_{x\%} \) = the critical z-value based on the normal distribution and the selected X\% probability, 
\( \sigma \) = the standard deviation of daily returns on a percentage basis.

OBJECTIVE OF THE STUDY
Due to diversification, the value at risk (VaR) of a portfolio will be less than or equal to the sum of the VaRs of the positions in the portfolio. Diversified VaR is simply the VaR of the portfolio where the calculation takes into account the diversification effects. Basic formula is 
\[ \text{VaR}_p = Z_c \times \sigma \times P \]
Where \( Z_c \) = the z-score associated with the level of confidence c 
\( \sigma = \) standard deviation for two asset portfolio is 
\[ \sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \rho_{1,2}} \]
\( P = \) the nominal value invested in the portfolio

We can square \( Z_c \) and \( P \) and put them under the square root sign. This allows us to express \( \text{VaR}_p \) as a function of the VaRs of the individual positions, which we express as \( \text{VaR}_i \) for each position \( i \). For a two-asset portfolio we have \( \text{VaR}_1 \) and \( \text{VaR}_2 \). If the correlation is Zero, \( \rho_{1,2} = 0 \), then the third term under the radical is zero and VaR for uncorrelated positions will be 
\[ \text{VaR for uncorrelated positions} = \text{VaR}_p = \text{VaR}_1^2 + \text{VaR}_2^2 \]

The other extreme is when the correlation is one, \( \rho_{1,2} = 1 \). If the correlation equals one, then there is no benefit from diversification. For the two-asset portfolio, Undiversified VaR will be 
\[ \text{Undiversified VaR} = \text{VaR}_p = \text{VaR}_1^2 + \text{VaR}_2^2 + 2 \text{VaR}_1 \text{VaR}_2 = \text{VaR}_1 + \text{VaR}_2 \]

Evaluating VaR using both a correlation of zero and a correlation of one will place a lower and upper bound on the portfolio VaR. Total VaR will be less if the positions are uncorrelated and greater if the positions are correlated.

Paper under study tests the above concept of diversified portfolio VaR by analyzing return data of three asset classes namely:
1. Equity (BSE Sensex) 
2. Gold (Goldman Sachs Gold Exchange Traded Scheme) 
3. Agri Commodity (Agri Index Dhaanya)

RESEARCH METHODOLOGY
Study is based on Secondary data. Five year publicly available data is taken in terms of daily closing prices form 1-07-2008 to 30-06-2013 for the three asset classes under study. Data points for BSE Sensex (Equity) and Goldman Sachs Gold exchange scheme (Gold) comes to 1240 trading day but since commodity markets trade on Saturday, data points comes to 1492. To make data point’s comparable, assumption is included for Agri Commodity Index i.e. Saturday index closing is taken as Friday closing. This assumption will not impact return calculation because the volumes are generally muted on Saturday in commodity markets and data points confirms to 1240.

Daily return’s data for three asset classes under study conforms to normal distribution; tool being used SPSS version 16.0. Calculation of VaR requires the assumption of data confirms to normal distribution, for e.g. in calculating a daily VaR we calculate the standard deviation of daily returns in the past and assume it will be applicable in future.
### Statistics

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Gold</th>
<th>Agri Commodity</th>
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<tbody>
<tr>
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<td>1240</td>
</tr>
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</tr>
<tr>
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<td>.000899</td>
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<tr>
<td>Kurtosis</td>
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<td>Std. Error of Kurtosis</td>
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<td>.139</td>
<td>.139</td>
</tr>
</tbody>
</table>

Normal Distribution Curve for three asset classes under study

**Equity**

![Equity Normal Distribution Curve](image)

**Gold**

![Gold Normal Distribution Curve](image)
One-Sample Kolmogorov-Smirnov Test

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Gold</th>
<th>Agri Commodity</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1240</td>
<td>1240</td>
<td>1240</td>
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<td>.085</td>
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<tr>
<td></td>
<td>Positive</td>
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<tr>
<td></td>
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<td>Asymp. Sig. (2-tailed)</td>
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<td>.000</td>
<td>.028</td>
</tr>
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</table>

a. Test distribution is Normal.

For each asset classes Individual VaR is calculated at 95% confidence level ($Z = 1.645$) and 99% confidence level ($Z = 2.326$). Notional Amount of Rs 10,00,000 is considered as an investment in each asset class individually. Results are as follows:

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>Amount in Rs</th>
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<tr>
<td>Equity</td>
<td>27,899</td>
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<tr>
<td>Gold</td>
<td>18,204</td>
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<tr>
<td>Agri Commodity</td>
<td>14,862</td>
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</table>

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>Amount in Rs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>39,458</td>
</tr>
<tr>
<td>Gold</td>
<td>25,747</td>
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<tr>
<td>Agri Commodity</td>
<td>21,019</td>
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</table>
The above table reveals a negative correlation of returns between Equity & Gold (-0.079), Equity & Agri commodity (-0.16). If we combine two asset classes with negative correlation as a portfolio, we can see that Diversified VaR of the portfolio will be lesser than the sum of the Individual VaR.

**FINDINGS**

To prove the concept of “Benefit of Diversification” as a result of negative correlation, let us consider an equally weighted portfolio with 50% of Investment in BSE Sensex (Equity) and 50% investment in Goldman Sachs Gold exchange traded Scheme (Gold).

The results are as follows.

\[
\sigma_p = \text{the standard deviation of the portfolio} = 0.975\% \\
P = \text{Portfolio Value} = Rs \ 20,00,000 (Rs \ 10,00,000 \text{ in Equity} + Rs \ 10,00,000 \text{ in Gold}) \\
Zc = 1.645 (95\% \text{ Confidence level}) \\
VaR_p = 1.645 \times 0.975\% \times 20,00,000 = Rs \ 32,089 \\
\text{Individual VAR (Equity) + Individual VAR (Gold)} = Rs \ 27,899 + Rs \ 18,204 = Rs \ 46,103. \\
\text{Reduction in VaR (diversification benefit)} = \text{Rs } 46,103 - \text{Rs } 32,089 = \text{Rs } 14,014.
\]

\[
Zc = 2.326 (99\% \text{ Confidence level}) \\
VaR_p = 2.326 \times 0.975\% \times 20,00,000 = Rs \ 45,384. \\
\text{Individual VAR (Equity) + Individual VAR (Gold)} = Rs \ 39,458 + Rs \ 25,747 = Rs \ 65,205. \\
\text{Reduction in VaR (diversification benefit)} = \text{Rs } 65,205 - \text{Rs } 45,384 = \text{Rs } 19,821.
\]

Let us now consider another portfolio with negative correlation with equal weights, 50% of investment in BSE Sensex (Equity) and 50% investment in Agri Index Dhaanya (Agri commodity).

The results are as follows.

\[
\sigma_p = \text{the standard deviation of the portfolio} = 0.954\% \\
P = \text{Portfolio Value} = Rs \ 20,00,000 (Rs \ 10,00,000 \text{ in Equity} + Rs \ 10,00,000 \text{ in Agri commodity}) \\
Zc = 1.645 (95\% \text{ Confidence level}) \\
VaR_p = 1.645 \times 0.954\% \times 20,00,000 = Rs \ 31,393 \\
\text{Individual VaR (Equity) + Individual VaR (Agri Commodity)} = \text{Rs } 27,899 + \text{Rs } 14,862 = \text{Rs } 42,761. \\
\text{Reduction in VaR (diversification benefit)} = \text{Rs } 42,761 - \text{Rs } 31,393 = \text{Rs } 11,368.
\]

\[
Zc = 2.326 (99\% \text{ Confidence level})
\]
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CONCLUSION

Zero or a negative correlation among assets class or securities help in reducing the existing VaR. A portfolio manager can lower a portfolio VaR by increasing allocations to the positions which have a negative correlation with the existing asset classes / securities. It also means that if the manager keeps the total invested capital constant; this would mean increasing allocations to positions with lower or negative correlation. Thus one of the various applications of VaR is the usefulness in investment process i.e. decision with respect to strategic asset allocation.

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