IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 6, Issue 1 (May. - Jun. 2013), PP 109-116
www.iosrjournals.org

Analysis of Pseudogap in Superconductors

Abdul Quader[1], Md. Ashraful Islam[2], Imran Khan[3]
[1](Department of EEE, Eastern University, Dhaka, Bangladesh)
[2](Department of CSE, Jessore Science & Technology University, Jessore, Bangladesh)
[3](Department of EEE, Jessore Science & Technology University, Jessore, Bangladesh)

Abstract : *The effects of the hole content, p, on the temperature dependent static magnetic susceptibility, $\chi(T)$, of $YBa_2Cu_3O_{7-\delta}$ superconductor were investigated over a wide range of oxygen deficiency. The systematic variation in χ (T, p) was found to be governed by the pseudogap energy scale, $E_g(p)$. From the analysis of $\chi(T,p)$ data we have extracted the values of pseudogap energy scale. The extrapolated $E_g(p)$ tends to vanish at p ~ 0.19 and the pseudogap appears to be states-non-conserving in nature. These are indicative of a non-superconducting origin for the pseudogap.*
Keywords*- magnetic susceptibility, pseudogap, superconductor.*

I. INTRODUCTION

In 1911, Onnes [1] found surprisingly that dc resistivity of mercury drops suddenly to an immeasurably small value when the temperature falls below a certain critical temperature, which he found to be 4.2 K, the so-called transition temperature of mercury. He referred this phenomenon as superconductivity. That is a material becomes superconducting below a characteristic temperature, called the superconducting transition temperature, T_c, which varies from a very small values (millidegrees or microdegrees Kelvins) to values above 100K. The material is normal above T_c, which merely means that it is not superconducting. Elements and compounds that become superconductors are conductors – but not good conductors – in their normal state. The good conductors such as copper, silver and gold do not superconduct (or only superconductive at very low temperature).

Meissner and Oschsenfeld [2] found that the magnetic inductance inside a superconductor becomes zero when it is cooled below T_c in a weak external magnetic field: the magnetic flux is expelled from the interior of the superconductor. As the dc resistivity is zero the current induced in a superconductor can exist without external source of emf and will run over two and half years without any measurable decay [3]. This is known as persistence current. The superconductivity state can also be destroyed due to excess magnetic field known as the critical field and excessive current, known as the critical current density, through the superconductor. There are many kinds of superconductors, for example, Al (T_c ~ 9 K), Pb (T_c ~ 7 K) are metallic elements; NB_3Sn (T_c ~ 18 K) is a binary compound; $La_{2-x}Sr_xCuO_4$ (T_c ~ 38 K), $YBa_2Cu_3O_7$ (T_c ~ 92 K), $Tl_2Ca_2Ba_2Cu_3O_{10}$ (T_c ~ 125 K) are High-T_c – cuprates. Materials have recently been developed that exhibit superconductivity up to much higher temperatures that anyone dared to hope. Since 1911, when Kamarlingh Onnes discovered superconductivity in mercury (Hg) at 4.2 K, the highest observed value of T_c gradually moved upwards. In 1973, J. R. Gavaler [4] observed that sputtered films of Nb_3Ge became superconducting at 22.3 K, this was soon pushed up to 23.3 K by L. R. Testardi [5] by altering the sputtering condition slightly. Inspite of great efforts to increase this limit further, it stood as the record until 1986. In that year, J. G. Bednorz and K. A. Müller observed [6] that a Lanthanum Barium Copper Oxide compound becomes superconducting as it was cooled below 35 K. This discovery opened the way for all of the subsequent research work on High-T_c superconductors.

High temperature superconductivity denotes the superconductivity in materials, chiefly copper oxides, with high transition temperatures, accompanied by high critical currents and critical magnetic fields. In 1988 the long standing 23K ceiling of T_c in inter-metallic compounds had been elevated to 125 K in bulk superconducting oxides [7]; which passed the standard tests for superconductivity – the Meissner effect, ac Josephson effect, Persistent current of long duration and zero resistivity. In the following year W. Chu [8] found that the closely related material $YBa_2Cu_3O_{7-\delta}$, known as YBCO or Y123, has a T_c of about 93 K when δ ~ 0.10, well above the boiling point of liquid nitrogen. A few best of the known cuprate superconductors are listed in Table I. Many other closely related compounds with similar transition temperatures are now known: the compound $HgBa_2Ca_2Cu_3O_{8+\delta}$ for instance has a T_c as high as 150 K [9]. YBCO is now the most widely used system.

Table I: Some Common High Temperature Cuprate Superconductors (HTCS)

Compound	Abbreviation	Critical Temperature T_c (K)
$La_{2-x}Sr_xCuO_4$	LSCO (La214)	38
$YBa_2Cu_3O_7$	YBCO (Y123)	93
$Tl_2Ba_2Ca_2Cu_3O_{10-x}$	TBCCO (Tl2223)	125

The normal and superconducting state properties of high-T_c cuprates are sensitive to doped carriers per copper oxide plane. One of the most extensively investigated phenomena in the physics of the high-T_c superconductors is the so-called pseudogap. The pseudogap correlation is observed in T – p phase diagram over a certain dopping range, extending from unerdoped to overdoped region.

The Electron Density Of States (EDOS) as a function of energy is at the heart of any problem associated with the pseudogap and the static magnetic susceptibility (χ) as function of the temperature and doping concentration gives us an opportunity to study the p-dependent phenomena of HTCS. The study of $\chi(T)$ for a wide range of hole concentration gives the information about the temperature and p-dependencies of the electronic density of states. In this study we are going to investigate the effect of pseudogap on the static magnetic susceptibility of high-T_c superconductor $YBa_2Cu_3O_{7-\delta}$.

II. PSEUDOGAP

The left and right regions of the T-p phase diagram, shown in Fig. 1 & Fig. 2, are called the underdoped and the overdoped regions, respectively. The underdoped region of the High-T_c Superconductors is characterized by the presence of a normal state correlations, referred to as the pseudogap (PG) which reveals itself below a certain characteristic temperature T^* (Fig. 1) [10-12]. The metalic state above T_c in the underdoped region has been under intense study and exhibits many unusual properties. A PG appears on the underdoped side of the phase diagram and weakens as the optimal doping is approached. A weak PG is still present at the optimal doping but disappears not too far into the overdoped region. PG is not a well defined phase in that a definite finite phase boundary has never been found. There is now broad agreement that the High-T_c problem is synonymous with that of doping of Mott insulator.

The experiments such as ARPES [13], Transport propeties [14], Electronic Raman Scatterinig [15], Nuclear Magnetic Resonance (NMR) [16] etc. have given evidence of the presence of the PG in the normal state of all the cuprates. The PG is seemed to be related to the superconductig gap and has the same d-wave symmetry. Although many measurements are inductive of a PG, even in the strongly overdoped region, there are several contradictory observations [10-12]. Resistivity and magnetic susceptibility data in the PG region exhibit anomalous suppression well above T_c [14]. A PG is comletely absent in the spin-lattice relaxation rate in the underdoped region. One possible cause of this difference is the presence of paramagnetic centers.

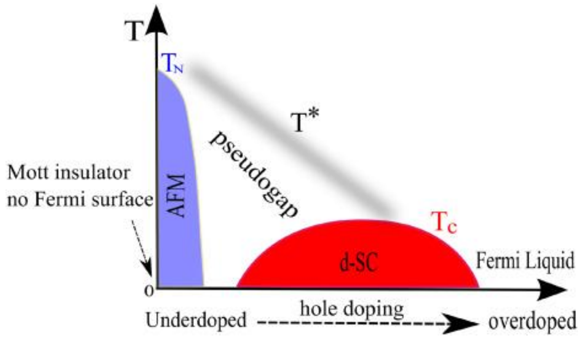


Figure 1. The boundary between the antiferromagnetically ordered state (denoted by AFM) and the d-wave superconductor (denoted by d-sc) is uncertain. The overdoped Fermi liquid has a full Fermi surface while the stoichiometric Mott insulator has a charge gap [17].

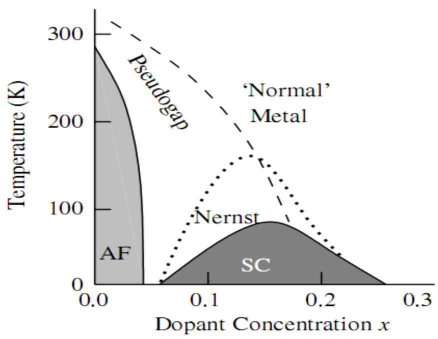


Figure 2. Schematic phase diagram of high T_c materials. The antiferromagnet (AF) is rapidly destroyed by doped holes. The d-wave superconductor is subject to strong phase fluctuations below the dotted line, where the proliferation of vortices has been detected by the Nernst effect. A PG region extends up to high temperatures in the underdoped region [18].

III. STATIC MAGNETIC SUSCEPTIBILITY

Here the experimental results of uniform static magnetic susceptibility measurements are analyzed. These were taken for different values of the oxygen deficiency, δ and consequently hole content p for Y123 given in Table II. Using these data we are going to analyze the effect of pseudogap on the static magnetic susceptibility of the sample.

The 1822 Quantum Design SQUID magnetometer was used to measure the static magnetic susceptibility, χ(T), of the sample. There were two quartz tubes of similar dimension in the system. These tubes were cleaned properly before each measurement to avoid contamination by any magnetic particles. In this measurement the sample was mounted between the quartz tubes and the whole configuration was attached to a sample probe rod. A dc magnetic field of 5 tesla was used to obtain the magnetic moment. Data were collected following a predefined sequence, usually on the range of 5 K to 400 K and for different values so as to check the linear field dependence. Here the scan length was 6 cm. The background signal was subtracted from the raw data to obtain the magnetic moment of the sample, from which susceptibility would be obtained. The total signal was actually the convolution of the two separate signals from the samples and the quartz tubes. Since the distance between the quartz tubes is exactly the same as the length of the sample, hence this subtraction technique hold good. The background signal was due to the absence of the quartz in the site of the sample which varied linearly with the separation between the two quartz tubes (up to a separation approximately 6 mm).

Table II: Hole Content, Oxygen Deficiency and Superconducting Transition Temperature for the Sample

Oxygen Deficiency, Δ (± 0.02)	Hole Concentration, P (± 0.004)*	T_c (K) (± 1)**
0.03	0.176	90.0
0.09	0.161	91.1
0.16	0.143	90.0
0.23	0.111	72.0
0.35	0.094	60.0

* From T_c(p) relation [19] and room temperature thermo power [20]
** From the zero resistivity point.

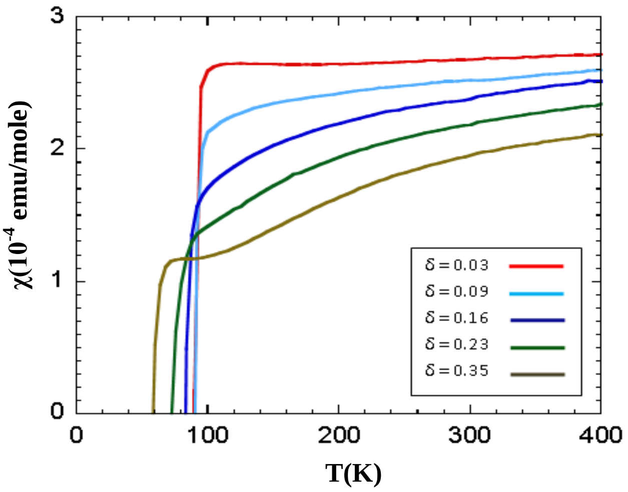


Figure 3. χ(T) data for different δ and p values as per Table II.

Fig. 3 shows the drastic effect of hole content on the static magnetic susceptibility. As p decreases χ(T) falls to systematically, while χ(T) for the slightly overdoped compound (p = 0.174) is almost T-independent in the normal state, χ(T) for the underdoped compounds develop a marked T-dependence and start falling from a doping dependent characteristic high temperature much above T_c. Such effects have been reported by earlier studies [19]. This observation is fairly universal. For example Johnston [21] reported this systematic decrease in χ(T) for LSCO and tried to explain the observed phenomenon in terms of a spin-1/2 square lattice Heisenberg antiferromagnetic model [22]. It is worth mentioning that the static susceptibility data closely resembles with the Knight shift measurements on Y123 [23].

IV. ANALYSIS

The normal state and superconducting state properties of high-T_c superconductors are very sensitive to the hole content, p, and the pseudogap (PG) is one of the most widely studied phenomena. Superconductivity is an instability of the normal state. Therefore, to understand the origin of superconductivity, one must understand the nature of the normal state from which it appears. Almost all of the normal state properties of high-T_c cuprates are dominated by the existence of the PG. In this chapter we will study the effect of the PG on χ(T).

A. X (T) and Pseudogap

The most important information that can be extracted from the analysis of χ(T) data is about the temperature and p dependences of low-energy electronic density of states (EDOS) for $YBa_2Cu_3O_{7-\delta}$. EDOS is a function of energy and at the centre of any problem associated with the pseudogap; χ(T) is indeed a powerful tool to study various p-dependent features of this phenomenon. It is in fact the intrinsic spin part (i.e., the Pauli spin susceptibility, χ_s) of χ(T) that represents the quasi-particle (QP) spectral density near the Fermi level. Pauli spin susceptibility arises from the coupling of intrinsic spins of the mobile carriers with the applied magnetic field and, for Fermi-liquids, can be expressed as-

$$\chi_s = \mu_B^2 N(E_F) \tag{1}$$

where, μ_B is the Bohr magneton and $N(E_F)$ is the EDOS at the Fermi energy. In cuprates, χ_s at a particular temperature T, approximately represents the thermal average value of the EDOS near the Fermi-level, $< N(E_F) >$, over an energy width of $\sim \varepsilon_F \pm 2k_BT$ [24].

The uniform magnetic susceptibility data consists of a number of contributions from different physical origins. It is therefore important that one is able to separate these different contributions from each other so that only the terms directly related to the PG (i.e., to the QP excitation spectrum) can be studied. Considering all these contributions to the uniform susceptibility, χ, we can express χ(T) as follows [25, 26]-

$$\chi(T) = \chi_s(T) + \chi_{Curie}(T) + \chi_{core}(T) + \chi_{VV} + \chi_{imp}(T) \tag{2}$$

where, the *Larmor* or the core susceptibility and the *Van Vleck* (VV) susceptibility are expected to be p- and T-independent over the experimental temperature range [25, 26]. $\chi_{Curie}(T)$ is the paramagnetic Curie susceptibility. We have ignored *Landau* diamagnetic susceptibility, which has the same T-dependence as $\chi_s(T)$, but is only from 2% to 5% of the $\chi_s(T)$ in magnitude [25]. $\chi_{imp}(T)$ is the contribution from possible impurity phases present in the compounds, even though X-ray diffraction profiles for the samples under study appeared phase-pure within the resolution of the X-ray diffractometer [27].

Information regarding the pseudogap have been extracted from χ(T) by using the correspondence between spin-susceptibility [24, 28] and the various other thermodynamical parameters obtained from the pioneering heat capacity works by Loram *et al.* [24, 28, 29]. Perhaps the most unambiguous measure of QP spectral weight is obtained from the electronic specific heat coefficient, γ. The striking qualitative and quantitative resemblance between $\chi_s(T)$ and γ(T) has been well documented [24, 28, 29, 30]. This gives one the confidence to interpret our χ(T, p) data in terms of the EDOS for the cuprates, in spite of the presence of strong electronic correlations in these compounds. $\chi_s(T)$ and S(T)/T (where S(T) is the electronic entropy, obtained by integrating γ(T)) show identical T dependences [24, 28]. Furthermore, $S/T\chi_s$ is very close to the free electron Wilson ratio, $a_0 = \frac{\pi^2 k_B^2}{3\mu_B^2}$ [24, 28].

In addition the T-dependent Knight shift data (which directly probes the Pauli paramagnetic spin susceptibility) from the NMR measurements [31] also corresponds quite nicely with the static susceptibility results. This lends further support to the assumption that the observed T-dependence of the χ(T) data comes primarily from a T-dependent EDOS.

B. Modelling of Pseudogap

The density of states for a d-wave superconductor with the carriers confined to a two dimensional planes (as is believed to describe the high-T_c cuprates) is given by integrating through a full rotation of 2π around Fermi surface as [32]-

$$N(E) = \frac{1}{2\pi}\int_0^{2\pi} R\left\{\frac{E d\theta}{\sqrt{E^2 - \Delta_o^2 \cos^2(2\theta)}}\right\} \tag{3}$$

where, θ = arctan (k_x / k_y) is the angle in k-space, and Δ_0 is the maximum of the superconducting energy gap.

In [24, 28] assume a states non-conserving form for the normal state PG. Ding *et al*. [33] also did the ARPES measurements in the normal state on high-T_c cuprate superconductors and the experiment showed that the linearly vanishing density of states with no coherence peaks at the gap edges, consistent with the above-noted states non-conserving gap. There is general consensus that this low-energy quasiparticle weight suppressed by the PG is transferred to much higher energies. The gap observed amounts to a triangular notch in the density of states centered at zero energy. The model (Fig. 4) of PG [34] in the limit as $T \to 0$ as follows:

$$N(E) = \begin{cases} N_0 \dfrac{E}{E_g} (E \le E_g) \\ N_0 (E > E_g) \end{cases} \tag{4}$$

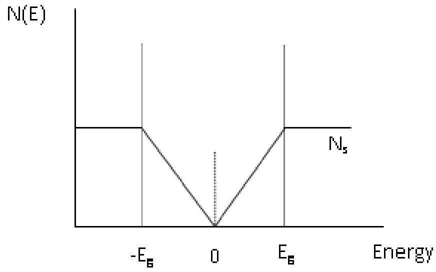


Figure 4. Model EDOS.

Hence the gap will be modeled in terms of two parameters, the width of the gap E_g and the limiting density of states N_0. Incidentally, this almost is the same model used for the PG by Balatsky *et al*. in [35], where they have studied the effect of non-magnetic impurity on the magnetization in cuprates.
The magnetic susceptibility χ_s for a Fermi liquid is given by-

$$\chi_s = 2\mu_B^2 \int_0^\infty N(E) \left(-\frac{\partial f}{\partial E} \right) dE \tag{5}$$

$$\chi_s = \frac{\mu_B^2}{2kT} \int_0^\infty N(E) \cosh^{-2} \left(\frac{E}{2kT} \right) dE \tag{6}$$

where, f is the Fermi-Dirac distribution function. Experimental data suggests that [22] the gap fills up with increasing temperature, with the bottom of the gap rising as-

$$b = N_0 \left[1 - \tanh \left(\frac{E_g}{2kT} \right) \right] \tag{7}$$

which in turn means [34]-

$$N(E) = \begin{cases} N_0 \left[1 - \tanh \dfrac{E_g}{2kT} \right] + N_0 \dfrac{E}{E_g} \tanh \dfrac{E_g}{2kT} (E \le E_g) \\ N_0 \qquad\qquad\qquad\qquad\qquad\qquad (E > E_g) \end{cases} \tag{8}$$

Substituting (8) into (6) and evaluating the integral predicts that, according to this model [22]-

$$\chi_s(T) = N_0 \mu_B^2 \left[1 - \left(\frac{E_g}{2kT} \right)^{-1} \tanh \frac{E_g}{2kT} \ln \left\{ \cosh \left(\frac{E_g}{2kT} \right) \right\} \right] \tag{9}$$

The only two parameters describing the T-dependence of $\chi_s(T)$ are N_0 and E_g. Here N_0 simply appears as a proportionality constant at the front of the expression. If the total static susceptibility data is fitted to (9) then N_0 takes into account of all the different T-independent contributions to $\chi(T)$.

We have shown the fits of $\chi(T)$ data to (9) in Fig. 5 with different hole contents. The important fitting parameters are listed in Table III. The quality of the fits are excellent and the values obtained for the PG energy scales agree quite well with earlier studies employing other experimental probes and methodology [22, 24, 28].

Table III: Fitting Parameters Obtained from (9)

Hole Content, P	$N_0\mu_B^2$ (10^{-4} Emu/Mole)	E_g/k_B (K)
0.174	2.6943	59.60
0.161	2.5960	140.6
0.143	2.5617	221.1
0.111	2.4338	277.2
0.094	2.2751	343.1

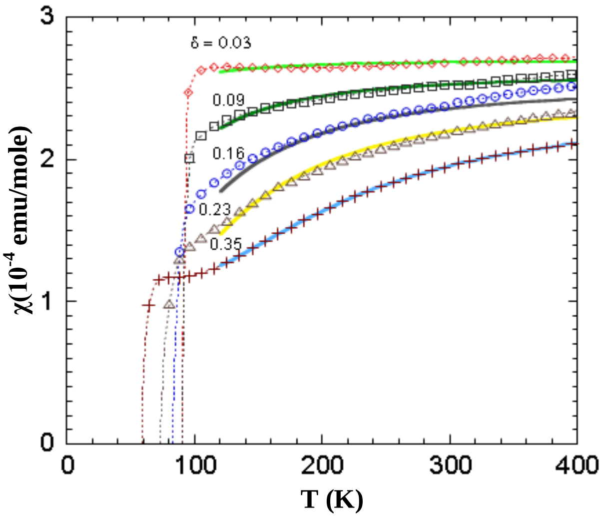


Figure 5. χ (T) for Y123 [thick full lines show the fits to (9)].

From the χ(T) data we have exerted the PG energy scales as a function of hole content. The p-dependent PG energy scale (expressed in temperature scale) with representative T_c(p) is shown in Fig. 6.

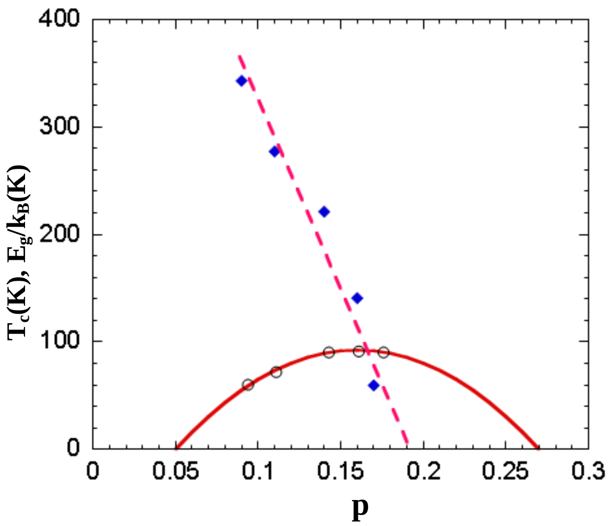


Figure 6. E_g/k_B(p) and T_c(p) of $YBa_2Cu_3O_{7-\delta}$.

Another important parameter is the value of N_0. From Table III it is seen that $N_0\mu_B^2$ decreases with decreasing p. We show this p-dependent behavior in Fig. 7.

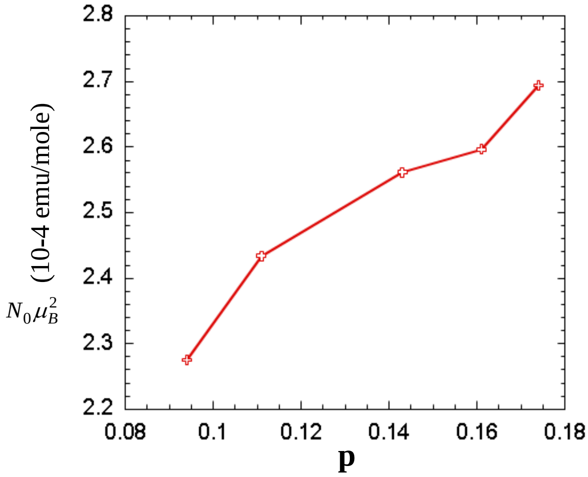
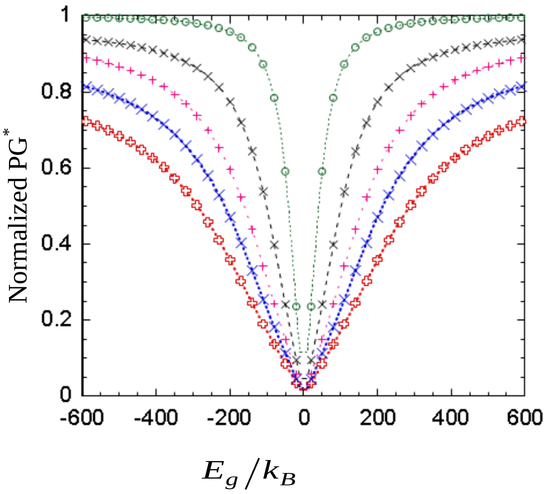


Figure 7. p-dependent behavior of $N_0\mu_B^2$

This has systematic behavior of $N_0\mu_B^2$ important theoretical consequence. This implies that the high energy density of states is actually not recovered. This in turn leads to a state-non-conserving PG. The states-non-conserving nature of the PG is clearly illustrated in Fig. 8, where PG is modeled using the fitting parameters. It worth noting that superconducting gap is always states-non-conserving.



*we have taken $N_0\mu_B^2 = 1$ for the p = 0.174 compound.

Figure 8. The profile of the PG as a function of energy (expressed in temperature scale) using (9). Negative energy corresponds to levels below the Fermi energy.

V. CONCLUSION

In this study we have analyzed the effect of the PG on the static magnetic susceptibility, χ(T), of Y123 compound. Using a simple PG model we have fitted the experimental χ(T) data which is quite satisfactory over a wide range of hole contents and oxygen deficiencies. From these fits the values of p-dependent PG energy scale, E_g, have been obtained. E_g values obtained here agree quite well with those found in other studies. The T-dependent contribution ($N_0\mu_B^2$ in (9)) to the χ(T) was also found to decrease with decreasing hole concentration.

This implies that the PG might the states-non-conserving nature (referring to Fig. 8). This states-non-conserving property and the fact that extrapolated E_g/k_B tends to vanish at p ~ 0.19 (Fig. 6) indicate that the pseudogap might be unrelated to superconducting correlations.

Acknowledgement

Authors would like to thank Dr. Saleh Hasan Naqib, Department of Physics, University of Rajshahi, Bangladesh for his support.

REFERENCES

[1] H. Kamarling Onnes, *Akad. Van Wetenschappen* (Amsterdam) 14, 113 (1911).
[2] W. Meissner and R. Ochsenfeld, *Narwiss* 21, 787 (1933).
[3] E. A. Lynton, *Superconductivity*, Wiley, New York, (1969).
[4] J. R. Gavalar, *App. Phys. Lett*. 23, 480 (1973).
[5] L. R. Testardi, J. H. Wernick and W. A. Royer, *Solid State Commun*. 15, 1 (1974).
[6] J. G. Bednorz and K. A. Müller, *Z. Phys*. B 64, 189 (1986).
[7] M. K. Wu., J. R. Ashburn, C. J. Torng, P. H. Hor, R. L. Meng, L. Gao, Z. J. huang, Y. Q. Wang and C. W.Chu. *Phys. Rev. Lett*. 58, 908 (1987)
[8] C. W. Chu. et al. *Nature* 365 323 (1993)
[9] B. Batlogg and C. Varma, Physics World 13, 33 (2000), C. Panagopouls, J. L. Tallon, B.D. Rainford, T. Xiang, J.R. Cooper, and C.A. Scott, *Phys. Rev*. B 66, 64501 (2002).
[10] J. L. Tallon and J. W. Loram, *Physica* C 349, 53 (2001).
[11] M. R. Norman, D. Pines and C. Kallin, *Advances in Physics* 54, 715 (2005).
[12] Z. X. Shen and D. S. Dessau, *Physics Reports* 253, 1 (1995).
[13] B. Bucher, P. Steiner, J. Karpiniski, E. Kaldis and P. Wachter, *Phys. Rev. Lett*. 70, 2012 (1993).
[14] C. Kendziora and A. Rosenberg, *Phys. Rev*. B 52, R9867 (1995).
[15] H. Alloul, T. Ohno and P. Mendels, *Phys. Rev. Lett*. 63, 1700 (1989).
[16] B. Batlogg, H. Y. Hwang, H. Takagi, R. J. Cava, H. L. Kao and J. Kwo, *Physica* 235, 130 (1994).
[17] T. M. Rice, Kai-Yu. Yang and F. C. Zhang, *Rep. Prog. Phys.* 75 (2012) 016502 pp. 36.
[18] P. A. Lee, *Rep. Prog. Phys.* 71, 012501 (2008).
[19] M. R. Presland, J. L. Tallon, R. J. Buckley, R. S. Liu, N. E. Fowler, Physica C 176, 95 (1991).
[20] S. D. Obertalli, J. R. Cooper, J. L. Tallon, Phy. Rev. B 46, 14928 (1991).
[21] D. C. Johnstosn, *Phys. Rev. Lett*. 62, 957 (1989).
[22] B. Batlogg, H. Y. Hwang, H. Takagi, R. J. Cava, H. L. Kao, J. Kwo, *Physica* C 235 – 240 (1994).
[23] A. Aurbach and D. P. Arovas, *Phys. Rev. Lett*. 61, 617 (1997).
[24] J. W. Loram, J. Luo, J. R. Cooper, W. Y. Liang, and J. L. Tallon, *J. Phys. Chem. Solids* 62, 59 (2001) and J. W. Loram, K. A. Mirza, and J. R. Cooper, *Research Review*, p.77 (IRCS) (1998).
[25] R. E. Walstedt, R. F. Bell, L. F. Schneemeyer, J. V. Waszczak, and G. P. Espinosa, *Phys. Rev*. B 45, 8074 (1992) and C. Allgeier and J. S. Schilling, *Phys. Rev*. B 48, 9747 (1993).
[26] S. Blundell, *Magnetism in Condendensed Matter*, Oxford University Press, 2001.
[27] S.H. Naqib, Ph.D. thesis, IRCS, University of Cambridge, 2003 (unpublished).
[28] J. R. Cooper and J. W. Loram, *J. Phys.* I France 6, 2237 (1996).
[29] J. W. Loram, K. A. Mirza, J. R. Cooper, and J. L. Tallon, *J. Phys. Chem. Solids* 59, 2091 (1998).
[30] J. W. Loram, K. A. Mirza, J. R. Cooper, W. Y. Liang, and J. M. Wade, *J. Supercon*.7, 243 (1994).
[31] G. V.M. Williams, J. L. Tallon, J. W. Quilty, H. J. Trodahl and N. E. Flower, *Phys. Rev. Lett*., 80, 377 (1998).
[32] A. A. Yurges, *Supercond. Sci. Technol*. 13, R85 - R100 (2000).
[33] H. Ding, T. Y. Yokaya, J. C. Campuzano, T. Takahashi, M. Randeria, M. R. Norman, T. Mochiku, K. Kadowaki and J. Giapinzakis, *Nature* 382, 51 (1996).
[34] T. M. Benseman Thesis, (unpublished).
[35] H. V. Kruis, I. Martin and A. V. Balatsky, *Phys. Rev*. B 64, 054501 (2001).