*IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)*
*e-ISSN: 2278-1676 Volume 4, Issue 3 (Jan. - Feb. 2013), PP 24-27*
*www.iosrjournals.org*

# Performance analysis of OFDM systems in the presence of Doppler-effect

Mohamed Tayebi[1], Merahi Bouziani[1]
[1]*(Telecommunications and Digital Signal Processing Laboratory, Djillali LiabesUniversity, Algeria)*

***Abstract :*** *Carrier Frequency Offset (CFO) in Orthogonal Frequency Division Multiplexing (OFDM) systems is generated partly by imperfections of the local oscillators (static CFO) and secondly, by the Doppler-effect (dynamic CFO). The aim of the present study is to investigate the impact of Doppler-effect on OFDM performances. A new expression of the global CFO is derived and Bit Error Rate performance is obtained by Monte-Carlo simulation.*

***Keywords -*** *Orthogonal Frequency Division Multiplexing (OFDM), Carrier Frequency Offset (CFO), Doppler-effect, Inter Carriers Interferences (ICI), Carrier to Interferences Ratio (CIR).*

## I. Introduction

OFDM is simply implemented using Fast Fourier Transform (FFT) and presents innumerable advantages such as high spectral efficiency and robustness against multipath fading. These advantages have enabled telecommunication systems to make a significant forward step and access gain to new perspectives. OFDM has found its place in optical communications [1] and in a large variety of broadcast standards such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB). However, its main disadvantage is its sensitivity to Carrier Frequency Offset (CFO). This is mainly due to imperfections in local oscillators and Doppler-effect present in mobile radio channels [2]. CFO gives rise to inter-carrier interference responsible for the degradation of system performance[3,4,5]. For a transmitter-receiver pair, the offset due to the imperfections of the local oscillators is constant, while the shift due to the Doppler- effect is variable and random. We can talk about static CFO and dynamic CFO. This letter concerns the dynamic CFO where a new expression of the global CFO is derived and in order to evaluate the OFDM performances, Bit Error Rates are obtained.

## II. Doppler-effect and Carrier Frequency Offset analysis

The OFDM symbol is transmitted by the radio frequency circuit. The carrier is noted $f_p$. Assume that local oscillators of the transmitter and of the receiver do not oscillate at the same frequency, the frequency at the receiver is:

$$f_r = f_p + f_{diff} \tag{1}$$

$f_{diff}$ is the frequency difference between the transmitter and the receiver. This frequency difference caused by the imperfections of the local oscillators is constant and still exists. The normalized value is equal to:

$$\varepsilon_{diff} = Tf_{diff} \tag{2}$$

When a mobile is moving at a speed $v$, a frequency shift $f_d$ occurs. This shift is called the Doppler-effect. Its mathematical expression is given by:

$$f_d = f_p \frac{v}{c} \cos\alpha \tag{3}$$

where $c$ is the light speed, $f_p$ is the carrier frequency and $\alpha$ is the angle between the velocity vector and the direction of the electromagnetic wave. The normalized value is:

$$\varepsilon_d = Tf_p \frac{v}{c} \cos\alpha \tag{4}$$

Figure 1 shows that the normalized Doppler frequency increases with velocity $v$ and the angle $\alpha$ determines its sign. For a mobile receiver, the received frequency is given by:

$$f_r = \left(f_p + f_{diff}\right)\left(1 + \frac{v}{c}\cos\alpha\right) \tag{5}$$

The total carrier frequency offset will be:

$$f_T = f_p \frac{v}{c}\cos\alpha + f_{diff}\left(1+\frac{v}{c}\cos\alpha\right) \tag{6}$$

Its normalized value is equal to:

$$\varepsilon_T = Tf_p \frac{v}{c}\cos\alpha + \varepsilon_{diff}\left(1+\frac{v}{c}\cos\alpha\right) \approx Tf_p \frac{v}{c}\cos\alpha + \varepsilon_{diff} \tag{7}$$

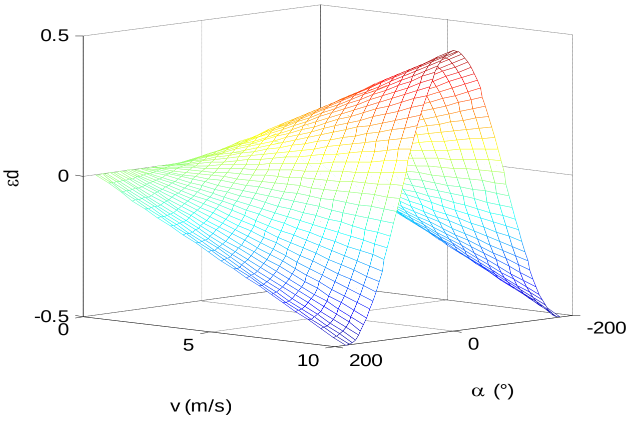

Fig.1: Normalized Doppler frequency

## III. Inter-Carrier Interference and performance analysis

### 3.1 Inter-Carrier Interference analysis

The orthogonality between the sub-carriers of the OFDM signal is broken, which leads to the rise of inter-carrier interference. The amplitude of ICI coefficients between the $l^{th}$ and the $k^{th}$ subcarriers is given by [4]:

$$|S(l-k)| = \left| \frac{\sin\pi\left(l-k+Tf_p\frac{v}{c}\cos\alpha+\varepsilon_{diff}\right)}{N\sin\frac{\pi}{N}\left(l-k+Tf_p\frac{v}{c}\cos\alpha+\varepsilon_{diff}\right)} \right| \tag{8}$$

These coefficients depend on the relative speed of the mobile, the angle $\alpha$ and the subcarrier indexes.

### 3.2 Carrier to Interference Ratio

This section examines the performances in the presence of the Doppler effect in terms of the CIR which is given by [4]:

$$CIR = \frac{|S(0)|^2}{\sum_{l=1}^{N-1}|S(l)|^2} \tag{9}$$

knowing that [5]:

$$\sum_{l=0}^{N-1}|S(l)|^2 = 1 \tag{10}$$

and:

$$|S(0)| = \left| \frac{\sin\pi\left(Tf_p\frac{v}{c}\cos\alpha+\varepsilon_{diff}\right)}{N\sin\frac{\pi}{N}\left(Tf_p\frac{v}{c}\cos\alpha+\varepsilon_{diff}\right)} \right| \tag{11}$$

We can rewrite Eq(9) in the following form:

$$CIR = \frac{|S(0)|^2}{1-|S(0)|^2} \tag{12}$$

Figure 2 shows the CIR as a function of speed $v$ and the angle $\alpha$ for $T$ = 224.10$^{-6}$ s and $f_p$ = 5 Ghz. The CIR decreases when the relative velocity v increases and when the angle $\alpha$ is in the range: $\alpha = ]90-0]$ and $\alpha = ]90-180]$. The CIR is maximum for $\alpha$ = 90 °, meaning that the Doppler-effect is cancelled.

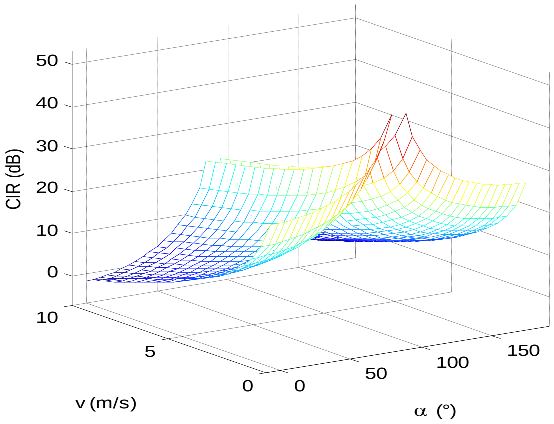

Fig.2: Carrier to Interferences Ratio

### 3.3 Bit Error Rate (BER)

Using the Monte Carlo method, Figure 3 shows the evolution of the BER for different values of Signal to Noise Ratio (SNR) given an angle of arrival $\alpha$. The simulation was performed with the following values (relative velocity $v$ = 100m/s and the normalized frequency offset $\varepsilon = 0.2$). For $\alpha = 0$, the Doppler effect is completely absent. For $\alpha < 90$, the shift due to the Doppler effect is positive, whereas for $\alpha > 90$, it is negative. In this case, the performance is improved since the global offset is reduced.

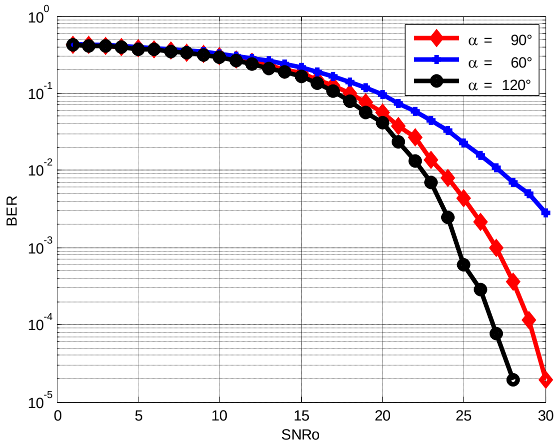

Fig.3: BER versus Signal to Noise Ratio for different values of $\alpha$ .

## IV. Conclusion

In this letter, a new expression for the global CFO is derived for OFDM systems. We deduce that the system performance, in terms of BER, is degraded by the CFO due to imperfections of the local oscillators and the CFO due to the Doppler effect contributes to an additional degradation if it is in the same direction as the first. However, performance will be improved if the two CFOs are in opposite directions.

## REFERENCES

### Journal Papers:

[1] K.Kikuchi, “Digital coherent optical communication systems: fundamental and future prospects”, *IEIEC Electronics Express, 8(20),* 2011, 1642-1662.

[2] Y. Zhao and S.-G. Häggman, “Sensitivity to Doppler shift and carrier frequency errors in OFDM systems—The consequences and solutions,” *Proc. 46th IEEE Vehicular Technology Conference, Atlanta*, GA, 1996, 1564–1568.

[3] R. Uma Mahesh and A. K. Chaturvedi, " Closed Form BER Expressions for BPSK OFDM Systems with Frequency Offset" *IEEE Communications Letters, 14(8)*, 2010, 731-733.

[4] Y. Zhao and S.-G.Haggman, “Intercarrier interference self-cancellation scheme for OFDM mobile communication systems,” *IEEE Transactions on Communications, 49(7)*, 2001, 1185–1191.

[5] J. Lee, H. Lou, D. Toumpakaris, and J. Cioffi, “SNR Analysis of OFDM Systems in the Presence of Carrier Frequency Offset for Fading Channels” *IEEE Transactions on wireless communications, 5(12)*, 2006, 3360–3364.