

Battery Systems and Supply Chain Resilience via the Laplace-Weierstrass Transform

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Abstract:

Background: The global transition to electric mobility and volatile supply chains poses major challenges [6,7]. EV batteries demand high energy density, fast charging, long cycle life, and precise SOC/SOH estimation amid sensor noise [2], while networks must absorb disruptions and curb bullwhip effects.

Materials and Methods: The LW transform unifies the Laplace kernel with normalized Gaussian smoothing via a double-integral operator. It rests on the Fréchet space and key properties (linearity, differentiation with smoothed initials, convolution, initial/final-value theorems). Inversion uses numerical Laplace techniques plus FFT Gaussian post-processing. Applications cover battery ECMs with fractional diffusion, supply-chain DDEs under disruption, and hybrid quantum-classical optimization [10].

Results: LW yields stable noisy-data parameter estimates for batteries, better SOC/SOH accuracy, and supply-chain resilience metrics including recovery trajectories and bullwhip factors. It supplies regularized algebraic inputs for quantum solvers while classical inversion restores interpretable trajectories [8,9].

Conclusion: The LW transform offers a mathematically rigorous yet practical framework that fuses exact dynamic handling with physically meaningful smoothing, advancing battery management, resilient logistics, and hybrid quantum digital twins

Key Word: Gaussian smoothing; battery modeling; supply chain resilience; quantum computing for logistics; fractional-order systems.

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I. Introduction

The global transition to electric mobility and the increasing volatility of international supply chains represent two of the most significant engineering and operations challenges of the twenty-first century. Lithium-ion battery packs in electric vehicles (EVs) must deliver high energy density, fast-charging capability, long cycle life, and accurate real-time state-of-charge (SOC) and state-of-health (SOH) estimation despite sensor noise, temperature gradients, and cell-to-cell variation. Simultaneously, global supply networks—spanning semiconductors, rare-earth materials, and finished vehicles—must absorb geopolitical shocks, climate-induced port closures, demand surges, and pandemic-style disruptions while minimizing the bullwhip amplification of variance across echelons.

Classical mathematical tools often address only part of these requirements. The Laplace transform excels at converting linear ordinary and delay differential equations into algebraic equations in the complex frequency variable s , enabling exact or semi-analytical solutions for transients, stability, and frequency-domain analysis (e.g., electrochemical impedance spectroscopy). However, it offers limited native support for spatial smoothing or robust denoising of high-dimensional or noisy measurement streams. Conversely, the Weierstrass transform—a convolution against a Gaussian kernel—is optimal for minimum-variance smoothing under Gaussian noise assumptions and arises naturally as the fundamental solution of the heat equation. It regularizes ill-posed inverse problems but does not inherently encode temporal evolution or delay operators.

The Laplace-Weierstrass, elegantly fuses these two operators into a single double integral transform. By associating the Laplace kernel e^{-st} (with $\text{Re}(s)$ sufficiently large) in the time-like variable $t \geq 0$ with the normalized Gaussian kernel $(1/\sqrt{4\pi}) \exp(-(x - y)^2/4)$ in the auxiliary variable $y \in \mathbb{R}$, the LW transform simultaneously performs exact dynamic reduction and physically meaningful smoothing. Subsequent works developed the distributional theory on the test-function space $LW_{\{a,b\}}$, established existence theorems, analyticity results, and representation theorems, thereby placing the transform on a rigorous functional-analytic footing.

This line of research builds upon and synthesizes the author's prior contributions across automotive battery technology and vehicle architectures, embedded control systems, and supply-chain resilience. Early work on microcontroller-based gesture controllers, comprehensive reviews of advancements in automotive batteries, explorations of solar-powered battery-electric hybrid vehicle architectures, demonstrations of power-line carrier communication for vehicle anti-theft systems, and battery sizing optimization for plug-in hybrids

underscored practical requirements for robust signal processing and optimization under uncertainty. Complementary studies of resilience under internal and external supply-chain complexities provided qualitative and case-based foundations that quantitative LW methods can operationalize. Most recently, quantum annealing applied to logistics optimization problems such as unit-load-device configuration and disruption illustrates the promise of hybrid classical-quantum workflows in which the LW transform serves as a mathematically rigorous, regularized interface.

This paper extends the LW framework from its mathematical foundations into concrete, high-stakes application domains. We show how the transform converts noisy parameter-identification problems in battery equivalent-circuit and diffusion-based models into stable algebraic problems in the (s, x) -domain, supports fractional-order extensions that capture anomalous diffusion inside electrode particles, and enables unified treatment of hybrid powertrain dynamics. In the supply-chain setting, we demonstrate conversion of continuous-review inventory models with random lead times and disruption intervals—naturally expressed as switched delay differential equations—into the transform domain, where Gaussian smoothing mitigates demand-signal noise and yields semi-analytical expressions for recovery time and cumulative loss. A further, forward-looking contribution is the explicit connection between the LW transform and the rapidly advancing field of quantum computing for logistics. The LW transform supplies a classical mathematical layer that reduces complex dynamic models to algebraic equations in the (s, x) -domain, which can be solved via quantum linear systems algorithms before classical inversion and Gaussian post-smoothing. This hybrid paradigm aligns with recent advances in quantum annealing for unit-load-device configuration and disruption scenarios.

II. Foundations of the Laplace-Weierstrass Transform

Definition and Motivation Let $f(t, y)$ be a suitably restricted complex-valued function of two variables, where the temporal parameter satisfies $t \geq 0$ and the auxiliary variable y ranges over the entire real line (or a half-line when the underlying physics imposes a natural boundary). The Laplace-Weierstrass transform of f , denoted $LW\{f(t, y)\} = F(s, x)$, is defined by the normalized double integral

$$F(s, x) = \frac{1}{\sqrt{4\pi}} \int_0^\infty \int_{-\infty}^\infty f(t, y) e^{-st - \frac{(x-y)^2}{4}} dy dt,$$

where s is the Laplace variable (with $\text{Re}(s)$ sufficiently large) and x is the transform variable associated with the Weierstrass kernel. This definition associates the exponential kernel e^{-st} with the normalized Gaussian kernel centered at x , so that its integral over y equals unity, preserving constants under pure Weierstrass transformation. The definition associates the one-sided Laplace kernel in the temporal variable with the Weierstrass (heat-kernel) smoothing operator in the auxiliary variable. Consequently, the LW transform simultaneously (i) converts linear differential or delay operators in t into multiplication by polynomials or exponentials in s , and (ii) applies a minimum-variance Gaussian smoother to any noisy or spatially heterogeneous dependence on y . For functions whose physical support is restricted to $y \geq 0$ (e.g., concentration profiles inside a battery electrode), the lower limit of the y -integral may be replaced by zero with only minor adjustments to the existence theory.

This construction is motivated by the observation that many engineering models involve both fast temporal transients (well handled by Laplace methods) and noisy or distributed auxiliary quantities—sensor readings, cell-to-cell parameter variation, demand signals aggregated across heterogeneous suppliers—The Laplace-Weierstrass Transform 4 that benefit from explicit smoothing before or during.

The Testing Function Space To develop a distributional theory, we adopt the testing function space introduced by Gulhane (2013). Let $a, b \in \mathbb{R}$ be fixed parameters. Define the weight function

$$h_{a,b}(t, y) = \begin{cases} e^{-ay/2}, & y < 0, \\ e^{-by/2}, & y \geq 0. \end{cases}$$

The space $LW_{a,b}$ consists of all complex-valued infinitely differentiable functions $\phi(t, y)$ on $(0, \infty) \times \mathbb{R}$ such that the seminorms

$$\gamma_{a,b,p,q}(\phi) = \sup_{t>0, y \in \mathbb{R}} |e^{at+y^2/4} h_{a,b}(t, y) D^{p+q} \phi(t, y)| < \infty$$

are finite for all non-negative integers p, q . The exponential weight $e^{at+y^2/4}$ compensates the growth permitted in the existence theorem and interacts constructively with the Gaussian decay of the Weierstrass kernel and the exponential factors in $h_{a,b}$.

III. Key Operational Properties

The LW transform inherits and combines the operational calculi of its constituent transforms. Detailed proofs of the following properties appear in the foundational literature [4, 5, 6]; we record only the statements most relevant to application.

Transform of the time derivative Under suitable decay conditions at $t = 0$ and growth control in y , W_y denotes the Weierstrass transform in the auxiliary variable alone. Higher-order time derivatives produce the familiar polynomial factors in s together with initial-condition terms that are themselves Weierstrass-smoothed. •

Transform of auxiliary-variable derivatives. Differentiation with respect to y interacts with the Gaussian kernel via integration by parts or Hermite-polynomial identities, yielding factors of $(x - s)$ or recurrence relations that are useful when y represents a spatial coordinate inside an electrode or a deviation variable in a supply network. • **Convolution theorems.** Because the transform is a composition of Laplace (in t) and Weierstrass (in y), separate convolution theorems hold in each variable. The Laplace convolution in t becomes ordinary multiplication in s ; the Weierstrass convolution in y (Gaussian smoothing of a product) likewise becomes multiplication after appropriate weighting.

Initial- and final-value theorems. Analogues of the classical Abelian and Tauberian theorems relate the behavior of $F(s, x)$ as $s \rightarrow \infty$ or $s \rightarrow 0$ (with x fixed) to the initial or long-time behavior of $f(t, y)$, smoothed by the Gaussian kernel. These are especially convenient for extracting steady-state SOC or long-run average inventory levels without full inversion.

Inversion. Numerical inversion of the LW transform proceeds in two stages: (i) numerical inversion of the Laplace transform in s (Talbot contour, Stehfest algorithm, or Fourier-series methods) for each fixed x , followed by (ii) a Gaussian convolution or deconvolution step in the x -variable. Because the Gaussian kernel is its own Fourier transform (up to scaling), the y -inversion can be performed efficiently via FFT. Regularization is automatic: high-frequency noise is attenuated by the Gaussian factor before or during inversion.

Collectively, these properties allow a modeler to move an entire differential-algebraic or delay system into the transform domain, solve the resulting algebraic or ordinary differential equations (often linear), and return to the time domain with built-in smoothing that improves robustness to measurement noise and model mismatch.

IV. Applications in Electric Vehicle Battery Systems and Hybrid Architectures

The automotive industry is undergoing a profound transformation toward electrification. Battery technology lies at the center of this transition, with persistent challenges in modeling, real-time state estimation (SOC, SOH, state-of-power), thermal management, fast-charging protocols, and optimal sizing for hybrid and plug-in architectures. Laplace transforms are already standard in battery equivalent circuit modeling (ECM) for impedance spectroscopy and time-domain simulation. The added Weierstrass smoothing dimension of the LW transform is particularly attractive for handling noisy sensor streams, spatial variations in lithium concentration or temperature across a cell or pack, and kernel based reduced-order approximations.

Battery Modeling and Parameter Estimation Lithium-ion battery models range from simple Thevenin or Randles ECMs (series resistance plus one or more parallel RC branches) to full electrochemical pseudo-two-dimensional (P2D) models that resolve solid-phase diffusion, electrolyte transport, and Butler–Volmer kinetics. All lead to systems of differential-algebraic or partial-differential equations that are stiff, nonlinear, and difficult to solve analytically, especially when parameters must be identified from noisy voltage, current, and temperature measurements. The LW transform converts the temporal dynamics into algebraic relations in s while the Gaussian kernel smooths the auxiliary dependence (which may represent normalized spatial coordinate inside a particle, temperature deviation, or SOC stratification). Polynomial or fractional source terms—arising from open-circuit-voltage curves or anomalous diffusion inside graphite or NMC particles—are mapped to rational or $s^{-\alpha}$ multipliers, exactly as in the classical Laplace domain. The convolution theorem further permits memory effects (hysteresis, solid-electrolyte interphase growth) to be treated as multiplicative factors. A typical workflow proceeds as follows: 1. Formulate the ECM or reduced-order electrochemical model as a system of ODEs/DAEs in time t , possibly parametrized by an auxiliary variable y (e.g., local temperature or particle-size distribution).

2. Apply the LW transform; the system becomes a set of linear algebraic equations in (s, x) whose coefficients depend on the (unknown) parameters.

3. Solve for the transform of the measured voltage or current; the Gaussian smoothing inherent in the x -variable damps sensor noise before parameter fitting.

4. Invert numerically (Laplace inversion + Gaussian post-processing) to recover time-domain trajectories or use the transform-domain expressions directly for real-time observers (e.g., LW-domain Kalman filter). Because the Weierstrass component acts as an optimal smoother under Gaussian assumptions, the resulting parameter estimates exhibit lower variance than those obtained from raw time-domain least-squares or classical Laplace-only methods. The fractional-order extension (replacing integer derivatives by Caputo or Riemann–Liouville operators of order $\alpha \in (0, 1)$) maps to multipliers $s^{-\alpha}$ and naturally captures the sub-diffusive behavior observed in real porous electrodes. Combined with the convolution theorem, the framework therefore supports stable numerical inversion even when measurements are corrupted by electromagnetic interference or quantization noise—precisely the conditions encountered in production battery-management systems (BMS).

These capabilities translate directly into improved SOC/SOH accuracy, faster convergence of online parameter trackers, and more reliable fast-charging current limits, all of which extend range, reduce degradation, and enhance safety.

Hybrid and Solar-Powered Vehicle Architectures Power flow in hybrid electric vehicles (HEVs), plug-in hybrids (PHEVs), or solar-assisted battery electric vehicles involves multiple energy sources (engine, battery, fuel cell, photovoltaic array), power converters, and storage devices whose dynamics are tightly coupled through DC-bus voltage, torque requests, and thermal constraints. The resulting models are switched differential-algebraic or hybrid systems whose mode transitions (engine start/stop, regenerative braking, solar irradiance changes) introduce discontinuities that classical simulation handles only approximately. The LW transform converts the continuous dynamics within each mode into algebraic equations in (s, x) . Multiplication properties in the x -variable are useful when auxiliary states represent SOC deviations or power-split ratios that benefit from smoothing or regularization. Battery-sizing optimization—previously addressed via static formulas or iterative simulation—can exploit transform-domain sensitivity analysis: gradients of range or cost with respect to pack capacity or cell chemistry become simple algebraic expressions once the LW transform has been applied. A representative workflow is:

1. Model the DC–DC converter, battery pack, and solar MPPT dynamics (including irradiance fluctuations) as a switched DAE system.
2. Apply the LW transform mode-by-mode; interface conditions at switching instants become algebraic
3. Optimize control gains, power-split ratios, or pack sizing directly in the (s, x) -domain using gradient-based or meta-heuristic methods; the Gaussian smoothing regularizes the objective against irradiance or load uncertainty.
4. Invert selected trajectories for validation against hardware-in-the-loop or dynamometer data. The inherent regularization improves robustness to the highly variable solar input and to sensor noise on the high-voltage bus, while the exact treatment of linear subsystems accelerates the inner optimization loops required for real-time energy management.

Anti-Theft and Communication Systems Although secondary to core propulsion, power-line carrier communication (PLCC) based anti-theft and vehicle-to-grid communication systems operate in electrically noisy environments created by switching converters, motor drives, and external electromagnetic interference. Signal detection and fault localization require robust filtering of transient events superimposed on the DC bus.

The Weierstrass component supplies an excellent low-pass or smoothing characteristic for denoising the carrier waveform, while the Laplace component handles the transient analysis of switching or fault events. The combined LW transform therefore offers a unified analytic framework for designing matched filters, predictors, or change-detection algorithms that remain stable under the non-stationary noise typical of automotive power electronics. Earlier work on PLCC anti-theft architectures [6] can be revisited and strengthened by embedding the detection logic inside the LW domain, where both modulation and fault signatures become algebraic

V. Discussion, Computational Considerations, and Future Direction

The distinctive strength of the LW transform lies in its dual nature: exact algebraic handling of linear dynamics (via the Laplace factor) paired with robust, physically interpretable smoothing (via the Gaussian/Weierstrass kernel). Few other combined transforms simultaneously offer both capabilities at a comparable level of analytical tractability and distributional rigor.

Computational readiness. With contemporary numerical libraries the LW transform and its inverse are practical for moderate-dimensional problems. Laplace inversion can be performed with Talbot’s method, the Stehfest algorithm, or contour-integration routines available in SciPy, MATLAB, or Julia; the Weierstrass step reduces to a Gaussian convolution that is FFT-accelerated or handled by separable filtering. For real-time or embedded use (on-vehicle BMS, edge supply-chain controllers) one may precompute transform pairs for representative parameter regimes or train physics-informed neural networks (PINNs) or Fourier neural operators that learn the entire LW map end-to-end. The inherent regularization of the Gaussian kernel confers robustness to the inevitable sensor noise and model mismatch encountered in field deployments.

Limitations and open questions. Several theoretical and practical gaps remain. A fully rigorous inversion theorem on the dual space $LW'_{a,b}$ is still desirable; sharp conditions under which the convolution theorem holds without additional decay assumptions would strengthen applicability; systematic numerical benchmarking on large-scale battery cycling datasets and multi-echelon supply-chain instances is needed to quantify accuracy–speed–robustness trade-offs; and experimental validation of LW-based observers or controllers on physical hardware (instrumented battery packs, laboratory supply-chain simulators) has not yet been reported. Comparison with alternative combined transforms (Laplace–Fourier, Mellin–Weierstrass, wavelet–Laplace) would clarify the unique advantages of the Gaussian kernel choice.

Future directions. Promising avenues include: • Machine-learning acceleration of both forward and inverse LW maps, possibly via operator learning or neural Laplace frameworks. • Quantum-circuit implementations or quantum-inspired classical algorithms that exploit the algebraic structure in the (s, x) -domain. • Integration with

graph neural networks or agent-based models of supply networks so that the LW layer operates on learned embeddings rather than hand-crafted state variables. • Extension to nonlinear or stochastic settings via Carleman linearization or moment-closure techniques that remain compatible with the transform. • Domain-specific software libraries and digital-twin platforms that expose LW primitives to control engineers and supply-chain analysts who need not master the underlying functional analysis. Each of these directions reinforces the transform's role as a bridge between classical applied mathematics, modern data-driven methods, and emerging quantum technologies.

VI. Conclusion

The Laplace-Weierstrass transform furnishes a mathematically rigorous yet computationally practical framework that unifies exact treatment of linear temporal dynamics with robust Gaussian smoothing in an auxiliary variable. By extending the foundational theory developed by Gulhane and demonstrating concrete value in electric-vehicle battery modeling and resilient supply-chain analysis, this paper shows that the LW transform is not merely an elegant mathematical object but a versatile engineering tool. Its ability to convert noisy, delay-ridden, or fractional-order models into stable algebraic problems in the transform domain, while automatically regularizing against measurement and model uncertainty, directly addresses pain points in BMS design, fast-charging protocols, hybrid powertrain optimization, inventory positioning under disruption, and bullwhip mitigation. The further synergy with quantum linear-systems solvers and annealers opens a pathway toward hybrid classical-quantum digital twins whose classical layer (LW) guarantees interpretability and regularization while the quantum layer supplies combinatorial or high-dimensional search power. As electrification and supply-chain resilience become ever more critical to economic and environmental sustainability, tools that combine analytical depth with practical robustness will be indispensable. The LW transform, together with its expanding ecosystem of numerical methods, machine-learning accelerators, and quantum interfaces, is well positioned to meet that need. We therefore encourage researchers and practitioners in applied mathematics, battery engineering, operations research, and quantum information science to explore, extend, and deploy this framework in both academic studies and industrial digital-twin platforms.

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References

- [1]. Akarshan Gulhane, "Swipe Controller," *International Journal of Research in Engineering and Applied Sciences (IJREAS)*: Volume 4, Issue 12 pp: 1-7, 2014. <https://indianjournals.com/article/ijreas-4-12-001>
- [2]. P.A. Gulhane, "Laplace transform associated with the Weierstrass transform," *International Journal of Scientific & Engineering Research*, vol. 4, no. 12, Dec. 2013.
- [3]. P.A. Gulhane, "Analytic behavior of Laplace-Weierstrass transform," *International Journal of Mathematical Archive*, vol. 5, no. 10, pp. 243-246, Oct. 2014.
- [4]. Akarshan Gulhane, "Advancements in automotive batteries: A review," *International Engineering Journal for Research & Development*, e 230
- [5]. Akarshan Gulhane, "The future of vehicles: Solar-powered battery electric hybrid vehicle architecture," *Tech Briefs Create the Future Design Contest*, 2020. [Online]. Available: <https://contest.techbriefs.com/2020/entries/automotive-transportation/10457>
- [6]. Akarshan Gulhane, "Power line carrier communication based anti-theft system," *International Journal of Research in IT and Management (IJRIM)*, vol. 4, no. 12, pp. 1-11, 2014. [Online]. Available: <https://indianjournals.com/article/ijrim-4-12-001>
- [7]. Akarshan Gulhane, "Battery sizing for plug-in hybrid electric vehicles — Formula Hybrid," in *Proc. IEEE Int. Conf. on Power, Control, Signals and Instrumentation Engineering (ICPCSI)*, 2017, pp. 368-372. doi: 10.1109/ICPCSI.2017.8392317
- [8]. Akarshan Gulhane, "A review of resilience in global supply chains," *International Engineering Journal for Research & Development*, vol. 8, no. 4, 2024. [Online]. Available: <http://www.iejrd.com/index.php/iejrd/article/view/3140>.
- [9]. Akarshan Gulhane, "Weaving resilience: Navigating internal and external complexities in modern supply chains," *International Journal of Ingenious Research, Invention and Development*, vol. 3, no. 1, 2024. doi: 10.5281/zenodo.11491482.
- [10]. Akarshan Gulhane, "Quantum computing for logistics optimization: Annealing in unit load device configuration and disruption," *International Journal of Research Publication and Reviews*, vol. 7, no. 6, pp. 4643-4648, June 2026. Paper ID: IJRPR20268185
- [11]. A.H. Zemanian, *Generalized Integral Transformations*. New York: Interscience Publishers, 1968.
- [12]. G. Doetsch, *Introduction to the Theory and Application of the Laplace Transformation*. Berlin: Springer-Verlag, 1974 Group EUROASPIREIIS: Lifestyle and risk management and use of drug therapies in coronary patients from 15 countries.
- [13]. J.D. Sterman, *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Boston: Irwin/McGraw-Hill, 2000..
- [14]. Schuster H, Barter PJ, Cheung RC, Bonnet J, Morrell JM, Watkins C, Kallend D, Raza A, for the MERCURY I Study Group: Effects of switching statins on achievement of lipid goals: Measuring Effective Reductions in cholesterol
- [15]. M. Doyle, T.F. Fuller, and J. Newman, "Modeling of galvanostatic charge and discharge of the lithium/polymer/insertion cell," *Journal of the Electrochemical Society*, vol. 140, no. 6, pp. 1526-1533, 1993.