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Research on Robust Optimal Control of Trajectory Tracking for Wheeled Mobile Robot using Disturbance Observer Combined with Reinforcement Learning Techniques

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Abstract: *In the motion control technique of wheeled mobile robot (WMR), the problem of trajectory tracking and fast impact is very necessary, the main task and attracts the attention of a large number of scientists. For a robot with holonomic constraints such as WMR, the set trajectory will include both its coordinates (x,y) and its direction angle θ. The task of the mobile robot tracing control problem here is to determine the torque applied to the wheels of the robot in the shortest time so that it can move in the correct trajectory with the smallest error. Therefore, the paper proposes a robust optimal control method for WMR to ensure the set quality criteria. This controller allows the robot to operate in conditions of slippery floors, affected by external disturbances.*

***Key Word**: Wheeled mobile robot; Reinforcement learning; Adaptive dynamic programming; Hamilton-jacobi-bellman; Disturbance observer.*

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I. Introduction

There have been many research results on the mobile robot trajectory tracking system published in the past decades, some researches on tracking control directly use Lyapunov function, others use the Lyapunov function, controller has state feedback, most of the solutions are designed on the basis of deterministic dynamic modeling. Recently, adaptive control methods are used to control mobile robots because this is an uncertain system with strong disturbances. The research focuses on methods such as sliding mode control, using Lyapunov adaptive control function, fuzzy logic system or neural network and some development methods in the direction of adaptive control according to the model.

Previous studies focused on designing controllers for kinematics or dynamics models separately. Usually, the relationship between the position on the initially chosen coordinate system and the speed of the robot is expressed through a kinematic equation. A few studies have focused on looking at the navigation position of the mobile robot as well as the control according to the robot's kinematic model [1]. Based on the kinematic model, a number of control methods have been developed to control the robot to follow the desired trajectory and limit it to a few conditions affected by the environment. Traction control for WMR has also been built and applied PID algorithm, like PI controller in [2, 3] to track motion trajectory. The simulation results give good tracking results, the actual trajectory of the robot follows the set trajectory, but in reality, the robot model is often affected by uncertain factors as well as the change of the parameter model, which kinematic equations cannot fully describe, so the experimental results often show that the robot's trajectory has deviations and fluctuates around the set value. In fact, due to the existence of nonlinear components including friction, vibration, wheel slip, etc., recent studies consider both kinematics and dynamic models to increase accuracy of specific applications in factories [4].
To facilitate the design and application of control algorithms as in [5, 6], or orbiting algorithms [7, 8] and image processing, the vision system for robots in [9] -11], the robot is modeled using a kinematic model and a dynamic model. Besides, Robot Operating System (ROS) has emerged as a support tool for simulation and programming of unified control systems [12-14].

Designing a control algorithm for a nonlinear system with uncertain components, model error, and external disturbance so that the closed system is not only stable and stable but also improves the control quality is a challenging problem, many researchers are interested. The methods to solve this problem are adaptive control [15] in which it is common to indirectly identify the system [16], then design the controller. The second method is to identify uncertain components in the system online using function approximators. Since the function approximation is limited by a finite number of parameters, the approximation error is unavoidable. This error together with disturbances can make the closed system unstable. Therefore, adding a robust control component to the adaptive control law to compensate for the approximation error and disturbance is necessary [17]. However, the adaptive control or robust adaptive control methods have not completely solved the optimization problem [18].

In that context, one of the new approaches in the 4.0 technology era is the application of artificial intelligence technology, helping the robot to improve its self-driving ability through learning the data and routes already established, provided in advance. These technology-based methods also demonstrate superiority in performance and computational cost over classical methods. Therefore, the topic focuses on researching and applying reinforcement learning techniques in mobile robot control, in order to ensure that the robot can learn new skills and improve its self-driving ability, as well as provide robots ability to make rational, intelligent decisions. In this way, the robot can learn to optimally adaptive to the uncertainty and unpredictable changes of random and constantly changing environmental factors.

II. Mathematical Model of WMR

Considering the WMR structure, two independent drive wheels at the rear and one rudder at the front, subject to nonholonomic constraints as shown in Figure.1.

Where G is the center of gravity of the WMR hardware, $M(x_M, y_M)$ is the midpoint of the axle connecting the two rear wheels, ψ is the direction angle of the WMR.

Let the angular velocities of the right and left wheel motors be $\dot{\varphi}_R$ and $\dot{\varphi}_L$, respectively. The axial slip of the right and left wheels is δ_R and δ_L, the axial slip is χ.

The kinematic equation of WMR when taking into account the effect of wheel slip is [19,20]:

$$\begin{cases} \dot{x}_M = \chi\cos\psi - \dot{\phi}\sin\psi \\ \dot{y}_M = \chi\sin\psi + \dot{\phi}\cos\psi \end{cases} \tag{1}$$

Where χ is the linear velocity in the direction perpendicular to the axis connecting the two rear wheels and ϖ is the angular velocity of the WMR:

$$\chi = \frac{r(\dot{\varphi}_R + \dot{\varphi}_L) + (\dot{\delta}_R + \dot{\delta}_L)}{2} \tag{2}$$

$$\varpi = \frac{r(\dot{\varphi}_R - \dot{\varphi}_L) + (\dot{\delta}_R - \dot{\delta}_L)}{2b}$$

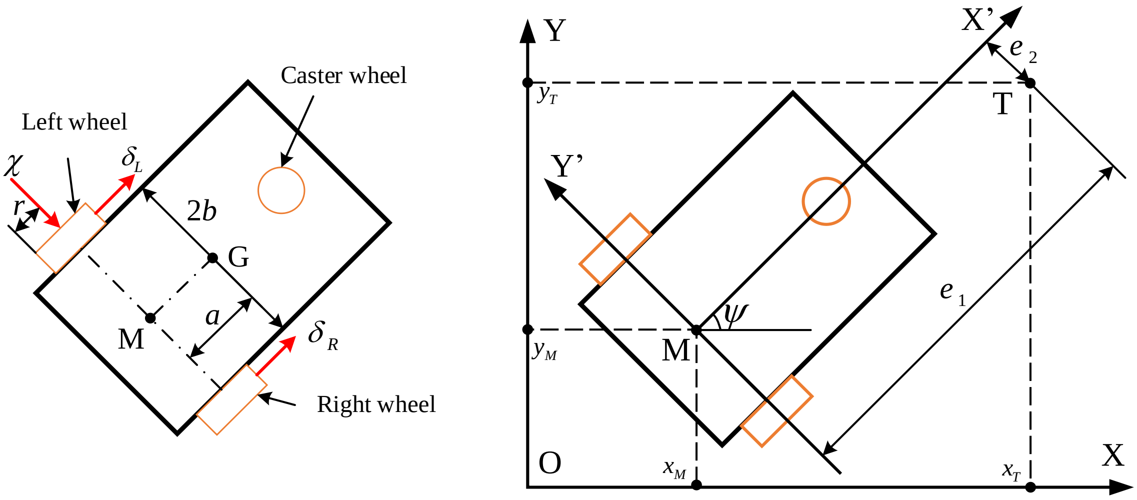

Figure 1: Model of WMR and coordinate system

The dynamic equation of WMR is [19,20]:

$$M\dot{v} + Bv + Q\ddot{\delta} + C\dot{\phi} + G\ddot{\phi} + \tau_d = \tau \tag{3}$$

$$v = [\dot{\varphi}_R \quad \dot{\varphi}_L]^T,\ \mu = [\dot{\delta}_R \quad \dot{\delta}_L]^T,\ M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

$$B = m_G \frac{r^2}{2b} \varpi \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2 & Q_1 \end{bmatrix}$$

$$C = m_G \frac{r}{2} \varpi \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ G = m_G \frac{ar}{2b} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

In which the parameters of WMR are presented in Table 1.

Variable	Meaning	Value
m_G	Hardware volume	10 (kg)
m_W	Mass of each wheel	2 (kg)
I_G	Moment of inertia of the WMR hardware about the vertical axis passing through G	4 (kgm^2)
I_W	Moment of inertia of each wheel about the wheel axis	0.1 (kgm^2)
I_D	Moment of inertia of each wheel about a vertical axis passing through the wheel center	0.05 (kgm^2)
a	Distance between point M and point G	0.2 (m)
b	Half the distance between the two rear wheels	0.3 (m)
r	Radius of each wheel	0.15 (m)

The problem is to control the WMR to follow a given trajectory. Let the moving target point of the WMR be $T(x_T, y_T)$, the position error between the point $\mathrm{M}(x_M, y_M)$ and the target point $T(x_T, y_T)$ in the *MX'Y'* coordinate system is calculated as follows:

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = H \begin{bmatrix} x_T - x_M \\ y_T - y_M \end{bmatrix} \tag{4}$$

$$H = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}$$

$$\dot{e} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \kappa v + \xi_1 \tag{5}$$

$$\kappa = \begin{bmatrix} \frac{(e_2 - b)r}{2b} & -\frac{(e_2 + b)r}{2b} \\ -\frac{e_1 r}{2b} & \frac{e_1 r}{2b} \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} \frac{\left(\dot{\delta}_R - \dot{\delta}_L\right)e_2}{2b} - \frac{\left(\dot{\delta}_R + \dot{\delta}_L\right)}{2} \\ \frac{-\left(\dot{\delta}_R - \dot{\delta}_L\right)e_1}{2b} - \dot{\phi} \end{bmatrix} + \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_T \\ \dot{y}_T \end{bmatrix}$$

We choose $x_1 = e$, $x_2 = \dot{x}_1 + \lambda x_1$

$$\dot{x}_1 = \dot{e} = \kappa v + \xi_1 \tag{6}$$

$$\ddot{x}_1 = \kappa \dot{v} + \dot{\kappa} v + \dot{\xi}_1 \tag{7}$$

$$\dot{v} = -M^{-1}Bv - M^{-1}\left(Q\ddot{\delta} + C\dot{\phi} + G\ddot{\phi} + \tau_d\right) + M^{-1}\tau = -M^{-1}Bv + M^{-1}\tau + \xi_2 \tag{8}$$

$$\xi_2 = -M^{-1}\left(Q\ddot{\delta} + C\dot{\phi} + G\ddot{\phi} + \tau_d\right)$$

$$\ddot{x}_1 = -\kappa M^{-1}Bv + \kappa M^{-1}\tau + \kappa\xi_2 + \dot{\kappa}v + \dot{\xi}_1 \tag{9}$$

$$\dot{x}_2 = \ddot{x}_1 + \lambda \dot{x}_1 \tag{10}$$

$$\dot{x}_2 = -\kappa M^{-1}Bv + \kappa M^{-1}\tau + \kappa\xi_2 + \dot{\kappa}v + \dot{\xi}_1 + \lambda\kappa v + \lambda\xi_1 = \mathrm{K}_1 v + \mathrm{N}\tau + \xi_3 \quad (11)$$

$$\mathrm{K}_1 = -\kappa M^{-1}B;\ \mathrm{N} = \kappa M^{-1};\ \xi_3 = \kappa\xi_2 + \dot{\kappa}v + \dot{\xi}_1 + \lambda\kappa v + \lambda\xi_1$$

$$v = \kappa^{-1}\dot{x}_1 - \kappa^{-1}\xi_1 = \kappa^{-1}\left(x_2 - \lambda x_1\right) - \kappa^{-1}\xi_1 = \kappa^{-1}x_2 - \kappa^{-1}\lambda x_1 - \kappa^{-1}\xi_1 \quad (12)$$

$$\dot{x}_2 = \mathrm{K}_1 v + \mathrm{N}\tau + \xi_3 = \mathrm{K}_1\kappa^{-1}x_2 - \mathrm{K}_1\kappa^{-1}\lambda x_1 - \mathrm{K}_1\kappa^{-1}\xi_1 + \mathrm{N}\tau + \xi_3 = \mathrm{K}x_2 - \mathrm{K}\lambda x_1 + \mathrm{N}\tau + d \quad (13)$$

$$\mathrm{K} = \mathrm{K}_1\kappa^{-1},\ d = \xi_3 - \mathrm{K}_1\kappa^{-1}\xi_1$$

$$\begin{cases} \dot{x}_1 = x_2 - \lambda x_1 \\ \dot{x}_2 = \mathrm{K}x_2 - \lambda\mathrm{K}x_1 + \mathrm{N}\tau + d \end{cases} \quad (14)$$

From this we have the nonlinear system

$$\dot{x} = f(x) + g_u\tau + g_{di}d \quad (15)$$

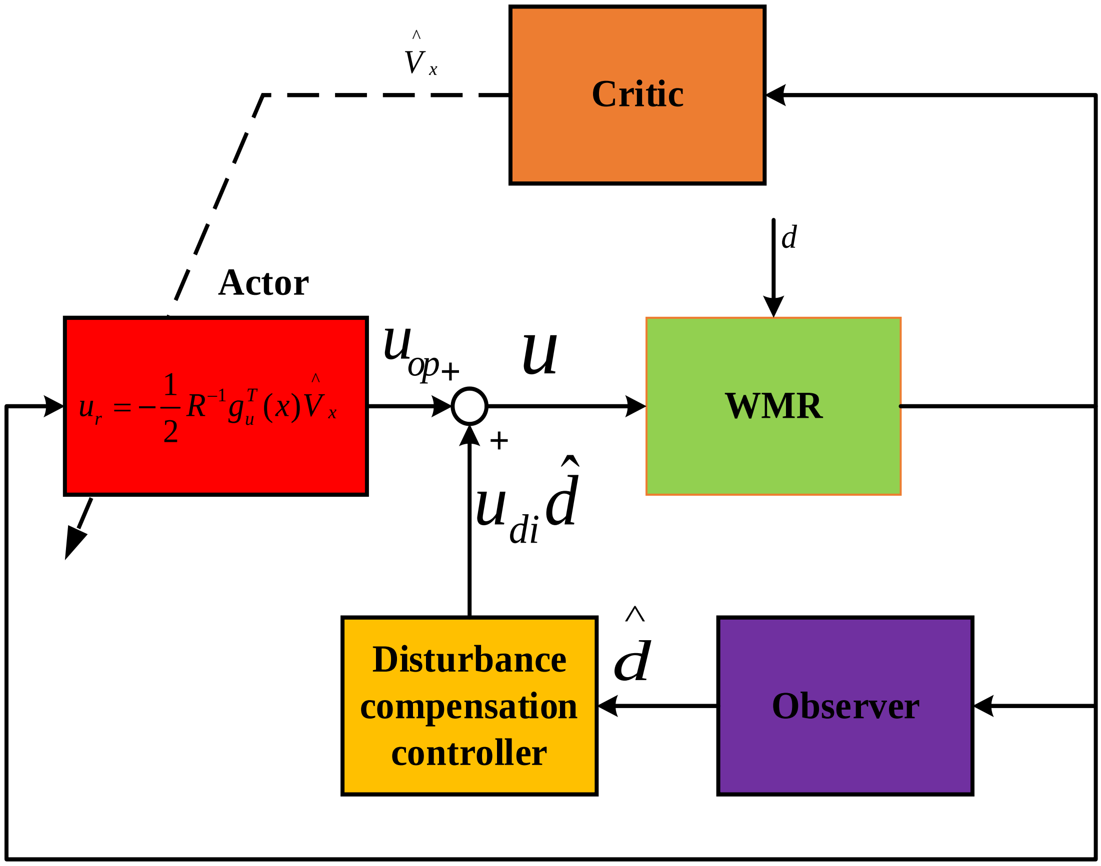

Figure 2: Control structure diagram

III. Controller Design for WMR

Optimal Controller

Consider the nonlinear system

$$\dot{x} = f(x) + g_u\tau \quad (16)$$

Cost function is defined [21-24]:

$$J(x) = \int_t^\infty r\left(x, u_{op}\right)dt \quad (17)$$

The Hamilton function is defined

$$H(x, u_{op}, J) = \left(\frac{\partial J}{\partial x}\right)^T \dot{x} + r\left(x, u_{op}\right) = \left(\frac{\partial J}{\partial x}\right)^T \dot{x} + x^T Q x + u_{op}^T R u_{op} \quad (18)$$

The HJB equation:

$$H(x,u_{op},V)=\left(\frac{\partial V}{\partial x}\right)^T \dot{x}+x^TQx+u_{op}^TRu_{op}=0 \tag{19}$$

Then, the optimal control signal is calculated according to the following formula:

$$u_{op}^*=-\frac{1}{2}R^{-1}g_u^T\frac{\partial V}{\partial x} \tag{20}$$

$V(x,u_{op})$ is approximated by a neural network of the following form:

$$V(x,u_{op})=W^T\Phi(x)+\varepsilon(x) \tag{21}$$

Then the optimal control signal becomes:

$$u_{op}^*=-\frac{1}{2}R^{-1}g_u^TW^T\frac{\partial \Phi}{\partial x} \tag{22}$$

$V(x,u_{op})$ is estimated by:

$$\hat{V}(x,u_{op})=\hat{W}^T\Phi(x) \tag{23}$$

Then update the control signal as follows:

$$u_{op}^{i+1}=-\frac{1}{2}R^{-1}g_u^T\left(\hat{W}^T\right)^i\frac{\partial \Phi}{\partial x} \tag{24}$$

The governing law for weight would be:

$$\dot{\hat{W}}=\begin{cases}\dot{\hat{W}}_1 \text{ if } x^T\left(f(x)+g_u(x)u_{op}\right)\le 0\\ \dot{\hat{W}}_1+W_{RB} \text{ contrast}\end{cases} \tag{25}$$

Disturbance Observer

Consider the nonlinear system

$$\dot{x}=f(x)+g_u(x)u+g_{di}(x)d \tag{26}$$

Since the disturbance of the system is unknown, we use an observer to approximate the noise of the system (26) as follows [25,26]:

$$\begin{cases}\hat{d}=\sigma+\varsigma(x)\\ \dot{\sigma}=-k(x)\left\{g_{di}(x)\left[\sigma+\varsigma(x)\right]+f(x)+g_u(x)u\right\}\end{cases} \tag{27}$$

The function $k(x)$ is chosen as:

$$k(x)=l_d\left[0_{n\times n}^T \quad M^{-1}(q_d-e_1)\right]^T \tag{28}$$

Then $\varsigma(x)$ and $P(x)$ are calculated as follows:

$$\varsigma(x)=\int k(x)dx \tag{29}$$

$$P(x)=k(x)g_{di}(x)=l_dI_{2\times 2} \tag{30}$$

The selected disturbance compensation components are:

$$u_{di}=-\alpha(x) \tag{31}$$

The proposed controller is designed as follows:

$$u=-\frac{1}{2}R^{-1}g_u^T\left(\hat{\mathrm{W}}^T\right)^i\frac{\partial \Phi}{\partial x}-\alpha(x)\hat{d} \tag{32}$$

The above is the orbital tracking control algorithm for WMR based on the use of noise observers combined with reinforcement learning techniques to improve tracking quality and robust adaptive when the robot has changing parameters and disturbances.

IV. Result

In order to verify the correctness of the sustainable optimal tracking algorithm based on the ADP algorithm with AC structure, the paper performs numerical simulation on Matlab-Simulink software with the selected parameters as follows:

$$Q = \begin{bmatrix} 40 & 2 & -4 & 4 & 8 \\ 2 & 40 & 4 & -6 & 0 \\ -4 & 4 & 4 & 0 & -4 \\ 4 & -6 & 0 & 4 & 2 \\ 8 & 0 & -4 & 2 & 8 \end{bmatrix}; \; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The simulation results are shown in Fig. 3, 4, 5, 6, 7, 8.

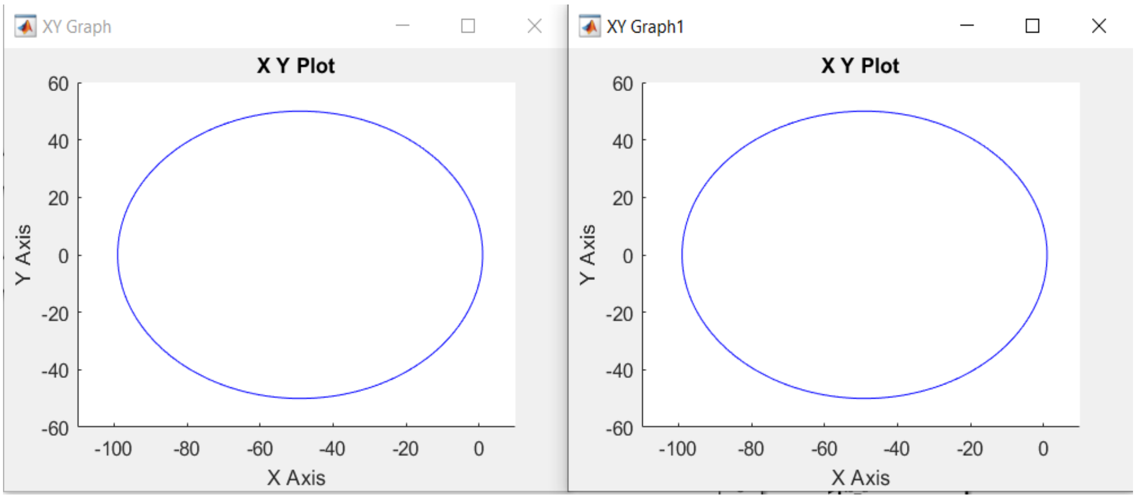

Figure 3: Set trajectory and actual trajectory of the robot

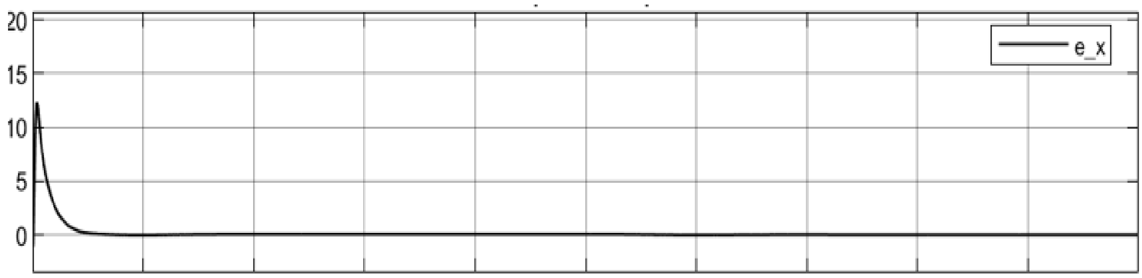

Figure 4: Trajectory tracking error in *x* axis

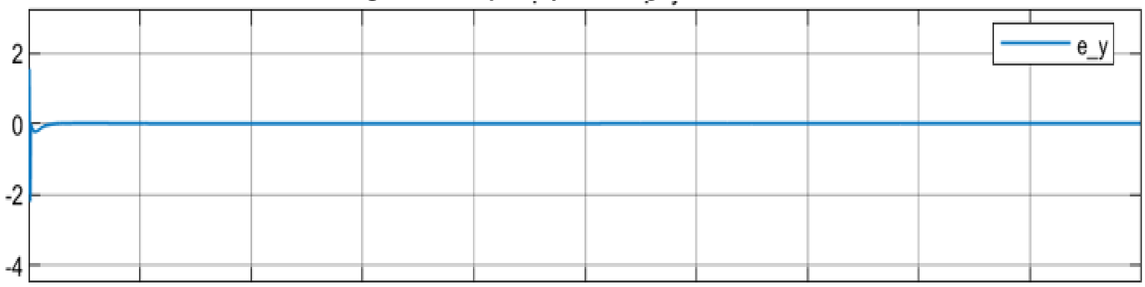

Figure 5: Trajectory tracking error in *y* axis

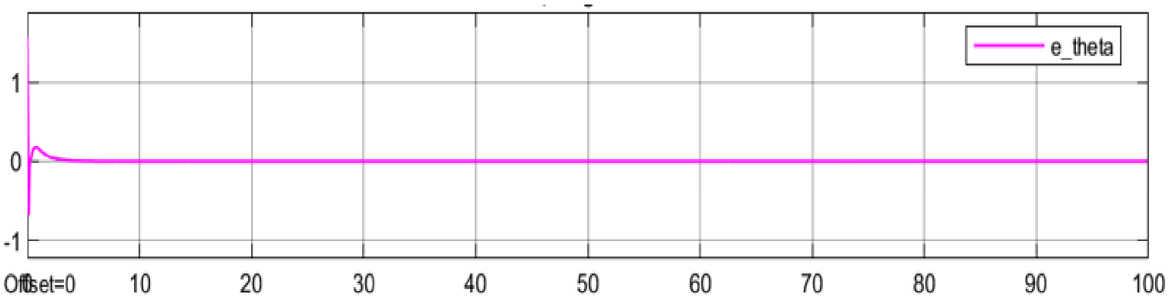

Figure 6: Trajectory tracking error in direction angle

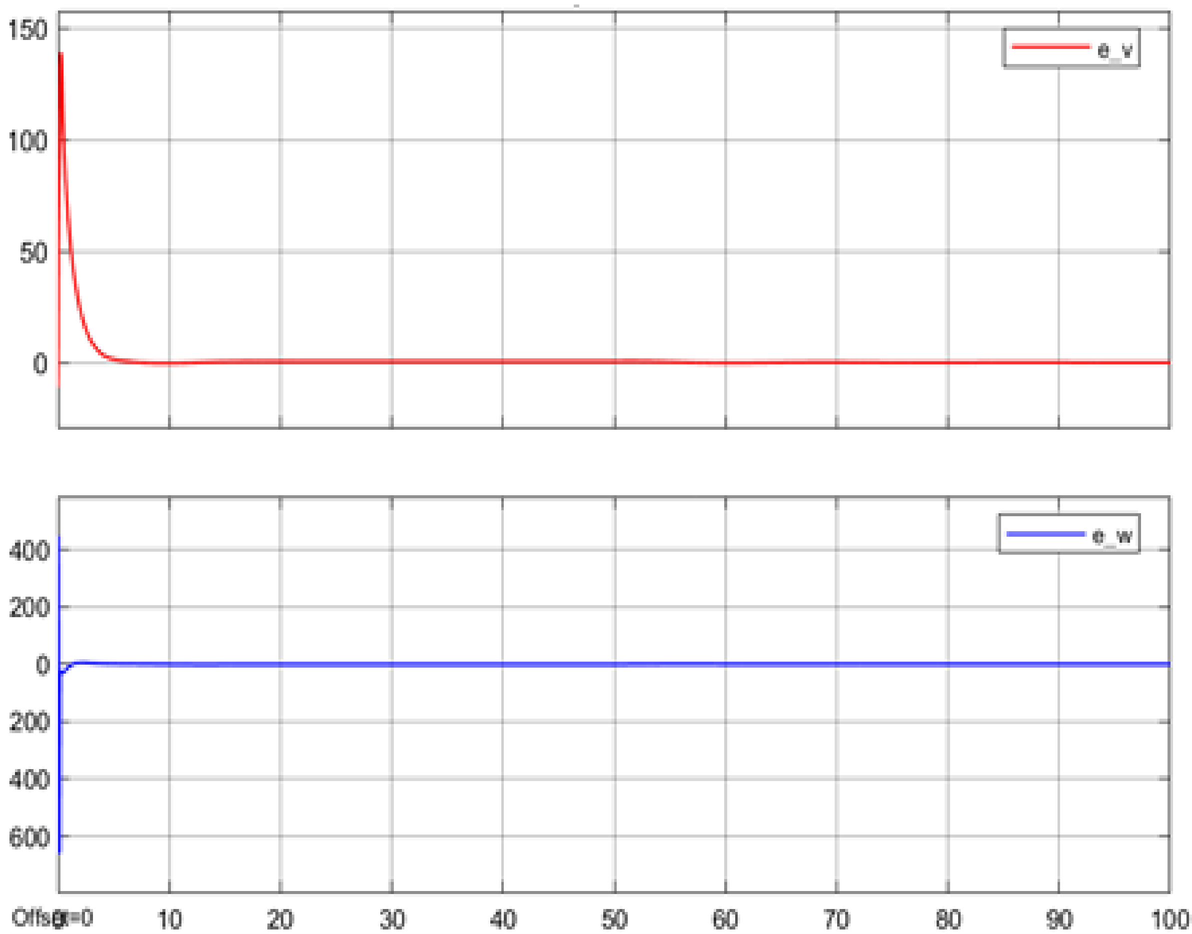

Figure 7: Error of linear and angular velocity

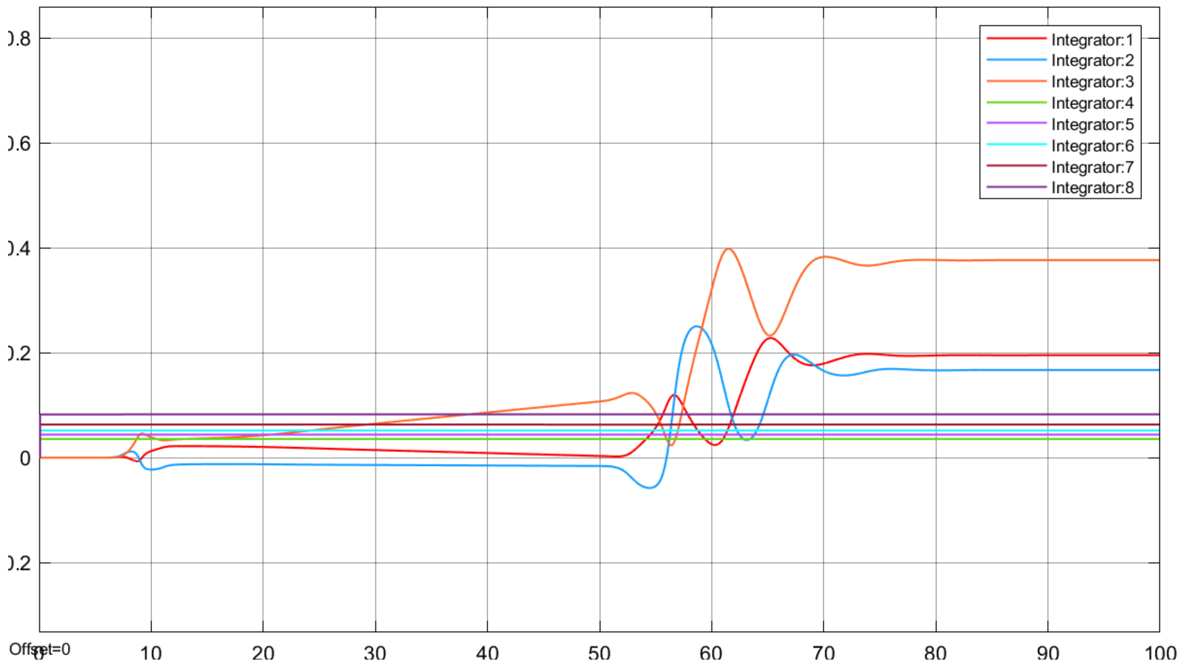

Figure 8: Convergence of the weight matrix NN

From the simulation results, it can be seen that at the beginning of the simulation, Critic is in the learning process. Responses of output states tend to follow reference values. After about 75s, the learning process ends, the weights of the neural network W converge to the ideal value.

V. Conclusion

The online adaptive dynamic process method combined with the noise observer is proposed to solve the sustainable optimization problem for WMR systems. The scheme with only one neural network provides better performance, i.e. reduces computation time, increases the quality of the system. The stability of the whole system consisting of Actor, Critic components and disturbance observer is proved mathematically through Lyapunov theory. The simulation results show that the observer-based OADP technique is capable of providing a good response for WMR even under unstable system conditions and the influence of external disturbance.

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