Geometrical Positioning Schemes for MS Location Estimation

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Abstract: To achieve more accurate measurements of the mobile station (MS) location, it is possible to integrate many kinds of measurements. In this paper we proposed several hybrid methods that utilized time of arrival (TOA) at seven base stations (BSs) and the angle of arrival (AOA) information at the serving BS to give location estimation of the MS in non-line-of-sight (NLOS) environments. From the viewpoint of geometric approach, for each a TOA value measured at any BS, one can generate a circle. The MS position is then given by the intersections of the circles from multiple TOA measurements. Rather than applying the nonlinear circular lines of position (LOP), the proposed methods are easier by using linear LOP to determine the MS. In addition, the proposed methods can mitigate the NLOS effect, simply by applying the weighted sum of the intersections between different linear LOP and the AOA line, without requiring priori knowledge of NLOS error statistics. Simulation results show that the proposed methods can always yield superior performance in comparison with Taylor series algorithm (TSA) and the hybrid lines of position algorithm (HLOP).

Keywords- time of arrival (TOA); angle of arrival (AOA), non-line-of-sight (NLOS).

I. Introduction

The definition of wireless location is to determine the position of the mobile station (MS) in wireless communication networks. It has received significant attention, and various location technologies have been proposed in the past few years. It is always desirable to achieve the higher accuracy in location applications as possible. However, the requirements and performances for different applications may differ due to various reasons such as the cost and the technology. For those emergency applications, such as emergency services E-911 and other applications based on location-sensitive billing, would require high accuracy. However, for the other location applications such as fleet management, one can utilize lower-accuracy location techniques [1].

There are various techniques for wireless location, which can be classified into two categories -- handset and network-based techniques. Handset-based solutions generally require a handset modification to calculate its own position when fully or partially equipped with a global positioning system (GPS) receiver. From the technical aspect, the handset-based techniques are more accurate to determine the MS. The drawbacks of this technique include not only the high cost for developing a suitable low-power, but economical integrated technology for applying in the handsets. Moreover, a GPS receiver needs to have at least four satellites constantly visible. Therefore, the GPS-based solution is a feasible option for outdoor positioning, but not for indoor positioning within dense urban environments. The network-based methods require the estimation of the mobile location based on the received signals between the MS and a set of base stations (BSs). The existing wireless communications infrastructure without supplementary technology has been utilized in MS location estimation. One of the goals of the location solution is to allow carriers to locate current users by the existing network without expensive modifications, as well as be adaptable to complement satellite handset-based techniques. A robust estimate of location in all environments should be provided by the optimum location technology.

On the other hand, the primary network-based techniques include signal strength [2], angle of arrival (AOA) [3], time of arrival (TOA) [4], and time difference of arrival (TDOA) [5] techniques. Signal strength is a well known location method that uses a known mathematical model describing the path loss attenuation with distance. The AOA scheme utilizes an antenna array and a directive antenna to estimate the direction of arrival signal. TOA location scheme measures the arrival time of the radio signal coming from various BS. The TDOA scheme measures the time difference between the radio signals.

The accuracy of MS location estimation highly depends on the propagation conditions of the wireless channels. The propagation signals in wireless location system are usually corrupted by additive noise, multipath
propagation, multiple access interference, and line-of-sight (LOS) blockage. Many procedures are necessary to reduce the effects of impairments.

The additive noise is relatively easy to control comparing to the other wireless channel impairment. Usually, it is modeled as a zero mean Gaussian white noise with variance determined by the signal to noise ratio (SNR), measurement resolution, and other factors. It can be effectively suppressed by higher SNR and improved resolution of measurements. Compared to conventional code division multiple access (CDMA) systems, the higher chip rate of wideband CDMA system facilitates higher accuracy in time-based measurements.

In wireless locating systems, the transmitted signals are frequently corrupted with multipath propagations even when there is LOS path between the MS and BS. First of all, we define multipath as the presence of multiple signal paths between the MS and BS. The reflections and diffractions from buildings in urban areas or mountains in rural terrains can cause significant path blockages and multipath time dispersions [7]. Even when the LOS propagation exists, the multipath propagations can induce errors in the timing estimations of the time-based location systems [6]. The multipath propagations may also cause serious problems in the signal strength measurements [6]. In the case of AOA schemes, multipath in the propagation channels would significantly degrade the performance of the estimation of direction-of-arrival [20]. High-resolution frequency estimator has been developed to mitigate the effects of multipath on delay estimation [3].

In cellular CDMA, users share the same frequency band with different spreading codes. Since the signals from the different MSs are received with unequal power level by a BS, it is difficult to recover the weaker users. Thus, the primary obstacle to high capacity in CDMA cellular systems is the near-far effect. To reduce near-far interference, power control schemes should be applied on the reverse link. Therefore, the user’s signal is received with the same level power at the BS [22]. The near-far resistant delay estimators based on the subspace technique, or in conjunction with multi-user detectors and interference cancellation techniques, can yield superior location estimation in the presence of multiple access interference [6].

The dominant error for wireless location systems is usually due to the NLOS propagation effect. High accuracy can be achieved in location estimation if LOS propagation exists between the MS and each participating BS. However, in practice LOS paths are not always available, especially in urban or suburban areas. Due to the extra path length traveled by the reflection or diffraction of the signal between the MS and the BSs, NLOS propagation can introduce a bias in time-based measurements, even when multipath is absent and when high-resolution techniques are used. The NLOS propagation will heavily degrade the precision of MS location estimation. Therefore, it is very important to reduce and mitigate the effects of NLOS propagation. In the past few years, extensive research on NLOS-effect mitigation for location estimation has been carried out. In [8], the residual weighting algorithm was used to mitigate the location error caused by measurement and NLOS noises. Similar residual schemes were proposed for AOA systems in [17] and for TDOA system in [18]. These approaches failed to work with multiple NLOS BSs, as the outliers tend to bias the final estimate and precision to reduce the residuals.

To enhance the precision of location estimation, it is reasonable to consider hybrid methods of integration two or more schemes. In [19] Sprito proposed the combination of TDOA and TOA, and showed that it is better than pure TDOA on the location estimation in GSM systems. If two BSs can provide both TOA and AOA measurements simultaneously, we have proposed the geometrical positioning methods utilizing the intersection of two circles and two lines to estimate MS location [15]. In addition, we expanded the methods in [15] to locate MS when three BSs are available for location purposes [21]. TOA measurements from three BSs and the AOA information at the serving BS can be used to give a location estimate of the MS. These methods can provide much better accuracy but with higher complexity, because the related circle equations are nonlinear.

In order to reduce the computational load, the linear lines of position (LOP) formed by the intersection points of two circles is employed to replace the circular LOP for estimating the MS location in this paper. This technique is more simple than applying nonlinear circle LOP. Due to NLOS error, the MS may be located at various intersections of different LOP and AOA line. Since the NLOS range errors are always positive, TOA measurements are greater than the true values. Therefore the true MS location should lie in the overlap region of the seven circles. The above intersections are defined as the feasible intersections, By acquiring the feasible intersections of various linear LOP and AOA line, it is possible to locate the desired MS under the constraint that the MS can be heard by seven BSs. The proposed methods have the advantage of simpler computation of MS location. Simulation results show that the proposed methods always perform better than Taylor series algorithm (TSA) [10][11] and the hybrid lines of position algorithm (HLOP) [13].

The rest of this paper is organized as follows. Section II describes the positioning methods by using TSA and HLOP. Section III introduces numerous approaches that use the intersections of various linear LOP and the AOA line to estimate the position of MS. Section IV applies the simulations to evaluate the algorithms and analyze the results obtained. Finally, conclusions are drawn in Section V.
II. Taylor Series Algorithm (TSA) And Hybrid Lines Of Position Algorithm (HLOP)

If both the TOA and AOA measurements are accurate, only one BS is required to locate the MS [15]. In reality, TOA and AOA measurements contain errors due to NLOS propagation. Since the NLOS errors would seriously degrade location accuracy, so more than one BS is required for MS location of reasonable accuracy. Let \( t_i \) denote the propagation time from the MS to BS \( i \), the distances between BS \( i \) and the MS can be expressed as

\[
    r_i = c \cdot t_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2}
\]

where \( c \) is the propagation speed of the signals, \((x, y)\) is the MS location, and the coordinates of BS \( i \) are given by \((X_i, Y_i)\). We assume that BS1 is the serving BS, and let \( \theta \) be the angle between MS and its serving BS, which is defined as:

\[
    \theta = \tan^{-1}\left(\frac{y - Y_1}{x - X_1}\right)
\]

2.1. Taylor Series Algorithm (TSA)

TOA and AOA measurements are inputs to the Taylor series position estimator when seven BSs are available, as shown in Fig. 1. The Taylor series approach can achieve high accuracy, but requires an initial location guess. TSA may suffer from the convergence problem if the initial guess is not accurate enough. Let \((x, y)\) be the true position and \((x_i, y_i)\) be the initially estimated position. Assume that \( x = x_i + \delta x, \ y = y_i + \delta y \).

By linearizing the TOA and AOA equations of (1) and (2) with Taylor series expansion and retaining second-order terms, we have

\[
    A\delta \approx z
\]

where

\[
    A = \begin{bmatrix}
        a_{11} & a_{12} \\
        a_{21} & a_{22} \\
        \vdots & \vdots \\
        a_{71} & a_{72}
    \end{bmatrix} \quad \delta = \begin{bmatrix}
        \delta_x \\
        \delta_y \\
        \vdots \\
        \delta_{\theta}
    \end{bmatrix} \quad z = \begin{bmatrix}
        r_1 - r_{11} \\
        r_2 - r_{12} \\
        \vdots \\
        r_7 - r_{17}
    \end{bmatrix}
\]

and

\[
    a_{11} = \frac{\partial r_i}{\partial x_{i_i}}, \quad a_{12} = \frac{\partial r_i}{\partial y_{i_i}}, \\
    a_{21} = \frac{\partial r_i}{\partial x_{i_2}}, \quad a_{22} = \frac{\partial r_i}{\partial y_{i_2}}, \\
    \vdots \\
    a_{71} = \frac{\partial r_i}{\partial x_{i_7}}, \quad a_{72} = \frac{\partial r_i}{\partial y_{i_7}},
\]

\[
    r_{ii} = \sqrt{(x_i - X_i)^2 + (y_i - Y_i)^2}, i = 1, 2, \ldots, 7, \quad b_{11} = \frac{\partial \theta}{\partial x_{i_1}}, \\
    b_{72} = \frac{\partial \theta}{\partial y_{i_7}}, \quad \theta = \tan^{-1}\left(\frac{y - Y_1}{x - X_1}\right).
\]

Then, the least-square (LS) estimation can be solved by

\[
    \delta = (A^TA)^{-1}A^Tz
\]

The process starts with an initial guess for the MS location and can achieve high accuracy. This method is recursive but tends to be computationally intensive. The convergence of this iterative process is not assured, which depends on the accuracy of the initial guess of the MS location [10] [11].

2.2. Hybrid Lines of Position Algorithm (HLOP)

The geometrical interpretation is presented in this section for which the linear LOP, rather than the circular LOP, are proposed to determine the position of the MS. The detail algorithm of the linear LOP approach can be acquired by using both the TOA measurements as in [12], and the hybrid linear LOP and AOA measurement (HLOP) in [13]. Given the linear LOP and AOA line, the equations that describe all the lines can be written in matrix form as

\[
    Gl = h
\]

where \( l = \begin{bmatrix} x \\ y \end{bmatrix} \) denotes the MS location,

\[
    G = \begin{bmatrix}
        X_1 - X_2 & Y_1 - Y_2 \\
        \vdots & \vdots \\
        X_i - X_j & Y_i - Y_j \\
        \tan \theta & -1
    \end{bmatrix}
\]

and

\[
    h = \begin{bmatrix}
        y_1 - Y_1 \\
        \vdots \\
        y_j - Y_j
    \end{bmatrix}
\]

where \( h \) is the position of the MS and \( G \) is a matrix containing the coefficients of the lines. The solution of the LS estimation can be solved by

\[
    l = (G^TG)^{-1}G^Th
\]
According to the LS, the solution to Eq. (5) is given by

$$l = (G^T G)^{-1} G^T h$$

(6)

Figure 1. Seven-cell system layout.

III. Proposed Hybrid TOA/AOA Methods

In the TOA approach, the distance between an MS and a BS is measured by finding the propagation time between an MS and a BS. Geometrically, TOA measurement generates a circle and the MS lie on a circle centered at the BS. Using the circles produces by TOA measurements at multiple BSs, the MS location can be found at the intersections of circles. The AOA method utilizes antenna arrays or directive antennas to estimate the direction of arrival of the signal. A single AOA measurement constrain the MS along a line. The equations of the seven TOA circles and the AOA line used in location estimation can be expressed as

**Circle 1-7:**

$$\left( x - x_i \right)^2 + \left( y - y_i \right)^2 = r_i^2, \; i = 1, 2, \ldots, 7$$

(7)

**Line 1:**

$$\tan \theta \cdot x - y = 0$$

(8)

We have proposed the methods that employ the intersections of the TOA circles and the AOA line to estimate the MS location in [15] [21]. These methods can achieve high accuracy of MS location, but the computational complexity is more intensive.

The complexity can be much reduced for solving the intersection of two linear lines rather than two nonlinear circles. Replacing conventional circular LOP, the linear LOP equation passes through the intersections of the two circular can be used in our proposed methods. The line which passes through the intersections of the two circular LOP, be found by squaring and subtracting the distances obtained by Eq. 1, can be expressed as

**Line L**

$$L_i: X + Y = \frac{1}{2}(r^2 - r_i^2 + X_i^2 + Y_i^2)$$

(9)

Calculating the intersections of the various LOP and AOA line produced the MS location estimate. Under the assumption of LOS propagation and there exists no measurement error, the intersections of these LOP and AOA line are at the same point. However, it is very often that the LOS does not exist for propagation of signals between an MS and some fixed BSs. Therefore, the NLOS effect could cause these LOP and AOA line to intersect at various points, which will be offset from the true MS location. With NLOS propagation, the measured TOA values are always greater than the true TOA values due to the excess path length. The true MS location should be inside the region, enclosed by the overlap of the seven circles. As mentioned previously, the intersections that are within this region are defined as feasible intersections. The feasible intersections must satisfy the following inequalities simultaneously:

$$\left( x - x_i \right)^2 + \left( y - y_i \right)^2 \leq r_i^2, \; i = 1, 2, \ldots, 7$$

(10)

In order to enhance the performance of MS location estimation with less complexity, the hybrid TOA/AOA methods are proposed, which integrate these linear LOP and AOA line to find all the feasible intersections to determine the MS location. The proposed positioning methods are based on the weighted sum of the feasible
intersections of these LOP and the AOA line. These methods are much less difficult since there is no need to compute the intersections of circles. The complexity and computation load can be reduced to find the solution of two linear line equations rather than nonlinear circle ones. The details are as follows.

1) Distance-Weighted Method
The simplest method to estimate the MS location is to calculate the average of those feasible intersections. However, not all feasible intersections apply the same weighting factors for MS location estimation. The closer to the averaged MS location, the higher the weight should be. In this method, the weighting factor of each feasible intersection is inversely proportional to the squared distance of the averaged MS location to the feasible intersection.

Step 1. Find all the feasible intersections of these LOP and AOA line.
Step 2. The average MS location \((\bar{x}_N, \bar{y}_N)\) is defined by
\[
\bar{x}_N = \frac{1}{N} \sum_{i=1}^{N} x_i \quad \text{and} \quad \bar{y}_N = \frac{1}{N} \sum_{i=1}^{N} y_i,
\]
where \((x_i, y_i)\) are feasible intersections.
Step 3. Calculate the distance \(d_i\) between each feasible intersection \((x_i, y_i)\) and the average location \((\bar{x}_N, \bar{y}_N)\):
\[
d_i = \sqrt{(x_i - \bar{x}_N)^2 + (y_i - \bar{y}_N)^2}, \quad 1 \leq i \leq N
\]
Step 4. Set the weighting factor for the \(i\)th feasible intersection as \((d_i^{-2})^{-1}\). Then the MS location \((x_*, y_*)\) is determined by
\[
x_* = \frac{\sum_{i=1}^{N} (d_i^{-2})^{-1} x_i}{\sum_{i=1}^{N} (d_i^{-2})^{-1}} \quad \text{and} \quad y_* = \frac{\sum_{i=1}^{N} (d_i^{-2})^{-1} y_i}{\sum_{i=1}^{N} (d_i^{-2})^{-1}}.
\]
Since some of the feasible intersections are too far away from the averaged MS location, these feasible intersections may not provide improved accuracy of MS location. Therefore, we proposed both sort averaging method and sort-weighted method, which does not consider those contributions far away from the feasible intersections.

2) Sort Averaging Method
Steps 1-3 are the same as those of the distance-weighted method.
Step 4. Rank the distances \(d_i\) in increasing order and re-label the feasible intersections in this order.
Step 5. The MS location \((\bar{x}_M, \bar{y}_M)\) is estimated by the mean of the first \(M\) feasible intersections.
\[
\bar{x}_M = \frac{1}{M} \sum_{i=1}^{M} x_i \quad \text{and} \quad \bar{y}_M = \frac{1}{M} \sum_{i=1}^{M} y_i \quad (M = 0.5^{*}N \leq N)
\]

3) Sort-Weighted Method
Steps 1-4 are the same as those of the sort averaging method.
Step 5. The MS location is estimated by a weighted average of the first \(M\) feasible intersections with weighting factor \((d_i^{-2})^{-1}\).
\[
x = \frac{\sum_{i=1}^{M} (d_i^{-2})^{-1} x_i}{\sum_{i=1}^{M} (d_i^{-2})^{-1}} \quad \text{and} \quad y = \frac{\sum_{i=1}^{M} (d_i^{-2})^{-1} y_i}{\sum_{i=1}^{M} (d_i^{-2})^{-1}} \quad (M = 0.5^{*}N \leq N)
\]

4) Threshold Method
In the previously proposed method, the decision of each weight is based on how close the feasible intersections are. The closer the feasible intersections, the more weight will be assigned. In addition, if the distance is larger than a threshold value, then the contribution to the location estimation is neglected. The detailed steps are as follows:
Step 1. Find all the feasible intersections of these LOP and AOA line.
Step 2. Calculate the distance \(d_{mn}\), \(1 \leq m,n \leq N\), between any pair of feasible intersections.
Step 3. Define a threshold value \(D_{mn}\) as the average of all the distances \(d_{mn}\) calculated in Step 2.
Step 4. Set the initial weight \(I_{mn}\), \(1 \leq k \leq N\), to be zero for all feasible intersections. If \(d_{mn} \leq D_{mn}\), then \(I_m = I_m + 1\) and \(I_n = I_n + 1\) for \(1 \leq m,n \leq N\).
Step 5. The MS location \((x_*, y_*)\) is estimated by
IV. Simulation Results

Assuming no knowledge of the NLOS errors in advance, computer simulations are performed to demonstrate the performance of the proposed location scheme. A number of 10,000 independent trials are performed for each simulation. The serving BS, that is, BS1, is located at (0, 0). Each cell has a radius of 2000 m and the MS locations are uniformly distributed in the center cell [23], as shown in Fig. 1. Regarding the NLOS effects in the simulations, three different propagation approaches were used to model the measured ranges and angle, distance-dependent model [9], the uniformly distributed noise model [9] and the circular disk of scatterers model (CDSM) [9] [14].

The first NLOS propagation model is based on the distance-dependent NLOS error model [9]. The NLOS range error for the $i$-th range $i$ is taken to be $\xi_i = \chi_i \cdot R_i$, where $\chi_i$ is a proportional constant and $R_i$ is the true range between $i$-th BS and MS. The AOA measurement error is assumed to be $f_i = w \cdot \tau_i$, where $\tau_i = 5^\circ$ and $w$ is a uniformly distributed variable over [-1, 1] [16]. Figure 2 shows the cumulative distribution function (CDF) of the average location error of different algorithms with distance-dependent NLOS error for $\chi_i = 0.13$. Because of the different operation principle, HLOP can cancel the part of the squared NLOS errors. One can see that HLOP gives better performance than TSA. But the proposed methods can give much better location estimation as compared with the other existing algorithms.

![CDFs of the location error using the distance-dependent NLOS error model for $\chi_i = 0.13$.](image)

The second NLOS propagation model is based on the uniformly distributed noise model [9], in which the TOA measurement error is assumed to be uniformly distributed over $(0, U_i)$, where $U_i$ is the upper bound. The AOA measurement error is assumed to be $f_i = w \cdot \tau_i$, where $\tau_i = 5^\circ$ [16]. Figure 3 shows how the average location error versus the upper bound of the uniform NLOS error. As expected, the location error increases with the upper bound of the uniform NLOS error. When the NLOS situation gets worse, both TSA and HLOP provide relatively poor estimation performance. The performance degradation of the proposed methods is not pronounced for large NLOS errors. The superior MS location prediction for the proposed hybrid TOA/AOA still can reduce the RMS errors effectively and estimate the MS location accurately.
Figure 3. Performance comparison of the location estimation methods versus the upper bound of the uniform NLOS.

The BS1 serving a particular MS is called the serving BS which can provide more accurate measurements. The variables of this model are chosen as follows: \( U_1 = 200 \) m, \( U_2 = 600 \) m, \( i = 2, 3, \ldots, 7 \), and \( \tau_i = 5^\circ \). Figure 4 shows CDF of the average location error of different algorithms for the cases when the range errors were using the uniformly distributed noise model. The distance-weighted method provides the best results, followed by the threshold method. Furthermore, the performance of the proposed method is significantly better than TSA and HLOP.

Figure 4. Comparison of error CDFs when NLOS errors are modeled as the upper bound.

The final NLOS propagation model is based on CDSM [9] [14]. The CDSM assumes that there is a disk of scatterers around the MS and that signals traveling between the MS and the BSs undergo a single reflection at a scatterer. The measured AOA at the serving BS would in reality be the angle between the BS1 and a scatterer. The measured ranges are the sum of the distances between the BS and the scatterer and between the MS and the scatterer. The radius of the scatterers for BS1 and the other BSs were taken to be 100 m and 300 m, respectively. As illustrated in Fig. 5 we can see the improvement in location accuracy by using the proposed method from the CDF curves of the location error. The proposed methods can always generate more accurate MS location estimates than TSA and HLOP.

V. Conclusions

In this paper we presented several hybrid location methods by using one AOA measurement together with the TOA measurements to get improved MS location estimation. The traditional geometrical approach for computing MS location is to solve for the intersections of the circular LOP. However, in this paper the linear LOP is derived to reduce the complexity. Based on the NLOS situation and without the knowledge of NLOS
error statistics in advance, the proposed hybrid methods utilize all the feasible intersections of these linear LOP and the AOA line to locate the MS. The proposed methods mitigate the NLOS effect simply by applying the weighted sum of the intersections between different linear LOP and the AOA line. Simulation results show that the proposed methods consistently achieve the better performance than the conventional TSA and HLOP in the MS location estimation, regardless the NLOS propagation model. Simulation results demonstrate that the accurate MS location estimate of the proposed methods is possible even in severe NLOS conditions.

![Figure 5. Comparison of error CDFs when CDSM is used to model the NLOS errors.](image-url)

References


