IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 14, Issue 3 Ser. II (May. – June. 2019), PP 43-52
www.iosrjournals.org

Adaptive Output Feedback Control Strategy Based Chaotic Synchronization with Parametric Uncertainty

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Abstract *: This paper explores the master-slave synchronization approach using output feedback control with parametric uncertainty. Initially state feedback controller and updation laws are developed using Lyapunov stability theory considering uncertainty in all parameters. Further, output feedback control scheme is proposed by assuming some limited set of parameters as uncertain. Thus, the controller and updation laws are developed that depend only on the output state. Sufficient stability conditions are obtained to ensure chaotic synchronization in master-slave configuration. The validation of proposed technique is depicted using simulations of chaotic Chua's circuit, which belongs to Lur'e class of nonlinear systems.*

Keywords*: Lyapunov stability, Output feedback control, Parametric uncertainty, Synchronization.*

Date of Submission: 20-05-2019 Date of acceptance: 05-06-2019

I. Introduction

In the area of control system, synchronization of two systems is very essential process. It is very important phenomena in almost all the fields of science and engineering. Many researchers have evolved many systematic synchronization ways for different schemes like chaotic systems which are sensitive to parametric variations and beginning conditions [1]-[3]. The Chaotic synchronization has gained appreciable attention of scientists from multifaceted scrutinize groups [4], [5] because of its several utilizations in secure communication [6], [7], in the field of biological and chemical systems [8], neural networks [9], signal processing [10] etc.

The work of Pecora and Carroll on developing the technique of synchronization of two similar chaotic system in 1990's has led to introduction and exploration of several synchronizing scheme [11]. The seminal work done by many researchers in the field of chaos synchronization problem has become popular in chaos theory. Linear output-feedback control, linear state-feedback control, state feedback linearization method, back-stepping technique, parameter adaptive control approach, sliding-mode control method, optimal control, nonlinear $H\infty$ controller etc. are some of the various techniques of chaotic synchronization [12]-[18]. These techniques have been explored an excellent attention in the last two decades.

One of the important type of chaotic system synchronization is master-slave synchronization. The key idea behind this synchronization scheme is to make the trajectories of slave system states follow the trajectories of master system states. In this field, quite a lot of research have been done in developing an adaptive feedback control for chaotic synchronization including and excluding uncertain parameters [19], [20]. Also there has been a considerable deal of research in synchronizing two chaotic systems using different control approaches [21]-[23]. In state feedback control method, the system's states must be well known. But practically it's almost impractical to gather all the state information of a system because all the state variables may not be approachable in many practical applications. So, Chaotic synchronization become further challenging because in some implementations only output state of system is available for feedback [24], [25]. Because of its wide application, output feedback control approach has been recognized as very convenient and efficient method for stabilization of chaotic system. In master-slave synchronization theory, formulation of error system can be done by taking difference of master system and slave system so that two systems can be synchronized. Feedback controller is to be developed so as to stabilize the error system by utilizing the available output. Synchronization can be achieved when states of error system go to zero.

In this paper, initially master-slave synchronization of specific class of system with parametric uncertainty is introduced. Further, output feedback control is developed by considering only those parameters as uncertain which are present in dynamics of state which is reflected in output. The designed output feedback controller and adaptation laws depend on the estimates and output state which are known. The limitation of this type of scheme is that the parameters estimates are available for limited set of parameters which appearing in the dynamics of output state only. However, for the presented class of systems, the approach leads to simpler control function which requires available output only, whereas for addressing similar synchronization with parametric uncertainty, more number of control functions with complex structure are generally required. Using Adaptation laws and Lyapunov stability theory, the requisite for error system converging to zero are proposed under given set of parametric uncertainty.

The remaining part of paper is arranged accordingly. The system description is presented in section II which also gives strategy for development of controller and adaptation laws based on Lyapunov stability method. In section III, the presented scheme is illustrated on Lur'e system using cubic Chua's circuit. The final remarks are contained in Section IV.

II. System Description

Let us consider two chaotic systems in master-slave configuration with dynamics of master system be given as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{\Phi}\big(\boldsymbol{H}\boldsymbol{x}(t)\big)$$

$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) \tag{1}$$

And the dynamics of slave system be given as

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}\boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big) + \boldsymbol{U} \tag{2}$$

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{H}\hat{\boldsymbol{x}}(t)$$

Above systems consist of master and slave system with control input $\boldsymbol{U} \in R^{nx1}$ being associated with slave system. Here $\boldsymbol{x}(t)$, $\hat{\boldsymbol{x}}(t) \in R^n$ are the state vectors of drive and response system, respectively. The matrices $\boldsymbol{A} \in R^{nxn}$, $\boldsymbol{B} \in R^{nxp}$ and $\boldsymbol{H} \in R^{pxn}$ are constant matrices of suitable dimensions. Here, A and B matrices are associated with parameters of system and have either constants or system parameters as part of them. $\boldsymbol{y}(t) \in R^p$ is the output of master system and $\hat{\boldsymbol{y}}(t) \in R^p$ is output of slave system. $\boldsymbol{\Phi}(.) \in R^{px1}$ is nonlinear function and depends on output state of the system. The goal is to design feedback controller considering the case of uncertainty associated with parameters.

Case I: Design of feedback controller with complete parametric uncertainty

In this case, design of a feedback controller and adaptation laws is proposed when all the parameters are assumed to be uncertain. Synchronization of the systems given in equation (1) and (2) with parametric uncertainty is proposed. For the present case, master system dynamics is same as in equation (1), whereas the dynamics of slave system gets modified as

$$\dot{\hat{\boldsymbol{x}}}(t) = \hat{\boldsymbol{A}}\hat{\boldsymbol{x}}(t) + \hat{\boldsymbol{B}}\boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big) + \boldsymbol{U} \tag{3}$$

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{H}\hat{\boldsymbol{x}}(t)$$

here $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ are the estimated value of uncertain parameters $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. Error is defined as the difference between true value and estimated value and is given by:

$$\tilde{\boldsymbol{A}} = \boldsymbol{A} - \hat{\boldsymbol{A}};$$

$$\tilde{\boldsymbol{B}} = \boldsymbol{B} - \hat{\boldsymbol{B}} \tag{4}$$

Further, synchronization error can be written as $\quad \boldsymbol{e} = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \qquad (5)$

Using equations (1) and (3), the error dynamics is provided by

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t)$$

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{\Phi}(\boldsymbol{H}\boldsymbol{x}(t)) - \hat{\boldsymbol{A}}\hat{\boldsymbol{x}}(t) - \hat{\boldsymbol{B}}\boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big) - \boldsymbol{U}$$

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{x}(t) - \hat{\boldsymbol{A}}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}\boldsymbol{\Phi}(\boldsymbol{H}\boldsymbol{x}(t)) - \hat{\boldsymbol{B}}\boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big) - \boldsymbol{U}$$

Using (4), it can be described as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{x}(t) - \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}\hat{\boldsymbol{x}}(t) + \hat{\boldsymbol{B}}\boldsymbol{\Phi}\big(\boldsymbol{H}\boldsymbol{x}(t)\big) + \tilde{\boldsymbol{B}}\,\boldsymbol{\Phi}(\boldsymbol{H}\boldsymbol{x}(t)) - \hat{\boldsymbol{B}}\boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big) - \boldsymbol{U}$$

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} + \tilde{\boldsymbol{A}}\hat{\boldsymbol{x}}(t) + \tilde{\boldsymbol{B}}\,\boldsymbol{\Phi}(\boldsymbol{H}\boldsymbol{x}(t)) + \hat{\boldsymbol{B}}\big[\boldsymbol{\Phi}\big(\boldsymbol{H}\boldsymbol{x}(t)\big) - \boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big)\big] - \boldsymbol{U} \tag{6}$$

Theorem1**:** For the given master and slave system in (1) and (3) with error dynamics stated as (6), the feedback controller is selected as

$$\boldsymbol{U} = -\boldsymbol{H}^T\boldsymbol{k}\boldsymbol{e} + \hat{\boldsymbol{B}}\big[\boldsymbol{\Phi}\big(\boldsymbol{H}\boldsymbol{x}(t)\big) - \boldsymbol{\Phi}\big(\boldsymbol{H}\hat{\boldsymbol{x}}(t)\big)\big] \tag{7}$$

Where $\boldsymbol{k} \in R^{pxn}$ represents the feedback control gain matrix.

The parameter update laws are selected as $\quad \dot{\tilde{\boldsymbol{A}}} = -\boldsymbol{e}\,\hat{\boldsymbol{x}}^T$

$$\dot{\tilde{\boldsymbol{B}}} = -\boldsymbol{e}\,\boldsymbol{\Phi}^T(\boldsymbol{H}\boldsymbol{x}(t)) \tag{8}$$

The above control function in (7) along with update laws in (8) leads to asymptotic synchronization of slave system (3) with master system (1) if selection of gain matrix $\boldsymbol{k}$ is made such that $(\boldsymbol{A} + \boldsymbol{H}^T\boldsymbol{k})$ be negative definite.

Proof: Let us consider positive definite and quadratic Lyapunov function as:

$$V = \frac{1}{2}e^T e + \frac{1}{2}\tilde{A}^T\tilde{A} + \frac{1}{2}\tilde{B}^T\tilde{B} \tag{9}$$

For error system to be asymptotically stable, time derivative of Lyapunov function must be negative semi-definite.

$$\dot{V} = \frac{1}{2}(e^T\dot{e} + \dot{e}^T e + \tilde{A}^T\dot{\tilde{A}} + \dot{\tilde{A}}^T\tilde{A} + \tilde{B}^T\dot{\tilde{B}} + \dot{\tilde{B}}^T\tilde{B})$$

$$= \frac{1}{2}\{e^T[Ae + \tilde{A}\hat{x}(t) + \tilde{B}\,\Phi(Hx(t)) + \hat{B}[\Phi(Hx(t)) - \Phi(H\hat{x}(t))] - U] + [Ae + \tilde{A}\hat{x}(t) + \tilde{B}\,\Phi(Hx(t)) + \hat{B}[\Phi(Hx(t)) - \Phi(H\hat{x}(t))] - U]^T e + \tilde{A}^T\dot{\tilde{A}} + \dot{\tilde{A}}^T\tilde{A} + \tilde{B}^T\dot{\tilde{B}} + \dot{\tilde{B}}^T\tilde{B}\}$$

with control function U be selected as (7), the above equation reduces to

$$\dot{V} = \frac{1}{2}\{e^T(A + A^T + H^T k + k^T H)e + e^T\tilde{A}\hat{x}(t) + \hat{x}^T(t)\tilde{A}^T e + e^T\tilde{B}\,\Phi(Hx(t)) + \Phi(H\hat{x}(t))\tilde{B}^T e + \tilde{A}^T\dot{\tilde{A}} + \dot{\tilde{A}}^T\tilde{A} + \tilde{B}^T\dot{\tilde{B}} + \dot{\tilde{B}}^T\tilde{B}\}$$

using adaptation laws as per (8), the $\dot{V}$ will become

$$\dot{V} = e^T(A + H^T k)e$$

Which can be expressed as

$$\dot{V} = e^T Q e \quad \leq 0 \tag{10}$$

where $Q = (A + H^T k)$ is negative definite by making suitable selection of feedback control gain matrix k.
This completes the proof.

Case II: Design of output feedback controller with parametric uncertainty

In this case, output feedback control is developed by considering only those parameters uncertain which are present in dynamics of output state. So, the designed output feedback controller and adaptation laws utilizes the estimates of states and available output state only. For this purpose, the master system (1) can be rewritten in the following manner.

$$\begin{bmatrix}\dot{x}\\ \dot{\xi}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}x\\ \xi\end{bmatrix} + \begin{bmatrix}0\\ \Theta\end{bmatrix}\Phi(\xi) \tag{11}$$

where scalar output y is considered as

$$y = \xi$$

The dynamics of slave system is

$$\begin{bmatrix}\dot{\hat{x}}\\ \dot{\hat{\xi}}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ \hat{a}_{21} & \hat{a}_{22}\end{bmatrix}\begin{bmatrix}\hat{x}\\ \hat{\xi}\end{bmatrix} + \begin{bmatrix}0\\ \hat{\Theta}\end{bmatrix}\Phi(\hat{\xi}) + \begin{bmatrix}0\\ U\end{bmatrix} \tag{12}$$

and the corresponding output is

$$\hat{y} = \hat{\xi}$$

The state vectors of master system are $x \in R^{n-1}$ and output $\xi \in R$. ξ is the state on which scalar output of the system depends. Master system contains the constant matrices $A_{11} \in R^{(n-1)x(n-1)}$, $A_{12} \in R^{(n-1)x1}$and uncertain parameters $a_{21} \in R^{1x(n-1)}$, $a_{22} \in R$ and $\Theta \in R^{(1\,x\,p)}$ with $\hat{\Theta}$ as estimate of parameter vector Θ. y and $\hat{y}$ are the outputs of master and slave system, respectively. $\Phi(.) \in R^{(px1)}$ is nonlinear function and is depend on output state of the system. The slave system is having similar type of structure with state vectors $\hat{x} \in R^{n-1}$ and $\hat{\xi} \in R$, is controlled by control vector $U \in R$.

Synchronization error system is given by

$$\begin{bmatrix}\dot{e}_x\\ \dot{e}_\xi\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}e_x\\ e_\xi\end{bmatrix} + \begin{bmatrix}0 & 0\\ \tilde{a}_{21} & \tilde{a}_{22}\end{bmatrix}\begin{bmatrix}\hat{x}\\ \hat{\xi}\end{bmatrix} + \begin{bmatrix}0\\ \tilde{\Theta} + \hat{\Theta}\end{bmatrix}\Phi(\xi) + \begin{bmatrix}0\\ -\hat{\Theta}\end{bmatrix}\Phi(\hat{\xi}) + \begin{bmatrix}0\\ -U\end{bmatrix} \tag{13}$$

The results for deriving the suitable control for error system in (13) is described in the theorem given below.

Theorem II: For the systems in equation (11) as master and equation (12) as slave, the output feedback controller is selected as

$$U = -ke_\xi + \hat{\Theta}[\Phi(\xi) - \Phi(\hat{\xi})] \tag{14}$$

where k is output feedback controller gain selected such that matrix $M = \begin{bmatrix}A_{11} & A_{12}\\ a_{21} & a_{22} + k\end{bmatrix}$ is stable Hurwitz matrix and the parameter update laws are selected as

$$\dot{\tilde{a}}_{21} = -e_\xi\,\hat{x}^T$$

$$\dot{\hat{a}}_{22} = -e_\xi{}^T \hat{\xi} \qquad (15)$$
$$\dot{\hat{\boldsymbol{\theta}}} = -e_\xi \boldsymbol{\Phi}^T(\xi)$$

The above controller along with adaptation laws lead to the synchronization for the present case as of slave system with master system.

Proof**:** Let us consider positive definite and quadratic Lyapunov function as:

$$V = \frac{1}{2}([\boldsymbol{e}_x{}^T \boldsymbol{e}_x + e_\xi{}^2 + \tilde{\boldsymbol{a}}_{21}{}^T \tilde{\boldsymbol{a}}_{21} + \tilde{a}_{22}{}^2 + \tilde{\boldsymbol{\theta}}^T \tilde{\boldsymbol{\theta}}) \qquad (16)$$

Here a_{21}, a_{22} and θ are the uncertain parameters of the system and $\hat{a}_{21}$, $\hat{a}_{22}$ and $\hat{\theta}$ are the estimated parameters, respectively. Error is defined as the difference between true value and estimated value given by:

$\tilde{\boldsymbol{a}}_{21} = \boldsymbol{a}_{21} - \hat{\boldsymbol{a}}_{21}$;

$\tilde{a}_{22} = a_{22} - \hat{a}_{22}$;

and $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$

For error system in (13) to be asymptotically stable, time derivative of Lyapunov function must be negative semi-definite.

$$\dot{V} = \frac{1}{2}[\boldsymbol{e}_x{}^T \dot{\boldsymbol{e}}_x + \dot{\boldsymbol{e}}_x{}^T \boldsymbol{e}_x + 2e_\xi \dot{e}_\xi + \tilde{\boldsymbol{a}}_{21}{}^T \dot{\tilde{\boldsymbol{a}}}_{21} + \dot{\tilde{\boldsymbol{a}}}_{21}{}^T \tilde{\boldsymbol{a}}_{21} + 2\tilde{a}_{22}\dot{\tilde{a}}_{22} + \tilde{\boldsymbol{\theta}}^T \dot{\tilde{\boldsymbol{\theta}}} + \dot{\tilde{\boldsymbol{\theta}}}^T \tilde{\boldsymbol{\theta}}]$$

which can be expressed as

$$\dot{V} = \boldsymbol{e}_x{}^T \dot{\boldsymbol{e}}_x + e_\xi \dot{e}_\xi + \dot{\tilde{\boldsymbol{a}}}_{21}{}^T \tilde{\boldsymbol{a}}_{21} + \tilde{a}_{22}\dot{\tilde{a}}_{22} + \dot{\tilde{\boldsymbol{\theta}}}^T \tilde{\boldsymbol{\theta}}$$

Substituting the values from the error equation given in (13)

$$\dot{V} = \boldsymbol{e}_x{}^T [\boldsymbol{A}_{11}\boldsymbol{e}_x + \boldsymbol{A}_{12} e_\xi]$$
$$+ e_\xi \left[\boldsymbol{a}_{21}\boldsymbol{e}_x + a_{22} e_\xi + \tilde{\boldsymbol{a}}_{21}\hat{\boldsymbol{x}}(t) + \tilde{a}_{22}\hat{\xi} + \hat{\boldsymbol{\theta}}\left(\boldsymbol{\Phi}(\xi) - \boldsymbol{\Phi}(\hat{\xi})\right) + \tilde{\boldsymbol{\theta}}\boldsymbol{\Phi}(\xi) - U\right] + \dot{\tilde{\boldsymbol{a}}}_{21}{}^T \tilde{\boldsymbol{a}}_{21} + \tilde{a}_{22}\dot{\tilde{a}}_{22} + \dot{\tilde{\boldsymbol{\theta}}}^T \tilde{\boldsymbol{\theta}}$$

with control function U be selected as (14), the above equation reduces to

$$\dot{V} = \boldsymbol{e}_x{}^T \boldsymbol{A}_{11}\boldsymbol{e}_x + \boldsymbol{e}_x{}^T \boldsymbol{A}_{12} e_\xi + e_\xi \boldsymbol{a}_{21}\boldsymbol{e}_x + e_\xi(a_{22} + k)e_\xi + e_\xi \tilde{\boldsymbol{a}}_{21}\hat{\boldsymbol{x}}(t) + e_\xi \tilde{a}_{22}\hat{\xi} + e_\xi \tilde{\boldsymbol{\theta}}\boldsymbol{\Phi}(\xi)$$
$$+ + \dot{\tilde{\boldsymbol{a}}}_{21}{}^T \tilde{\boldsymbol{a}}_{21} + \tilde{a}_{22}\dot{\tilde{a}}_{22} + \dot{\tilde{\boldsymbol{\theta}}}^T \tilde{\boldsymbol{\theta}}$$

using adaptation laws as per (15), the $\dot{V}$ will become

$$\dot{V} = \boldsymbol{e}_x{}^T \boldsymbol{A}_{11}\boldsymbol{e}_x + \boldsymbol{e}_x{}^T \boldsymbol{A}_{12} e_\xi + e_\xi \boldsymbol{a}_{21}\boldsymbol{e}_x + e_\xi(a_{22} + k)e_\xi$$

which can be expressed as

$$= \boldsymbol{\zeta}^T \boldsymbol{M} \boldsymbol{\zeta} \le 0 \qquad (17)$$

where $\boldsymbol{\zeta} = [\boldsymbol{e}_x; e_\xi]$

and $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{a}_{21} & a_{22} + k \end{bmatrix}$ is negative definite matrix by making suitable selection of feedback control gain k.

This completes the proof.

III. Illustrative example and numerical simulation

The One of the subclasses of the system described in equation (1) is Lur'e class of nonlinear system. Consider the example of Cubic Chua system which belongs to Lur'e class and is used here for verifying the given control strategy. The master-slave synchronization of Cubic Chua circuit with following differential description is utilized for numerical validation of the results:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_3{}^3 - cx_1) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -bx_2 \\ y &= x_3 \end{aligned} \qquad (18)$$

The parameters are selected as $a = 10,\ b = 16$ and $c = -0.143$, for which it exhibits chaotic behavior.

The intention is to develop a controller with initial conditions x_0 and $\hat{x}_0$, respectively fulfil the necessary condition for synchronization. i.e.

$\lim_{t\to\infty} \|\boldsymbol{e}\| = \lim_{t\to\infty} \|\boldsymbol{x}(t, x_0) - \hat{\boldsymbol{x}}(t, \hat{x}_0)\| = 0$ Where ||.|| is Euclidian norm.

The above structure of Chua system can be modified by interchanging x_1 and x_3. The master system of transformed structure can be expressed in following way

$$\begin{aligned} \dot{x}_1 &= -bx_2 \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= \alpha x_2 - \beta x_3 - \gamma\Phi(x_3) \end{aligned} \qquad (19)$$

and the output is given as
$y = x_3$
Where x_1 , x_2 and x_3 are state variables and y is the output of the system.
$\Phi(x_3) = x_3{}^3$ is a sector bounded nonlinearity and the parameters being $b = 16, \alpha = 10, \beta = -1.43$ and $\gamma = 10$.

It can be easily checked that the Cubic Chua system given in (19) can be expressed in nonlinear system (1) with $A = \begin{bmatrix} 0 & -b & 0 \\ 1 & -1 & 1 \\ 0 & \alpha & -\beta \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -\gamma \end{bmatrix}$ and $H = [0 \quad 0 \quad 1]$
In first case, all the parameters (i.e. b, α, β and γ) are assumed to be uncertain. To make the response system globally synchronize the master system, the feedback controller $\boldsymbol{U_j}(j = 1,2,3)$ is given by

$$U_1 = 0$$
$$U_2 = 0$$
$$U_3 = -k_1 e_1 - k_2 e_2 - k_3 e_3 - \hat{\gamma}(\Phi(x_3) - \Phi(\hat{x}_3)) \tag{20}$$

And the parameter adaptation laws are chosen as

$$\dot{\tilde{b}} = e_1 \hat{x}_2$$
$$\dot{\tilde{\alpha}} = -e_3 \hat{x}_2 \tag{21}$$
$$\dot{\tilde{\beta}} = e_3 \hat{x}_3$$
$$\dot{\tilde{\gamma}} = e_3 \Phi(x_3)$$

where $\tilde{b} = b - \hat{b}, \tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta}$ and $\tilde{\gamma} = \gamma - \hat{\gamma}$.
$\hat{b}, \hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are the estimates of the parameter b, α, β and γ, respectively. The synchronization can be executed if we select feedback control gains $k_1 = 0,\ k_2 = 0, k_3 < -9.704$ where $k = [k_1 \quad k_2 \quad k_3]$.

In simulation, Ordinary Differential Equation solver is used to find the solution of the differential equation of master system in (19) and slave system with feedback controller $\boldsymbol{U_j}(j = 1,2,3)$ in (20) and updation laws given in (21). Master system's initial conditions are considered as $x_1(0) = 3,\ x_2(0) = -3$ and $x_3(0) = -3$, respectively and that of slave systems are considered as $\hat{x}_1(0) = -0.1,\ \hat{x}_2(0) = -0.1$ and $\hat{x}_3(0) = -1$, respectively. The feedback control gain k_3 is considered as -50. Further, the initial values of estimated parameters are assumed to be $b_0 = 15,\ \alpha_0 = 8, \beta_0 = -1.2$ and $\gamma_0 = 8$. Fig. (1), fig. (2) and fig. (3) show the plot of time response of first, second and third state of master and slave system with controller, respectively. Fig. (4) shows plot of the estimates of parameters b, α, β and γ, respectively. The synchronization error plot is illustrated in fig. (5). Parameter estimates are bounded and Synchronization error tends to zero using computed feedback controller and parameter updation laws.

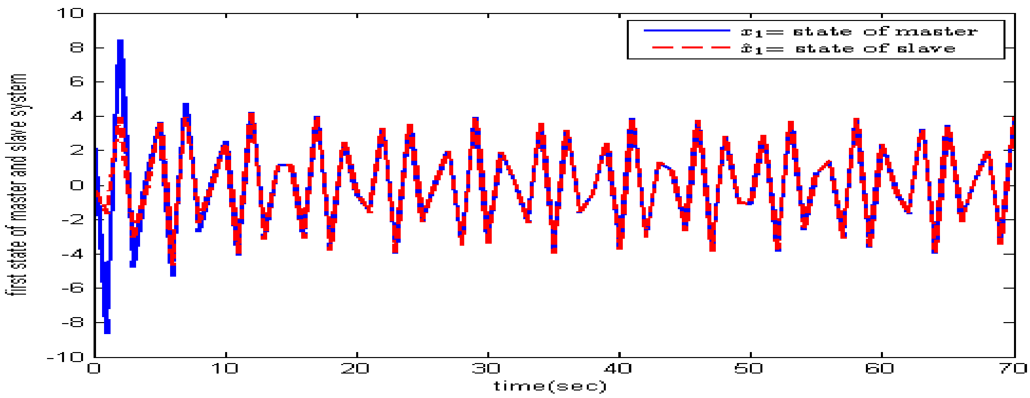

Fig.1. Plot of Synchronized trajectories of first state of master and slave systems

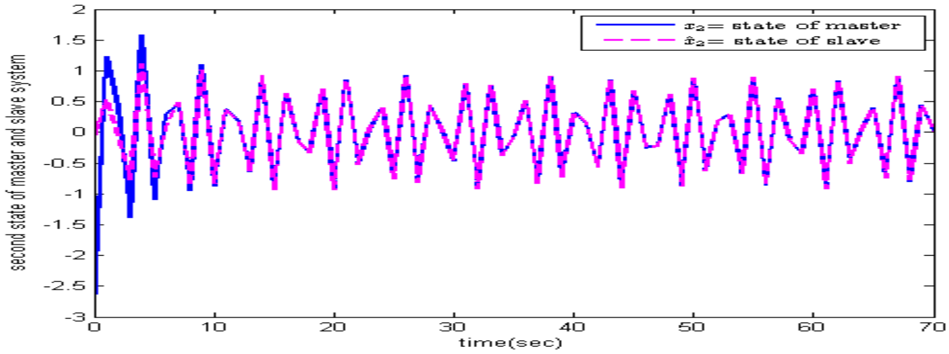

Fig.2. Plot of Synchronized trajectories of second state of master and slave systems

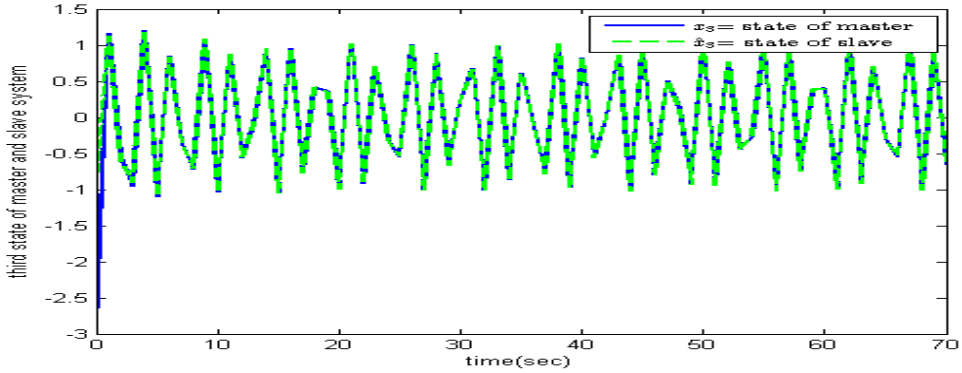

Fig.3. Plot of Synchronized trajectories of third state of master and slave systems

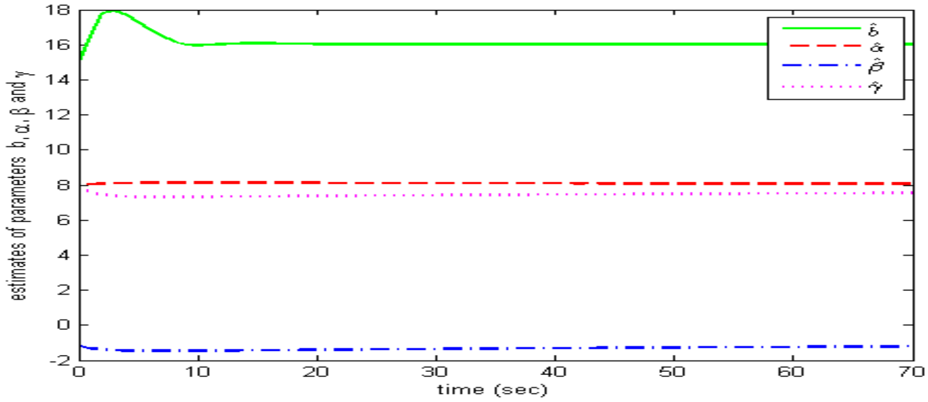

Fig.4. Plot of Estimates of parameters b, α, β and γ

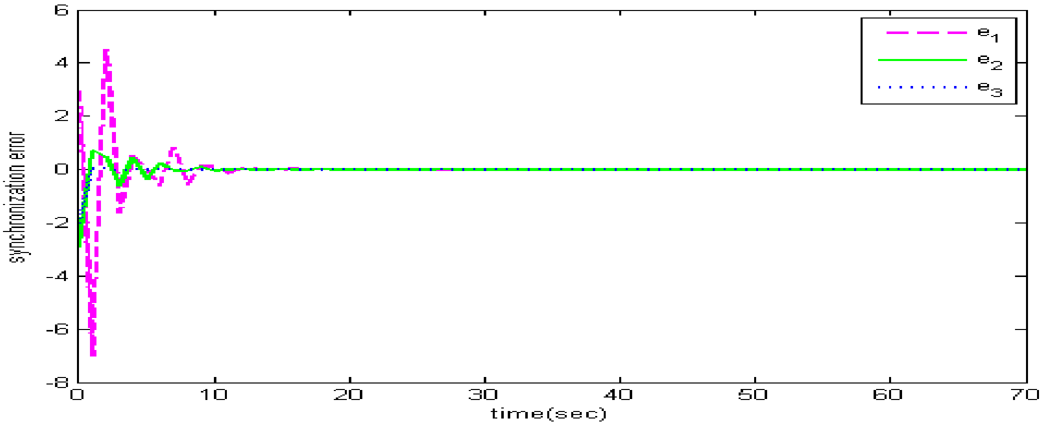

Fig.5. Plot of Synchronization error trajectories of master and slave system

In second case, output feedback control is developed by considering only those set of parameters uncertain which are present in dynamics of output state. To make the slave system globally synchronize with the master system, the feedback controller U is given by

$$U = -ke_3 - \hat{\gamma}[\Phi(x_3) - \Phi(x_3)] \tag{22}$$

And the parameter updation laws are given by

$$\begin{aligned} \dot{\hat{\alpha}} &= -e_3\hat{x}_2 \\ \dot{\hat{\beta}} &= e_3\hat{x}_3 \\ \dot{\hat{\gamma}} &= e_3\Phi(x_3) \end{aligned} \tag{23}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta}$ and $\tilde{\gamma} = \gamma - \hat{\gamma}$.

$\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are the estimates of the parameter α, β and γ, respectively. The synchronization can be executed if we select output feedback control gain $k < -9.704$.

In simulation, Ordinary Differential Equation solver is used to find the differential equation solution of master system in (19) and slave system with feedback controller U in (22) and updation laws given in (23). The initial conditions of master system are $x_1(0) = 0$, $x_2(0) = 0.1$ and $x_3(0) = 0$, respectively. The initial conditions of slave system are $\hat{x}_1(0) = -0.1$, $\hat{x}_2(0) = -0.01$ and $\hat{x}_3(0) = -3$, respectively. The output feedback control gain k is selected as -50. Further, the initial values of estimated parameters are considered to be $\alpha_0 = 9$, $\beta_0 = -1.2$ and $\gamma_0 = 8$. The plot time response of first, second and third states of master and slave system with controller are given in fig. (6), fig. (7) and fig. (8), respectively. Plot of estimates of parameters α, β and γ are shown in fig. (9), fig. (10) and fig. (11), respectively. The synchronization error plot is illustrated in fig. (12). Parameter estimates are bounded and Synchronization error reaches to zero with the help of computed output feedback controller and parameter updation laws.

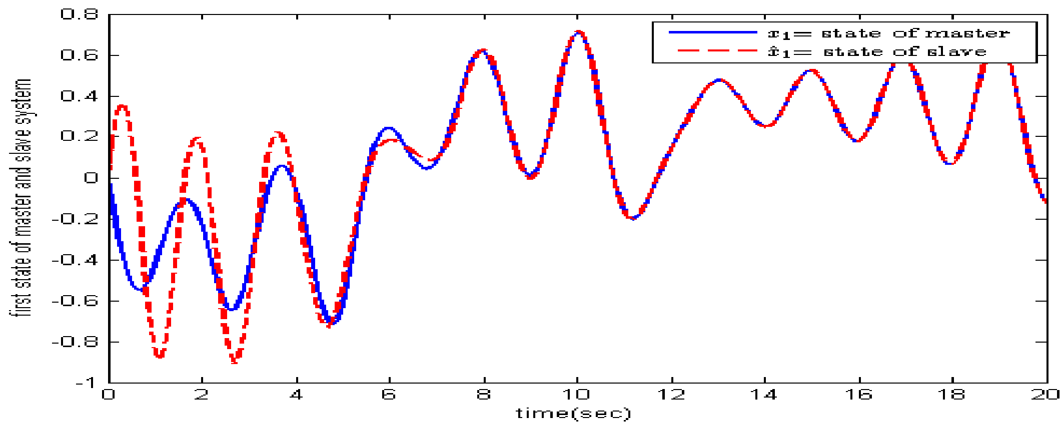

Fig.6. Plot of synchronized trajectories of first state of master and slave systems

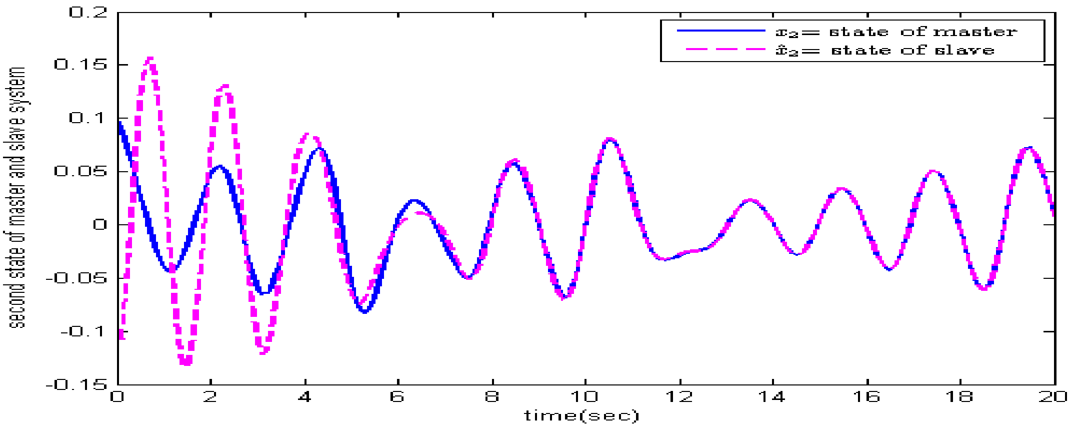

Fig.7. Plot of synchronized trajectories of second state of master and slave systems

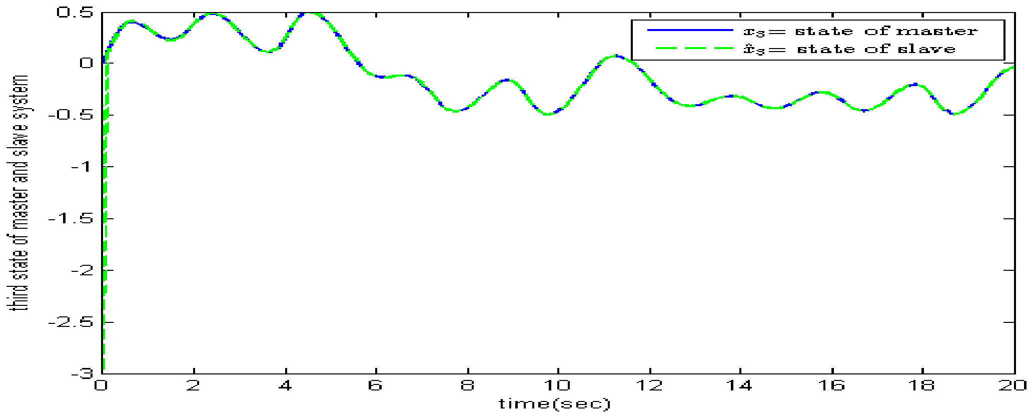

Fig.8. Plot of synchronized trajectories of third state of master and slave systems

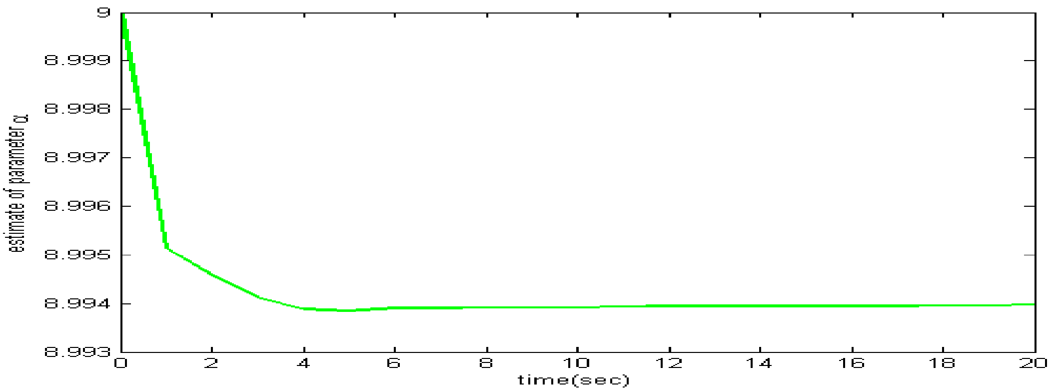

Fig.9. Plot of estimate of parameter α

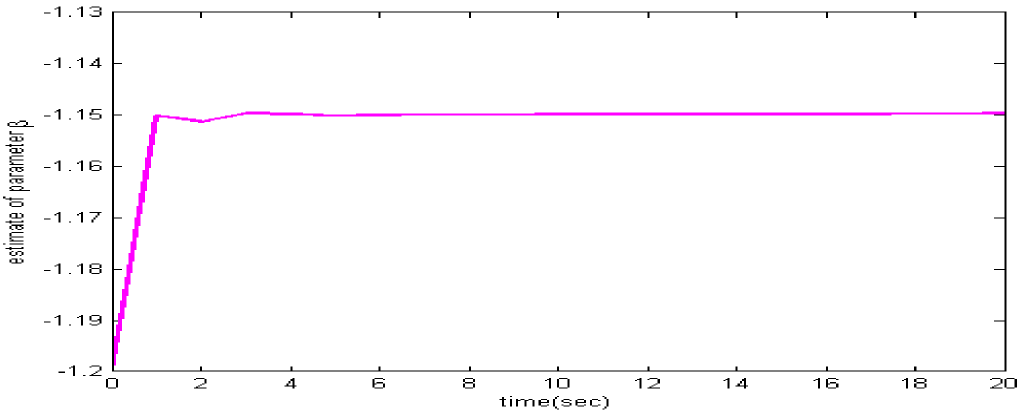

Fig.10. Plot of estimate of parameter β

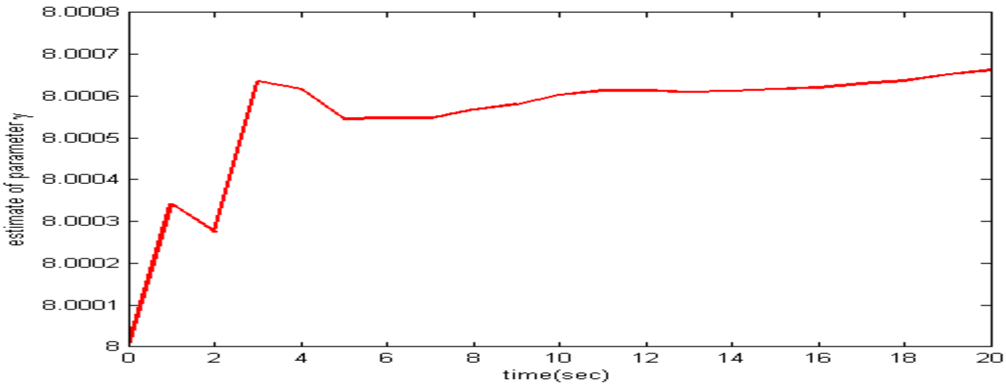

Fig.11. Plot of estimate of parameter γ

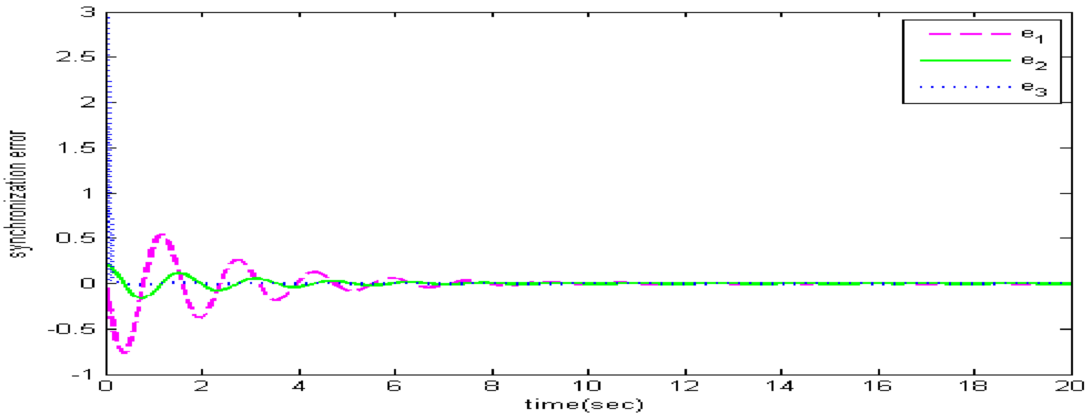

Fig.12 Synchronization error plot of master and slave system

IV. Conclusion

In this paper, Firstly, feedback controller and parameter updation laws are developed based on the Lyapunov stability theorem to synchronize master and slave system when all the parameters are assumed to be uncertain. In this case, the controller and the parameter updation laws are not only depend on the known output but also on other states of the system and control structure is complicated. Further, this synchronization scheme is converted to output feedback control scheme by assuming only those parameters uncertain which are present in the dynamics of output state. The proposed controller and adaptation laws only utilizes the available output state and estimates, though limited set of parametric uncertainty is considered for this case. The results could be further extended to enlarge the scope of class of system with complete parameter vector subjected to uncertainty. For simulations, Chua's system is used to verify that the method is effective.

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IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE) is UGC approved Journal with Sl. No. 4198, Journal no. 45125.

Megha Jiddewar. "Adaptive Output Feedback Control Strategy Based Chaotic Synchronization with Parametric Uncertainty." IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE) 14.3 (2019): 43-52.