# **Analysis of Compensated Ac Transmission Lines**

A.E.Anyalebechi

Electrical Engineering Department Nnamdi Azikiwe University Awka B. O. Anyaka Electrical Engineering Department, University of Nigeria Nsukka. Corresponding Author: A.E.Anyalebechi

**Abstract:** This paper presents the analysis of compensated ac transmission lines under two different forms of compensations: series and shunt compensations. The Thevenin equivalent circuit of each was drawn and analyzed. The result shows that at a transmission angle (that is  $\delta$ ) of  $180^\circ$ , a shunt compensated line would be able to deliver an active power double the amount achievable under series compensation.

**Keywords:** shunt compensation, series compensation, active power, Thevenin equivalent circuits, and transmission angle.

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## I. Introduction

Last fifteen years have witnessed tremendous efforts towards the understanding of power characteristics of alternating current transmission lines [1, 2, 3, 4, and 5]. Also, FACTS devices always provide fast control actions in comparison to conventional devices like switched compensation or phase shifting transformers with mechanical on-load tap changer [6 and 7]. It has always been desirable to transmit as much power as possible through transmission lines and cables, consistent with the requirements of stability and security of supply. Power transmission is limited mainly by thermal factors in cables, short transmission lines, transformers and generators; but in long lines and cables the variation of voltage and the maintenance of stability also constrain the power transmission. The voltage 'profile' and the stability of a transmission line or cable can be improved using 'reactive compensation'. In the early days reactive compensation took the form of fixed-value reactors and capacitors, usually controlled by mechanical switchgear.

Synchronous condensers and large generators were used in cases where it was necessary to vary the reactive power continuously. Since the 1970s power-electronic equipment has been developed and applied to extend the range of control, with a variety of methods and products. Bulk AC transmission of electrical power has two fundamental requirements:

Synchronism. The basis of AC transmission is a network of synchronous machines connected by A)transmission links, the voltage and frequency are defined by this network, even before any loads are contemplated. All the synchronous machines must remain constantly in synchronism: i.e. they must all rotate at exactly the same speed, and even the phase angles between them must not vary appreciably. By definition, the stability of the system is its tendency to recover from disturbances such as faults or changes of load. The power transmitted between two synchronous machines can be slowly increased only up to a certain level called the steady-state stability limit. Beyond this level the synchronous machines fall out of step, i.e. lose synchronism. The steady- state stability limit can be considerably modified by the excitation level of the synchronous machines (and therefore the line voltage); by the number and connections of transmission lines; and by the pattern of real and reactive power flows in the system, which can be modulated by reactive compensation equipment. A transmission system cannot be operated too close to the steady-state stability limit, because there *must* be a margin to allow for disturbances. In determining an appropriate margin, the concepts of *transient* and dynamic stability are useful. Dynamic stability is concerned with the ability to recover normal operation following a specified *minor* disturbance. Transient stability is concerned with the ability to recover normal operation following a specified *major* disturbance.

*B)* Voltage profile. It is obvious that the correct voltage level must be maintained within narrow limits at all levels in the network. Undervoltage degrades the performance of loads and causes overcurrent. Overvoltage is dangerous because of the risks of flashover, insulation breakdown, and saturation of transformers. Most voltage variations are caused by load changes, and particularly by the reactive components of current flowing in the reactive components of the network impedances. If generators are close by, excitation levels can be used to

keep the voltage constant; but over long links the voltage variations are harder to control and may require reactive compensation equipment.

Different techniques are used for controlling the voltage according to the underlying rate of change of voltage. Cyclic, diurnal load variation is gradual enough to be compensated by excitation control or the timely switching in and out of capacitors and reactors. But sudden overvoltages such as those resulting from disconnection of loads, line switching operations, faults, and lightning - require immediate suppression by means of surge arrestors or spark gaps. Between these extremes there are many possibilities for controlled reactive compensation equipment operating over time scales ranging from a few milliseconds to a few hours.

#### II. Voltage and Current Equations of a Long, Lossless Transmission Line

Figure 1 below shows one phase of a transmission line or cable with distributed inductance l H/m. The voltage and current phasors V(x) and I(x) both obey the transmission line equation:

Where  $\Gamma = \sqrt{\{(r + j\omega l)(g + j\omega l)\}}$ , and x = distance along the line; r = the resistance per unit length(ohm/meter) in series with *l*, and g = the shunt conductance per unit length (S/m) in parallel with c;  $\omega = 2\pi f$  is the radian frequency. If r and g are both small, then  $\Gamma = j\beta$ , where  $\beta = \omega \sqrt{(lc)}$  is the wavenumber. The propagation velocity  $u = 1/\sqrt{(lc)}$  is rather lower than the speed of light (3 x 10<sup>5</sup> km/s) and  $\beta = 2\pi f/u = 2\pi/\lambda$ , where  $\lambda = u/f$  is the wavelength [8].



Fig(1) Transmission line with distributed series inductance and shunt capacitance.

If "a" is the length of the line,  $\theta = \beta a$  is the electrical length. The solution for equation (1) for a lossless line is:  $V(x) = V_r \cos \beta(a - x) + jZ_o I_r \sin \beta(a - x)$ 

$$I(x) = j\frac{V_r}{Z_0} \sin \beta(a - x) + I_r \cos \beta(a - x)$$
.....3

Where  $Z_0 = \sqrt{(l/c)}$  is the surge impedance [ohm]. Also, if  $x_L = \omega l$  is the series inductive reactance [ohm/m] and  $x_c = 1/\omega c$  is the shunt capacitive reactance [ also in ohm/m] then we can write  $Z_0 = \sqrt{(x_L x_C)}$  and  $\beta = \sqrt{(x_L x_C)}$ .

#### **III.** Compensated Transmission Lines

Reactive compensation means the application of reactive devices

- (a) To produce a substantially flat voltage profile at all jevels of power transmission;
- (b) To improve stability by increasing the maximum transmissible power ; and/or
- (c) To supply the reactive power requirements in the most economical way.

Ideally the compensation would modify the surge impedance by modifying the capacitive and/or inductive reactance of the line, so as to produce a virtual surge- impedance loading  $P'_0$  that is always equal to the actual power being transmitted. According to the following equation:  $V(x) = V_r e^{j\beta(a-x)}$  and  $I(x) = I_r e^{j\beta(a-x)}$ 

this would ensure a flat voltage profile at all power levels. However, this is not sufficient by itself to ensure the *stability* of transmission, which depends also on the electrical line length  $\theta \left[P_{max} = \frac{E_s E_r}{x_L} = \frac{P_o}{\sin \theta}\right]$ . The electrical length can itself be modified by compensation to have a virtual value  $\theta'$  shorter than the uncompensated value, resulting in an increase in the steady-state stability limit  $P_{max}$ .

These considerations suggest two broad classifications of compensation scheme, *surge-impedance compensation* and *line-length compensation*. Line-length compensation in particular is associated with series capacitors used in long-distance transmission. A third classification is *compensation by sectioning*, which is achieved by connecting constant-voltage compensators at intervals along the line. The maximum transmissible power is that of the weakest section, but since this is necessarily shorter than the whole line, an increase in maximum power and, therefore, in stability can be expected.

#### 3a) PASSIVE AND ACTIVE COMPENSATORS

*Passive* compensators include shunt reactors and capacitors and series capacitors. They modify the inductance and capacitance of the line. Apart from switching, they are uncontrolled and incapable of continuous variation. For example, shunt reactors are used to compensate the line capacitance to limit voltage rise at light load. They increase the virtual surge impedance and reduce the virtual natural load  $P'_{0}$ . Shunt capacitors may be used to augment the capacitance of the line under heavy loading. They generate reactive power which tends to boost the voltage. They reduce the virtual surge impedance and increase  $P'_{0}$ . Series capacitors are used for line-length compensation. A measure of surge-impedance compensation may be necessary in conjunction with series capacitors, and this may be provided by shunt reactors or by a dynamic compensator.

usually shunt-connected Active compensators are devices which have the property of tending to maintain а substantially constant voltage at their terminals. They do this by generating or absorbing precisely the required amount of corrective reactive power in response to any small variation of voltage at their point of connection. They are usually capable of continuous (i.e. stepless) variation and rapid response. Control may be inherent, as in the saturated-reactor compensator; or by means of a control system, as in the synchronous condenser and thyristor-controlled compensators.

Active compensators may be applied either for surge-impedance compensation or for compensation by sectioning. In  $Z_0$ -compensation they are capable of all the functions performed by fixed shunt reactors and capacitors and have the additional advantage of continuous variability with rapid response. Compensation by sectioning is fundamentally different in that it is possible *only* with active compensators, which must be capable of virtually immediate response to the smallest variation in power transmission or voltage.

The automatic voltage regulators used to control the excitation of synchronous machines also have an important compensating effect in a power system. By dynamically maintaining constant voltage at the generator terminals they remove the Thévenin equivalent source impedance of the generator (i.e. the synchronous reactance) from the equivalent circuit of the transmission system.

Compensating equipment is often an economical way to meet the reactive power requirements for transmission. An obvious example is where the power can be safely increased without the need for an additional line or cable. But compensators bring other benefits such as management of reactive power flows; damping of power oscillations; and the provision of reactive power at conventional HVDC converter terminals. Both passive and active compensators are in growing use, as are all the compensation strategies: virtual surge-impedance, line-length compensation and compensation by sectioning.

### **IV. Static Shunt Compensation**

Shunt reactors are used to limit the voltage rise at light load. On long lines they may be distributed at intermediate substations as shown in Figure (2) below, typically at intervals of the order of 50-100 km.



Figure (2) Shunt reactors distributed along a long high voltage AC line.

#### 4a) THE MID-POINT SHUNT COMPENSATOR

The primary purposes of transmission system shunt compensation near load centers are voltage control and load stabilization. At the substation busbars where reactive demand increases, busbar voltage can be controlled by connecting capacitor banks in parallel with a lagging load. The capacitor banks supply part or full reactive power to the load, thus reducing the magnitude of source current necessary to supply the load. As a result, the voltage drops between the sending end and the load or receiving end gets reduced, improving power factor and increased active power output is available from the source [8]. Figure 3 shows a symmetrical line with a mid-point shunt compensator of admittance  $jB_{\gamma}$ . Each half of the line is represented by a  $\pi$ -equivalent circuit. The synchronous machines at the ends are assumed to supply or absorb the reactive power for the leftmost and rightmost half-sections, leaving the compensator to supply or absorb only the reactive power for the central half of the line. If the compensator can vary its admittance continuously in such a way as to maintain  $V_m = E$ , then in the steady state the line is sectioned into two independent halves with a power transmission characteristic given by

$$P = \frac{2E^2}{X_L} \sin \frac{\delta}{2} \dots 4$$

The maximum transmissible power is  $2E^2/X_L$ , twice the steady-state limit of the uncompensated line. It is reached when  $\delta/2 = \pi/2$ , that is, with a transmission angle  $\delta$  of 90° across each half of the line, and a total transmission angle of 180° across the whole line, Figure 4.



Figure (3) mid-point shunt compensator



Figure (4) Power transmission characteristic with dynamic shunt compensation.



Figure (5) Mid-point shunt compensator.



Figure (6) Thevenin equivalent circuit of line with mid-point shunt compensation.

The equivalent circuit can be simplified to the one shown in Figure 5. The compensating admittance is expressed in terms of the 'degree of compensation'  $k_m$ , which is positive if the compensator is inductive and negative if it is capacitive

The total shunt admittance in the centre is equal to

The equivalent circuit can be simplified still further by splitting the central admittance into two equal parallel admittances and then reducing each half to its Thevenin equivalent, as in Figure 6. The Thevenin equivalent voltage at the sending end is

$$U_{s} = \frac{\frac{\overline{B_{c}(1-K_{m})}}{4}}{\frac{1}{\frac{B_{c}(1-K_{m})}{4}} + \frac{B_{s}}{2}} E_{s} = \frac{E_{s}}{1-s} \quad \dots \dots 7$$

where

$$s = \frac{X_L}{2} \frac{B_c}{4} (1 - K_m)$$

and

$$Z = \frac{1}{jB_c \frac{1-k_m}{4} + \frac{1}{jX_L/2}} = \frac{jX_L/2}{1-s} \dots \dots 8$$

The parameter "s" is a potentiometer ratio determined by the relative values of  $X_L$ ,  $B_c$  and  $k_m$ . If we assume that  $E_s = E_r = E$ , the phasor diagram is as shown in Figure 7 and the mid-point voltage is given by



Figure (7)Phasor diagram of symmetrical line.

$$V_m = \frac{E\cos(\delta/2)}{1-s} \dots \dots \dots 9$$

If we now substitute for "s" in equation (9) we can determine the value of compensating susceptance  $B_{\gamma}$  required to maintain a given ratio  $V_m/E$ : thus

$$B_{\gamma} = -\frac{4}{X_L} \left[ 1 - \frac{E}{V_m} \cos \frac{\delta}{2} \right] + \frac{B_c}{2}$$
(10)

This equation tells how  $B_{\gamma}$  must vary with the transmission angle  $\delta$  in order to maintain a given value of mid-point voltage  $V_m$ . Naturally, through  $\delta$ ,  $B_{\gamma}$  varies with the power being transmitted. From Figure 7, using the analogy of a symmetrical line and the power equation of uncompensated line under load

 $(P = \frac{E_s E_r}{x_L} \text{ Sin } \delta), \text{ the power transmission can be deduced to be controlled by the equation}$  $P = \frac{E^2}{2} \sin \delta = \frac{E_m E}{2} \sin \delta = 2 \frac{E_m E}{2} \sin$ 

This establishes equation (4) which was earlier written down by inspection of Figure 3.

## V. Series Compensations

A series capacitor can be used to cancel part of the reactance of the line. This increases the maximum power, reduces the transmission angle at a given level of power transfer, and increases the virtual natural load. Since the effective line reactance is reduced, it absorbs less of the line-charging reactive power, so shunt reactors may be needed as shown in Figure 8. Series capacitors are most often used in very long distance transmission, but they can also be used to adjust the power sharing between parallel lines. A line with 100% series compensation would have a resonant frequency equal to the power frequency, and since the damping in power systems is very low, such a system would be hypersensitive to small changes. For this reason the degree of series compensation is limited in practice to about 80%. It is not practicable to distribute the capacitance in small units along the line. This makes for an uneven voltage profile. The line in Figure 8 is assumed to be a lossless, symmetrical line with a mid-point series capacitor with equal shunt reactors connected on either side. To permit the line to be analyzed in two halves, the capacitor is split into two equal series parts.



Figure (8) Series compensated transmission line

#### 5a) MAXIMUM TRANSMISSIBLE POWER

The general phasor diagram is shown in Figure 9. Note that  $V_2$  leads  $V_1$  in phase as a result of the voltage  $V_{CY}$  inserted by the capacitor. Considering the sending-end half, the conditions at its two ends are related by equations (2) and (3):

 $\mathbf{E}_{\mathrm{s}} = \mathbf{V}_{1} \cos \frac{\theta}{2} + \mathbf{j} \mathbf{Z}_{0} \mathbf{I}_{1} \sin \frac{\theta}{2} \dots \dots 12\mathbf{a}$ 

The receiving-end half behaves similarly. The capacitor reactance is  $X_{C\gamma} = 1/\omega C\gamma$  and the voltage across the capacitor is given by

 The currents  $\mathbf{I}_1$  and  $\mathbf{I}_2$  are given by

$$I_{\rm m} = I_1 + \frac{jV_1}{X} = I_2 - j \frac{V_2}{X} \dots \dots 15$$



Figure (9) Series compensated line: phasor diagram.

Using these relationships, and taking  $V_{\rm m}$  as reference phasor, it is possible to derive the basic power-transfer characteristic as

 $P = \frac{E_s V_m}{Z_0 \sin \frac{\theta}{2} - \frac{X_{CY}}{2} \left[ \cos \frac{\theta}{2} + \frac{Z_0}{X} \sin \frac{\theta}{2} \right]} \sin \frac{\delta}{2} \dots 16$ With

If  $V_m$  is substituted from equation (17) into equation (16), the following result is obtained for the symmetrical line, if  $E_s = E_r$ :

$$P = \frac{E_{s}E_{r}}{[Z_{o}\sin\theta - \frac{X_{c\gamma}}{2}(1 + \cos\theta)\mu]\mu} \sin\delta.....18$$

where

 $\mu = 1 + \frac{z_0}{x} \frac{\sin \theta}{1 + \cos \theta} = 1 + \frac{z_0}{x} \tan \frac{\theta}{2} \dots \dots 19$ 

With no shunt reactors,  $\mu = 1$ . With fixed terminal voltages,  $E_s = E_r = E$ , the transmission angle  $\delta$  can be determined from equation (18) for any level of power transmission below the maximum. Once  $\delta$  is known,  $V_m$  can be determined from equation (16). Then V<sub>1</sub>, V<sub>2</sub>, V<sub>Cγ</sub> and other quantities follow.

One simplification is to ignore the shunt capacitance of the line and remove the shunt reactors [9]. Then  $Z_o \sin \theta$  is replaced by  $X_L$  and  $\mu = 1$ , so that with  $E_s = E_r = E$ ,



Figure (10) Simplified equivalent circuit of series compensated line

If the degree of series compensation  $K_{se}$  is defined by  $K_{se} = \frac{X_{CY}}{X_L} = \frac{X_{CY}}{\omega al} \dots 21$ Then

#### VI. Conclusion

Essentially equations (4) and (22) are the summary of the entire analysis. At a transmission angle of  $180^{\circ}$  the series compensated line delivers zero power but the mid-point shunt compensated line will deliver a maximum power of  $2E^2/X_L$ .

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