Image Restoration By Kriging Interpolation Technique

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Abstract: Image restoration is the art of predicting damaged regions of an image. The manual way of image restoration is a time consuming. Therefore, there must be an automatic digital method for image restoration that recovers the image from the damaged regions. A novel statistical image restoration algorithm based on Kriging interpolation technique was proposed. Kriging technique automatically fills the damaged region in an image using the information available from its surrounding regions in such away that it uses the spatial correlation structure of points inside the kxk block. Kriging has the ability to face the challenge of keeping the structure and texture information as the size of damaged region heighten.

Keyword: Restoration, Kriging interpolation, PSNR

I. Introduction

The filling-in of missing or unwanted information is an extremely considerable topic in image processing. The most important applications of image restoration are objects removal, scratch removal, restoring missing areas, image repairing, etc. Actually, an image or photograph is sometimes damaged because of aging. Therefore, the exact definition of restoration is that the reconstruction of damaged images in such a way that is unnoticeable by the human eye. The manual work of restoration is most often a very time consuming process. Due to digitalization of this technique, it is automatic and faster. The most common restoration technique is the diffusion-based technique. this technique was proposed by M. Bertalmio, et.al. (2000 & 2001) in their paper [2], and by T. F. Chan, et.al. (2002) in their paper [5]. In these techniques, the missing blocks are filled by diffusing the image pixels from the observed blocks into the missing blocks. These techniques are based on the theory of partial differential equation (PDE). According to M. Bertalmio, et.al. (2000) in their paper [2], said that the holes are filled by procreating the isophote into the missing blocks. The isophote are lines of equal grey values. Furthermore, a Navier-Strokes equation in fluid dynamics have been utilized into the field of image restoration by M. Bertalmio, et.al. (2001) in their paper [2].

II. Need For Study And Description

1. Introduction

Image interpolation is a significant operation in image processing which can be used to resample the image either to decrease or increase the resolution. Image interpolation has been used sparciously by customary interpolation techniques. Recently, Kriging technique has been widely implemented in simulation area and geostatistics for prediction. Kriging technique was used instead of the classical interpolation methods to predict the unknown points in the digital image array. The efficiency of the proposed technique was proven using the PSNR, SSIM, and RMSE and compared with the traditional interpolation techniques.

Fig. 2.1: Before and after interpolation
2. Basics of image processing

Image processing is a general term for the wide range of techniques that exist for manipulating and modifying images in various ways. Photographers and physicists can perform certain image processing operations using chemicals or optical equipment. How digital images may be manipulated and enhanced, but also how they may be acquired, stored and represented in computer memory are considered.

Digital imaging actually predates modern computer technology; newspaper pictures were digitized for transatlantic transmission via submarine cable in the early 1920s. However, true digital image processing (DIP) was not possible until the advent of large-scale digital computing hardware.

2.1 Purpose of Image Processing

Very few engineers have experience in image processing or the lighting techniques required to capture images that can be processed quickly and accurately. Once images have been acquired, they are often not in an appropriate format for you to analyze and so they must be processed first. For the purposes of image analysis and pattern recognition, one often transforms an image into another better represented form. Image-processing techniques have been developed tremendously during the past five decades and among them, mathematical morphology has always received a great deal of attention because it provides a quantitative description of geometric structure and shape as well as a mathematical description of algebra, topology, probability, and integral geometry.

2.2 Digital Images

A two dimensional image that is recorded by the sensors is the mapping of the three-dimensional visual world. The captured two dimensional signals are sampled and quantized to yield digital images. The features of a digital image (such as shape, texture, colour, topology of the objects, etc.) can be used as index keys for search and retrieval of pictorial information from large image database. Once the images are formed (which is a two-dimensional analog signal), the next process involves sampling and digitization of the analog image. The digital images so formed after all these processes need to be represented in appropriate format so that they may be processed and manipulated by a digital computer for various applications. Types of Digital Images: Binary image, Gray scale image, Indexed image, True colour image.

2.2.1 Binary image

In binary image the value of each pixel is either black or white. The image only have two possible values for each pixel either 0 or 1, we need one bit per pixel.

![Fig. 2.2.1 Example of Binary Image](image)

2.2.2 Grayscale image

In grayscale image each pixel is shade of gray, which have value normally 0 [black] to 255 [white]. This means that each pixel in this image can be shown by eight bits, that is exactly of one byte. Other grayscale ranges can be used, but usually they are also power of 2.

![Fig. 2.2.2 Example of Grayscale](image)
2.2.3 Indexed image
Mostly all the colour images have a subset of more than sixteen million possible colours. For ease of storage and handling of file, the image has a related colour map, or the colours palette, that is simply a list of all the colours which can be used in that image. Each pixel has a value associated with it but it does not give its colour as for as in an RGB image. Instead it give an index to the colour in map. It is convenient for an image if it has 256 colours or less. The index values will require only one byte to store each. Some image file formats such as GIF which allow 256 colour only.

2.2.4 True Colour image
Each pixel in the RGB image has a particular colour; that colour in the image is described by the quantity of red, green and blue value in image. If each of the components has a range from 0–255, this means that this gives a total of 2563 different possible colours values. That means such an image is “stack” of three matrices; that represent the red, green and blue values in the image for each pixel. This way we can say that for every pixel in the RGB image there are corresponding 3 values.

2.3 Some Types of Image Processing
2.3.1 Image Enhancement
Processing an image so that the result is more suitable for a particular application. Such as sharpening or de-blurring an out of focus image, highlighting the edges of image, improving the contrast of image or increase the brightness level of an image, remove the noise from noisy image.
   a. Used for Contrast Enhancement
   b. Intensity, saturation and hue transformations
   c. Edge enhancement
   d. Producing the synthetic stereo image

2.3.2 Image Analysis
Image analysis is concerned with making a quantitative measurement from an image to produce a description of image. Image analysis techniques extract the certain features that aid in the recognition of an object.
   Quantitative measurement of an object features allows description and classification of the image.
   a. Produce principal component images
   b. Producing the ratio images
   c. Multi-spectral classification
   d. Produced change detection images
2.3.3 Image Restoration

Compensate for noise, data errors, and the geometric distortions that is introduced while recording, scanning, and the playback operations.

- It restores the periodic line dropouts
- Used for restoring periodic line striping
- Good for filtering of random noise
- Enhance geometric distortions

III. Proposed Methodology

- Read the image
- Pre processing
  - This is the steps which are done before interpolation to get a noise free enhanced image.
  - Noise removal: Noises are removed using suitable filters.
  - Image enhancement: It is done to increase the contrast by equalizing the histogram. Image Enhancement is one of the most important and difficult techniques in image research. The aim of image enhancement is to improve the visual appearance of an image, or to provide a “better transform representation for future automated image processing.

![Block Diagram]

- Image interpolation technique
  - Two methods are done for a comparison between them. Image interpolation refers to the “guess” of intensity values at missing locations.
  - Median diffusion
  - Kriging interpolation
  - Comparative analysis

- It is done using PSNR, SSIM and RMSE methods to get the better method. Kriging interpolation have high PSNR, SSIM and low RMSE.

\[
\text{PSNR} = 20 \log_{10} \sqrt{\frac{255^2}{\text{MSE}}} \quad \text{3.1}
\]

where the MSE is:

\[
\text{MSE} = \frac{1}{NM} \sum_{x=1}^{N} \sum_{y=1}^{M} [f(x, y) - g(x, y)]^2 \quad \text{3.2}
\]

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{j=1}^{n} (Y_j - y_j)^2} \quad \text{3.3}
\]

\[
\text{SSIM}(x, y) = \left( \frac{2\mu_x\mu_y + c_1}{\sigma_x^2 + \sigma_y^2 + c_2} \right) \left( \frac{2\sigma_{xy} + c_3}{\sigma_x^2 + \sigma_y^2 + c_4} \right) \quad \text{3.4}
\]

Result

Showing both the images and tabled the results of comparative analysis.
IV. Pre Processing

These are the steps which are done before interpolation to get a noise free enhanced image.

4.1 Noise

Noise is a random variation of image Intensity and visible as grains in the image. It may arise in the image as effects of basic physiologic like photon, nature of light or thermal energy of heat inside the image sensors produce at the time of capturing or image transmission. Noise means, the pixels in the image show different intensity values instead of true pixel values. Noise removal algorithm is the process of removing or reducing the noise from the image. The noise removal algorithms reduce or remove the visibility of noise by smoothing the entire image leaving areas near contrast boundaries. But these methods can obscure fine, low contrast details.

Noise is introduced in the image at the time of image acquisition or transmission. Different factors may be responsible for introduction of noise in the image. The number of pixels corrupted in the image will decide the quantification of the noise. The principal sources of noise in the digital image are:

a. The imaging sensor may be affected by environmental conditions during image acquisition.
b. Insufficient Light levels and sensor temperature may introduce the noise in the image.
c. Interference in the transmission channel may also corrupt the image.
d. If dust particles are present on the scanner screen, they can also introduce noise in the image.

4.1.1 Amplifier Noise (Gaussian noise)

The standard model of amplifier noise is additive, Gaussian, dependent at each pixel and dependent of the signal intensity, caused primarily by Johnson–Nyquist noise (thermal noise), including that which comes from the reset noise of capacitors ("kTC noise"). It is an idealized form of white noise, which is caused by random fluctuations in the signal. In colour cameras where more amplification is used in the blue colour channel than in the green or red channel, there can be more noise in the blue channel. Amplifier noise is a major part of the noise of an image sensor, that is, of the constant noise level in dark areas of the image. In Gaussian noise, each pixel in the image will be changed from its original value by a (usually) small amount. A histogram, a plot of the amount of distortion of a pixel value against the frequency with which it occurs, shows a normal distribution of noise. While other distributions are possible, the Gaussian (normal) distribution is usually a good model, due to the central limit theorem that says that the sum of different noises tends to approach a Gaussian distribution.

\[ p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \] .......................... 4.1.1

Fig.4.1.1 Example of Gaussian Noise

4.1.2 Salt-and-Pepper Noise (Impulse Noise)

Salt and pepper noise is sometimes called impulse noise or spike noise or random noise or independent noise. In salt and pepper noise (sparse light and dark disturbances), pixels in the image are very different in colour or intensity unlike their surrounding pixels. Salt and pepper degradation can be caused by sharp and sudden disturbance in the image signal. Generally this type of noise will only affect a small number of image pixels. When viewed, the image contains dark and white dots, hence the term salt and pepper noise. Typical sources include flecks of dust inside the camera and overheated or faulty (Charge-coupled device) CCD elements. An image containing salt-and-pepper noise will have dark pixels in bright regions and vice versa. This type of noise can be caused by dead pixels, analog-to-digital converter errors and bit errors in transmission.

for \( z = a \)
\[ p(z) = \begin{cases} p_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \] .......................... 4.1.2
4.1.3 Shot Noise

The dominant noise in the lighter parts of an image from an image sensor is typically that caused by statistical quantum fluctuations, that is, variation in the number of photons sensed at a given exposure level; this noise is known as photon shot noise. Shot noise has a root-mean-square value proportional to the square root of the image intensity, and the noises at different pixels are independent of one another. Shot noise follows a Poisson distribution, which is usually not very different from Gaussian. In addition to photon shot noise, there can be additional shot noise from the dark leakage current in the image sensor; this noise is otherwise known as "dark shot noise" or "dark-current shot noise".

4.1.4 Speckle Noise

The speckle noise is commonly found in the ultrasound medical images. It is a granular noise that inherently exists in and degrades the quality of the Active Radar and Synthetic Aperture Radar (SAR) images. Speckle noise in conventional radar results from random fluctuations in the return signal from an object that is no bigger than a single image-processing element. It increases the mean grey level of a local area. Speckle noise in SAR is generally more serious, causing difficulties for image interpretation. It is caused by coherent processing of backscattered signals from multiple distributed targets. In SAR oceanography, for example, speckle noise is caused by signals from elementary scatters, the gravity-capillary ripples, and manifests as a pedestal image, beneath the image of the sea waves.

4.1.5 Uniform Noise

The uniform noise cause by quantizing the pixels of image to a number of distinct levels is known as quantization noise. It has approximately uniform distribution. In the uniform noise the level of the gray values of the noise are uniformly distributed across a specified range. Uniform noise can be used to generate any different type of noise distribution. This noise is often used to degrade images for the evaluation of image restoration algorithms. This noise provides the most neutral or unbiased noise

\[ p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \]

4.1.6 Rayleigh Noise

Radar range and velocity images typically contain noise that can be modelled by the Rayleigh distribution.

\[ p(z) = \begin{cases} \frac{2}{b^2} (z-a) e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \]

4.1.7 Poisson noise

Poisson noise or shot noise isa type of electronic noise that occurs when the finite number of particles that carry energy, such as electrons in an electronic circuit or photons in an optical device, is small enough to give rise to detectable statistical fluctuations in a measurement.
4.2 Noise removal techniques

Filtering in an image processing is a basis function that is used to achieve many tasks such as noise reduction, interpolation, and resampling. Filtering image data is a standard process used in almost all image processing systems. The choice of filter is determined by the nature of the task performed by filter and behaviour and type of the data. Filters are used to remove noise from digital image while keeping the details of image preserved is a necessary part of image processing.

4.2.1 Mean Filter

The mean filter is a simple spatial filter. It is a sliding-window filter that replace the center value in the window. It replaces with the average mean of all the pixel values in the kernel or window. The window is mostly square but it can also be of any shape.

Unfiltered Values

<p>| | | |</p>
<table>
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<tr>
<td>8</td>
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<td>2</td>
<td>1</td>
<td>9</td>
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<tr>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
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\[
8+4+7+2+1+9+5+3+6=45 \\
45 / 9 = 5
\]

![Mean filtered](image)

Fig.4.2.1 Example of mean filtering

4.2.2 Median Filter

Median Filter is a simple and powerful non-linear filter which is based on order statistics. It is easy to implement method of smoothing images. Median filter is used for reducing the amount of intensity variation between one pixel and the other pixel. In this filter, we do not replace the pixel value of image with the mean of all neighbouring pixel values, we replace it with the median value. Then the median is calculated by first sorting all the pixel values into ascending order and then replace the pixel being calculated with the middle pixel value. If the neighbouring pixel of image which is to be considered contains an even numbers of pixels, then the average of the two middle pixel values is used to replace. The median filter gives best result when the impulse noise percentage is less than 0.1 %. When the quantity of impulse noise is increased the median filter not gives best result.

![Median filtered](image)

Fig. 4.2.2 Method of Median Filtering

4.2.3 Wiener Filter

The purpose of the Wiener filter is to filter out the noise that has corrupted a signal. This filter is based on a statistical approach. Mostly all the filters are designed for a desired frequency response. Wiener filter deal with the filtering of an image from a different view. The goal of wiener filter is reduced the mean square error as much as possible. This filter is capable of reducing the noise and degrading function. One method that we assume we have knowledge of the spectral property of the noise and original signal. We used the Linear Time Invariant filter which gives output similar as to the original signal as much possible.

Characteristics of the wiener filter are:

a. Assumption: signal and the additive noise are stationary linear-random processes with their known spectral characteristics
b. Requirement: the wiener filter must be physically realizable, or it can be either causal
c. Performance Criteria: There is minimum meansquare[MSE] error.
4.2.4 Fuzzy Filter

Fuzzy filters provide promising result in image-processing tasks that cope with some drawbacks of classical filters. Fuzzy filter is capable of dealing with vague and uncertain information. Sometimes, it is required to recover a heavily noise corrupted image where a lot of uncertainties are present and in this case fuzzy set theory is very useful. Each pixel in the image is represented by a membership function and different types of fuzzy rules that considers the neighbourhood information or other information to eliminate filter removes the noise with blurry edges but fuzzy filters perform both the edge preservation and smoothing. Image and fuzzy set can be modelled in a similar way. Fuzzy set in a universe of X is associated with a membership degree. Similarly, in the normalized image where the image pixels ranging from \{0, 1, 2... 255\} are normalized by 255, the values obtained are in the interval [0, 1].

4.2.5 Adaptive Filter

The wiener function applies a Wiener filter (a type of linear filter) to an image adaptively, tailoring itself to the local image variance. If the variance is large, wiener performs little smoothing. If it is small, wiener performs more smoothing. This approach often produces better results than linear filtering. The adaptive filter is more selective than a comparable linear filter, preserving edges and other highfrequency parts of an image. In addition, there are no design tasks; the wiener2 function handles all preliminary computations and implements the filter for an input image. wiener2, however, does require more computation time than linear filtering. Wiener works best when the noise is constant-power ("white") additive noise, such as Gaussian noise. Another method for removing noise is to evolve the image under a smoothing partial differential equation similar to the heat equation which is called anisotropic diffusion.

4.3 Image enhancement

It is done to increase the contrast by equalizing the histogram. Image Enhancement is one of the most important and difficult techniques in image research. The aim of image enhancement is to improve the visual appearance of an image, or to provide a "better transform representation for future automated image processing. Many images like medical images, satellite images, aerial images and even real life photographs suffer from poor contrast and noise. It is necessary to enhance the contrast and remove the noise to increase image quality. One of the most important stages in medical images detection and analysis is Image Enhancement techniques which improves the quality (clarity) of images for human viewing, removing blurring and noise, increasing contrast, and revealing details are examples of enhancement operations. The enhancement technique differs from one field to another according to its objective. The existing techniques of image enhancement can be classified into two categories: Spatial Domain and Frequency domain enhancement.

The principal objective of image enhancement is to process a given image so that the result is more suitable than the original image for a specific application.

- It accentuates or sharpens image features such as edges, boundaries, or contrast to make a graphic display more helpful for display and analysis
- The enhancement doesn't increase the inherent information content of the data, but it increases the dynamic range of the chosen features so that they can be detected easily.

Spatial domain enhancement methods:

Spatial domain techniques are performed to the image plane itself and they are based on direct manipulation of pixels in an image. The operation can be formulated as \( g(x,y) = T[f(x,y)] \), where \( g \) is the output, \( f \) is the input image and \( T \) is an operation on \( f \) defined over some neighborhood of \((x,y)\). According to the operations on the image pixels, it can be further divided into 2 categories: Point operations and spatial operations (including linear and non-linear operations). These methods enhance an image \( f(x,y) \) by convoluting the image with a linear, position invariant operator. The 2D convolution is performed in frequency domain with DFT.

Spatial domain: \( g(x,y) = f(x,y) * h(x,y) \)
Frequency domain: \( G(w_1,w_2) = F(w_1,w_2) \cdot H(w_1,w_2) \)

Histogram processing

- The histogram of a digital image with gray levels in the range [0,L-1] is a discrete function \( p(r_k) = n_k/n \), where \( r_k \) is the kth gray level \( n_k \) is the number of pixels in the image with that graylevel \( n \) is the total number of pixels in the image, and \( k=0,1,..,L-1 \).
- \( P(r_k) \) gives an estimate of the probability of occurrence of gray level \( r_k \).
The shape of the histogram of an image gives us useful information about the possibility for contrast enhancement.

A histogram of a narrow shape indicates little dynamic range and thus corresponds to an image having low contrast. Histogram equalization: The objective is to map an input image to an output image such that its histogram is uniform after the mapping.

Let \( r \) represent the gray levels in the image to be enhanced and \( s \) is the enhanced output with a transformation of the form \( s=T(r) \).

Assumption:
1. \( T(r) \) is single-valued and monotonically increasing in the interval \([0,1]\), which preserves the order from black to white in the gray scale.
2. \( 0 \leq T(r) \leq 1 \) for \( 0 \leq r \leq 1 \), which guarantees the mapping is consistent with the allowed range of pixel values.

V. Existing Methodology

A simple digital image restoration technique using median filter, one of the most popular nonlinear (order statistics) filters is done to have a comparative analysis with kriging method. The median is maximum likelihood estimate of location for the Laplacian distribution. Hence, in this algorithm it diffuses median value of pixels from the exterior area into the inner area to be inpainted. The median filter preserves the edge which is an important property needed to inpaint edges. This technique is stable.

This technique is introduced with reference to Bertalmio et al. The input to the algorithm is the image to be restored and a mask that delimits the region to be inpainted. As a pre-processing step, the whole original image undergoes anisotropic diffusion smoothing to minimize the influence of noise on the estimation of the direction of the isophotes arriving at restoration region boundary. After this, the image enters the restoration loop, where only value inside restoration regions modified. The inpainted image pixel is given by: \( I(i,j) = \text{median}(I(x,y)) \) for all \((i,j)\) in restoration region where \( I(x,y) \) indicates the region around the pixel to be inpainted.

Here, restoration is interpreted as median filtering, a nonlinear order statistics filtering. The median is the maximum likelihood estimate (MLE) of location for the Laplacian distribution. The parameter \( p \) which determines the region around the restoration pixel is chosen according to the width of restoration area and is an odd number. This determines the influence of pixel exterior to the restoration area. If median propagation is adopted for the restoration process, then the inpainted pixel will have probability greater than or equal to 0.5. It is important to have larger range of values across the restoration area to estimate the pixel distribution correctly.

5.1 Detailed algorithm

Consider an original image \( I(i,j); 1 \leq i \leq M, 1 \leq j \leq N \) and is an input to our algorithm.

\[ \begin{align*}
\text{Start} \\
\text{Select the parameter } p \text{ which is proportional to the width of restoration area.} \\
\text{For every pixel } (i,j) \text{ belonging to the restoration area} \\
\text{Select Range } = (i-(p-1)/2: i+(p-1)/2, \ (j-(p-1)/2: j+(p-1)/2) \quad \square \text{Apply median filtering:} \\
\text{Median of selected region} \\
\text{Diffuse the median value to the restoration pixel.} \\
\text{Repeat above steps for all pixels } (i,j) \text{ in the restoration area} \\
\text{End.}
\end{align*} \]

This technique is well-suited to small restoration region and require large number of iterations to converge for thick restoration area. For larger area restoration, blur and blocky effects creep in. The drawback of this technique is, that the images with texture background leave distinct edges around the mask area and the program of this method explained in the appendix.

Consider \( p=3 \), median filtering in the region is:

![Fig.5.1.1 Example of median diffusion (a) original image (b) inpainted image](image-url)
VI. Kriging Interpolation

6.1 Introduction

Kriging is a geostatistical interpolation method that takes into account both the distance and the degree of variation between known points when predicting values in unknown locations. Kriging is aiming to estimate unknown values at specific points in space by using data values from its surrounding regions. Kriging yields optimal aftermaths compared with the traditional interpolation methods [1]. It must be mentioned that, Kriging is an exact interpolator technique because it ensures that the original observed values will stay as it, i.e. the old values will not get affected by the interpolation technique. Kriging predictions are treated as weighted linear combinations of the known locations. According to Kriging technique, the closer the input, the more positively correlated predictions [9]. Now, let's bring the previously mentioned thoughts into digital image processing. According to R. C. Gonzalez, (2008) [5], the pixels within the same kxk block are highly correlated, therefore; the application of Kriging inside the kxk block will yields high positively correlated predictions. Kriging gives weights for each point inside kxk block in accordance to its distance from the unknown value. Actually, these predictions treated as weighted linear combinations of the known values. The weights should provide a Best Linear Unbiased Estimator (BLUE) of the predicted point [20]. The essential characteristic of Kriging over conventional interpolation methods is that it uses the spatial correlation structure of points inside kxk block being interpolated in order to compute the unknown point [18]. There is a robust connection between image denoising and image restoration especially scratch removal. Both fields are sharing the same principles in finding and removing the unwanted areas [2].

The basic formula of Kriging technique may be represented as follows:

\[ p^* = \sum_{i=1}^{N} \lambda_i p_i \]  6.1.1

where N is the total number of the non-scratched pixels inside the kxk block. Moreover, \( p^* \) stands for the predicted pixel and \( p_i \) are the representation for the non-scratched pixels inside the kxk block. The weights of the non-scratched pixels \( \lambda_i \) must satisfy: \( \sum_{i=1}^{N} \lambda_i = 1 \)..................Equation 6.1.2

The Kriging estimate is obtained by choosing \( \lambda_i \) that minimize variance of the estimator under the unbiasedness constraint:

\[ \sigma^2 = (p - p^*)^2 \] 6.1.3

There are several Kriging types, differ in their treatments for the weighted components (\( \lambda_i \)). The most preferred Kriging type and it is considered to be the best one is ordinary Kriging because it minimizes the variance of the prediction error [9]. It must be mentioned that, variogram is one of the most supporting functions to indicate spatial correlation in observations measured at observed points. The variogram is a function of the distance and direction separating two locations that is used to quantify dependence. The variogram is defined as the variance of the difference between two variables at two locations [18]. The variogram generally increases with distance and is described by nugget, sill, and range parameters. If the data is stationary, then the variogram and the covariance are theoretically related to each other. It is commonly represented as a graph that demonstrates the variance with respect to the distance between all points of the observed locations [19]. The variogram describes the variance of the difference of samples within the data set and is calculated by the following equation:

\[ 2\gamma(h) = \frac{1}{n} \sum_{i=1}^{n} [p(x_i) - p(x_i + h)]^2 \] ..........................6.1.4

where \( P(x_i) \) and \( P(x_i+h) \) are the pixel values at locations \( x \) and \( x_i+h \), respectively. In this paper, Kriging was treated as a supporting scheme that helps to reach the goal which is image restoration. Hence, there is no need to discuss the variogram in a detailed manner. An exhaustive discussion and analysis about variogram could be found through recommended readings [19][18].

6.2 Proposed technique

In this work, a novel image restoration method based on Kriging interpolation technique was proposed. The proposed method starts with identifying the queer pixels within the kxk block from the contaminated image. The contamination may be thin scratch, thick scratch, text, bad areas generated by aging, or even unwanted objects that may be eliminated from the original image. These contaminated areas will be marked according to its corresponding mask. After that, the kxk block will be dispatched to Kriging interpolation technique to predict the contaminated areas using the accurate prediction feature of Kriging. As mentioned previously, Kriging method uses variogram to express the spatial variation, and it minimizes the error of predicted values which are estimated by spatial distribution of the predicted values. The resulted predictions seem to be very close to the original pixels. Therefore, Kriging is very suitable to estimate the mask's pixels accurately.
VII. Comparative Parameters

7.1 PSNR (Peak Signal to Noise Ratio)

The term is an expression for the ratio between the maximum possible value (power) of a signal and the power of distorting noise that affects the quality of its peak signal-to-noise ratio (PSNR) representation. Because many signals have a very wide (ratio between the largest and smallest possible values of a changeable quantity) range of values this usually expressed in terms dynamic range PSNR of the logarithmic decibel scale.

Image enhancement or improving the visual quality of a digital image can be subjective. Saying that one method provides a better quality image could vary from person to person. For this reason, it is necessary to establish quantitative/empirical measures to compare the effects of image enhancement algorithms on image quality.

Using the same set of tests images, different image enhancement algorithms can be compared systematically to identify whether a particular algorithm produces better results. The metric under investigation is the If we can show that an algorithm or set of algorithms can enhance a degraded known image to more closely resemble the original, then we can peak-signal-to-noise ratio more accurately conclude that it is a better algorithm.

The mathematical representation of the PSNR is as follows:

\[
\text{PSNR} = 20 \log_{10} \frac{255}{\sqrt{\text{MSE}}} 
\]

where the MSE is:

\[
\text{MSE} = \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} [f(x, y) - g(x, y)]^2 \] ....7.1.2

Legend:
- \( f \) represents the matrix data of our original image
- \( g \) represents the matrix data of our degraded image
- \( m \) represents the numbers of rows of pixels of the images
- \( i \) represents the index of that row
- \( n \) represents the number of columns of pixels of the image
- \( j \) represents the index of that column

The mean squared error (MSE) for our practical purposes allows us to compare the “true” pixel values of our original image to our degraded image. The MSE represents the average of the squares of the "errors" between our actual image and our noisy image. The error is the amount by which the values of the original image differ from the degraded image.

The proposal is that the higher the PSNR, the better degraded image has been reconstructed to match the original image and the better the reconstructive algorithm. This would occur because we wish to minimize the MSE between images with respect the maximum signal value of the image.

7.2 SSIM (structural similarity)

The structural similarity (SSIM) index is a method for measuring the similarity between two images. The SSIM index is a full reference metric; in other words, the measuring of image quality based on an initial uncompressed or distortion-free image as reference. SSIM is designed to improve on traditional methods likepeak signal-to-noise ratio (PSNR) and mean squared error (MSE), which have proven to be inconsistent with human eye perception. The difference with respect to other techniques mentioned previously such as MSE or PSNR is that these approaches estimate perceived errors; on the other hand, SSIM considers image degradation as perceived change in structural information. Structural information is the idea that the pixels have strong inter-dependencies especially when they are spatially close. These dependencies carry important information about the structure of the objects in the visual scene. The SSIM metric is calculated on various windows of an image. The measure between two windows and of common size N×N is:

\[
\text{SSIM}(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma^2_x + \sigma^2_y + c_2)} 
\]

- \( \mu_x \) the average of \( x \)
- \( \mu_y \) the average of \( y \)
- \( \sigma_x^2 \) the variance of \( x \)
- \( \sigma_y^2 \) the variance of \( y \)
- \( \sigma_{xy} \) the covariance of \( x,y \)
- \( c_1 = (k_1L)^2 \), \( c_2 = (k_2L)^2 \) two variables to stabilize the division with weak denominator
7.3 RMSE (Root mean square error)

The root-mean-square deviation (RMSD) or root-mean-square error (RMSE) is a frequently used measure of the differences between values (sample and population values) predicted by a model or an estimator and the values actually observed. Basically, the RMSD represents the sample standard deviation of the differences between predicted values and observed values. These individual differences are called residuals when the calculations are performed over the data sample that was used for estimation, and are called prediction errors when computed out-of-sample. The RMSD serves to aggregate the magnitudes of the errors in predictions for various times into a single measure of predictive power. RMSD is a good measure of accuracy, but only to compare forecasting errors of different models for a particular variable and not between variables, as it is scale-dependent.

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}
\]

\text{Equation 8.3}

VIII. Result And Discussions

8.1 Experimental result and analysis

![Fig. 8.1: Before and after interpolation with different masks.](image)

8.2 Comparative Analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Kriging/Interpolation</th>
<th>Median Diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR</td>
<td>46.68</td>
<td>46.17</td>
</tr>
<tr>
<td>RMSE</td>
<td>43.01</td>
<td>45.86</td>
</tr>
<tr>
<td>SSIM</td>
<td>0.9966</td>
<td>0.996</td>
</tr>
</tbody>
</table>

\text{Table 8.1.1 Comparison of parameters}

IX. Conclusion

In our project, a novel approach for removing scratches and text from contaminated images has been presented. The proposed technique use Kriging in a way that removes unwanted regions from image which is known as image restoration. Despite Kriging being more computationally expensive, it has been shown that it gives very sophisticated output when repairing digital images that have scratches or unwanted text. Experimental results reveal that the proposed Kriging technique having high PSNR, SSIM and low RMSE values when implemented on a variety of test images.

There are several types of interpolation techniques. One of the most effective technique is kriging interpolation technique. Inverse distance weighted interpolation technique can also be used in this, but it doesn’t provide a detailed information. Both the technique produces similar result. The analyse from Kriging technique shows that there are many major maxima in studies area, hence making geological interpretation inaccurate. A technique has to be chosen which not only takes distance and degree of variation to predict values in unknown
locations into account but also other parameters for its applicability to a wide range of spatial scales inorder to obtain perfect result.

References