

A novel analysis of Optimal Power Flow with TCPS

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Abstract: Optimal power flow (OPF) is one of the most important algorithms available to utility for generating least cost generation patterns in a power system satisfying transmission and operational constraints. A wide variety of conventional techniques are available to obtain solution. In day to day life, the forecasted loads used in classical OPF algorithms are increasing with time and are also not completely free from errors. By varying the load demands leads to overloading of the transmission lines and forecasted errors cause loss of optimal system. So in this modern optimal power flow algorithms may not be able to provide optimal solutions. This paper presents solution of optimal power flow problem for 30bus system via a simple genetic algorithm analysis. Our objective is to minimize the fuel cost and keep the power outputs of Alternators, Line bus voltages, line shunt capacitors/reactors and transformers tap-setting in their secure limits.

Keywords: FACTS; Genetic algorithm; Optimal power flow.

I. Introduction

The Optimal Power Flow (OPF) problem was introduced by Carpentier in 1962 as a network constrained economic dispatch problem. OPF formulation is aimed to minimize the operating costs while satisfying constraints including voltage limits, generation capability etc. It determines the optimal setting of generation operating units. It is necessary importance to solve this problem as quickly and accurately as possible [1].

There are a number of conventional methods like Lambda iteration method, Gradient method, Newton's method etc [2]. However, these methods suffers from some shortages like enormous computational efforts and time consumption, sensitiveness to starting points, frequent convergence to local optimum solution, non applicable with discrete variables etc[3].

It is well known that FACTS devices can improve the steady state as well as transient performance of power systems. The degree of non convexity on OPF problems are further increased with the inclusion of FACTS devices on the network and the usual conventional methods will not produce optimal result. Hence, it is necessary to employ a new, reliable and modern method. It is thus necessary to solve OPF using one of the modern methods- Genetic Algorithm.

GA, invented by Holland [4] in the early 1980s, is a stochastic global search method that mimics the metaphor of natural biological evaluation. GAs are an attractive alternative to other optimization methods because they overcome the limitations of the conventional methods and are robust [5].

II. Optimal Power Flow Problem Formulation

The mathematical formulation of the OPF problems is a well known optimization problem. The basic formulation of any optimization can be represented as minimizing a defined objective function subject to any physical or operational constraints of the system. Hence an optimization problem consists of objective function subjected to equality constraints and inequality constraints. The main objective is to maximize the total potential energy of water stored in all the reservoirs. The formulation must take into account the fact that the water stored in one reservoir will be used in all its downstream reservoirs, hence, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir [1]:

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2-1. Objective function:

$$\max \sum_{i=1}^n E_p(x_i^{k_f}) + \sum_{k=a}^{k_f} E_p(u_{mi}^k)$$

Where:

k_f : Last period of the planning horizon.

$E_p(x_i^{k_f})$: Potential energy of water stored in reservoir i at the end of the planning horizon k_f . This energy depends on the amount of water $x_i^{k_f}$ stored in the reservoir i , on its effective water head and on the effective water head of the downstream reservoirs.

$\sum E_p(u_{mi}^k)$: Total potential energy of the released water u_{mi}^k from reservoir m , which will reach later the downstream reservoir i after the last period of the planning horizon k_f .

$$a = k_f - S_{mi}$$

m : The reservoir immediately preceding the reservoir i .

S_{mi} : Duration period for the water discharged from reservoir m to reach its direct downstream reservoir i , in hours.

2-2. Operational constraints:

The principal operational constraints of the system are the following [1-2, 5-8]:

- Hydraulic continuity constraint:

For each reservoir i at each time period k , the following constraint establishes the water balance equation:

$$x_i^k = x_i^{k-1} + y_i^k - u_i^k - v_i^k$$

Where:

x_i^k : Content of the reservoir i at period k , in Mm^3 .

u_i^k : Turbine discharge of hydroelectric plant i in period k , in Mm^3 .

v_i^k : Spillage of hydroelectric plant i in period k , in Mm^3 .

y_i^k : Total inflows to reservoir i in period k , in Mm^3 .

Taking into account hydraulic coupling, the total inflow to reservoirs is determined as follows:

$$y_i^k = q_i^k + u_{mi}^{k-S_{mi}}$$

q_i^k : Natural inflows to reservoir i in period k , in Mm^3 .

$u_{mi}^{k-S_{mi}}$: Turbine discharge from hydroelectric plant m , which will reach later the downstream reservoir i at period $k - S_{mi}$.

- Minimum and maximum storage limits:

$$\underline{x}_i \leq x_i^k \leq \bar{x}_i$$

$\underline{x}_i, \bar{x}_i$: Lower and upper limits on reservoir storage capacity, respectively, for reservoir i , in Mm^3 .

- Minimum and maximum discharge limits:

$$\underline{u}_i \leq u_i^k \leq \bar{u}_i$$

$\underline{u}_i, \bar{u}_i$: Minimum and maximum limits on water discharge, respectively, of hydroelectric plant i .

- Demand-generation balance:

The total power generated by all the hydroelectric plants must satisfy the system load demand at each period of the planning horizon. In mathematical terms, this has the following form:

$$\sum_{i=1}^n P_i^k = D^k$$

Where:

D^k : The demand for electrical power at each period k , in Mw.

P_i^k : Electric power generated by hydro plant i at period k , in Mw.

n : Number of reservoirs of the system.

III. Solution method

In mathematical terms, we formulate the short term scheduling problem of a hydroelectric plant system as follows [1]:

$$\max \sum_{i=1}^n E_p(x_i^{k_f}) + \sum_{k=a}^{k_f} E_p(u_{mi}^k) \quad (1)$$

Subject to the following constraints:

$$x_i^k = x_i^{k-1} + y_i^k - u_i^k - v_i^k \quad (2)$$

$$\sum_{i=1}^n P_i^k = D^k \quad (3)$$

$$0 \leq x_i^k \leq \bar{x}_i \quad (4)$$

$$0 \leq u_i^k \leq \bar{u}_i \quad (5)$$

$$v_i^k \geq 0 \quad (6)$$

This problem can be solved by applying the discrete maximum principle as follows [1, 9-10]:

Associate the constraint (2) to the criterion (1) with the dual variable λ_i^k . To satisfy the balance between electric energy demand and generation we associate the constraint (3) to the criterion (1) with the Lagrange multiplier β^k , and then we define the function H^k , called the Hamiltonian function, which has the following form:

$$H^k = \sum_{i=1}^n [\lambda_i^k (x_i^k + y_i - u_i^k + u \sum_m u_{mi}^{k-s_{mi}})] + \beta^k (\sum P_i^k - D^k)$$

The problem (1)-(6) becomes:

$$\max H^k \quad (7)$$

Subject to constraints (4)-(6), with the conversion equation of the associate variable [9]:

$$\lambda_i^{k-1} = \frac{\partial H^k}{\partial x_i^{k-1}} \quad (8)$$

When constraints (4), (5) and (6) are inactive, the optimal trajectory u_i^k will be reached when the following optimality conditions for each hydroelectric plant and at each period are satisfied:

$$\frac{\partial H^k}{\partial u_i^k} = 0 \quad (9)$$

In order to solve these equations, we need to know the operating limits, which are:

- The first one is the initial state, which is specified, i.e., at the initial time, the initial content of all reservoirs are known, thus:

$$x_i^0 = b_i \quad (10)$$

- The second one is the terminal condition for the adjoint equation:

$$\lambda_i^{k_f} = \frac{\partial E_p(x_i^{k_f})}{\partial x_i^{k_f}} \quad (11)$$

Consequently, equations (2) and (8)-(11) constitute a two-point boundary value problem whose solution determines the optimal state and control variables. This problem is solved iteratively by using the gradient method.

To take into account possible violations of constraints (4) and (5) we proceed respectively as follows:

- If the value of some u_i^k which satisfies the optimality condition (8) violates the constraint (5), we'll fix them to their boundary limits, the others are left free. Then a new research for the optimum is made but with only the free variables.

- To deal with possible violations of constraint (4), we use the augmented Lagrange method [1,3,11], which consists in adding a function R_i^k to the Hamiltonian H^k , which penalizes the violation of constraints (5).

Then the Hamiltonian H^k becomes:

$$H^k = \sum_{i=1}^n [\lambda_i^k (x_i^k + y_i - u_i^k)] + \beta^k (\sum P_i^k - D^k) + R_i^k \quad (12)$$

The function R_i^k is defined as follows:

$$R_i^k = \rho_i^k \Psi_i^k + r(\Psi_i^k)^2 \quad (13)$$

Where:

r : Penalty weight.

ρ_i^k : Lagrange multipliers, updated as follows:

$$\rho_i^k = \rho_i^k + 2r \max(x_i^k - \bar{x}_i, -\frac{\rho_i^k}{2r}) \quad (14)$$

The operating function Ψ_i^k is determined as follows:

$$\Psi_i^k = \max(x_i^k - \bar{x}_i, -\frac{\rho_i^k}{2r}) \quad (15)$$

IV. Model Of TCPS

Figure 1 shows the schematic diagram of a TCPS. The series transformer injects a voltage in series with the system. The active and reactive power injected by the series transformer is taken from the shunt connected transformer. Here the losses in the transformers and the converter are neglected. Thus the net complex power (active and reactive power) exchange between the TCPS and the system is zero. The injected complex power of the series transformer depends on the complex injected voltage and the line current.

Figure 2 shows the equivalent circuit of Figure 1, where V_s and V_{sh} represent by the voltage of the series and shunt transformer, respectively. X_s and X_{sh} represent the leakage reactance of the series and shunt transformers, respectively. X_s' represents the leakage reactance seen from the primary side of the series transformer [7] and is given by

$$X_s' = X_s + n^2 X_{sh} \quad (16)$$

where n is the turns ratio of the shunt transformer

The shunt voltage source and the associated leakage reactance X_{sh} can be represented by a shunt injected current source (I_{sh}) as shown in Figure 3. The shunt injected current has two components: in phase component (I_p) and quadrature component (I_q) with respect to the bus voltage V_m . Thus I_{sh} can be expressed as

$$I_{sh} = (I_p - jI_q) e^{j\delta_m} \quad (17)$$

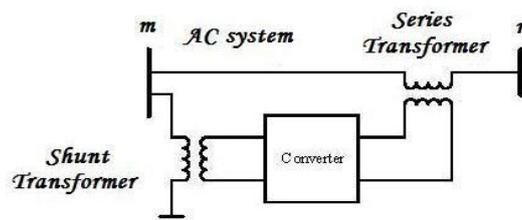


Fig.1.TCPS schematic diagram

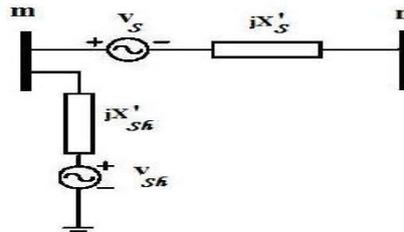


Fig.2. TCPS equivalent circuit

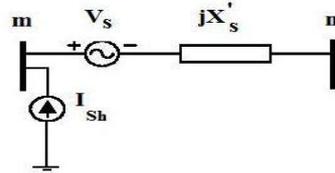


Fig.3. TCPS current injection model.

V. Severity Index

Here this system operating under normal and contingency cases can be described by a real power line flow performance index as

$$PI = \sum_{m=1}^{N_s} \frac{W_m}{2n} \left(\frac{P_{lm}}{P_{lm}^{\max}} \right)^{2n} \quad (18)$$

where P_{lm} is the real power flow and P_{lm}^{\max} is the rated capacity of line-m, n is the exponent and W_m a real nonnegative weighting coefficient which may be used to reflect the importance of line connected system. In this operating PI will be small when all the lines are within their limits and reach a high value when there are high levels operating loads. Thus, it provides a good measure of severity of the line overloads for a given state of the power system.

In solving the OPF problem, two types of variables need to be determined by the optimization algorithm: generator active power generation P_{gi} and generator bus voltages V_{gi} which are continuous variables and transformer tap setting t_k which are discrete variables. The discrete variables are in the formulation due to the discrete nature of the transformer tap positions. Conventional methods are not efficient in handling problems with discrete variables. In this paper Genetic Algorithm is applied to solve the OPF problem.

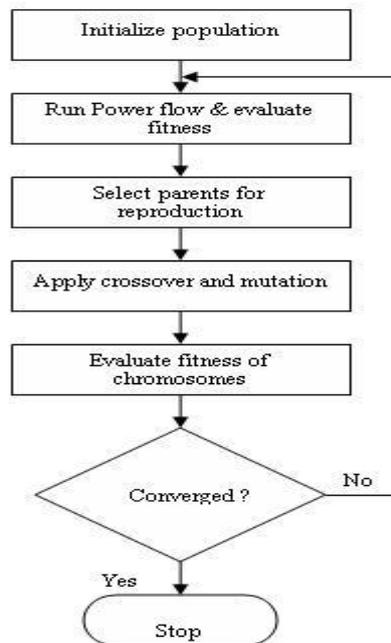


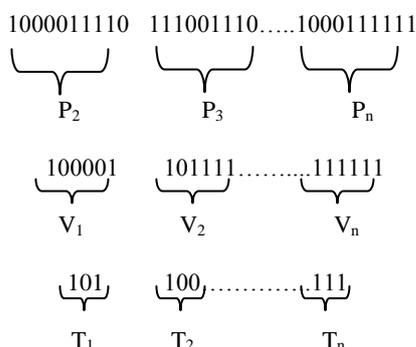
Fig.4. Flowchart of GA-OPF Algorithm.

While applying GA, to solve the OPF problem, either it can be used to obtain the optimal values of all the control variables or to search the discrete variables alone, leaving the continuous variables to be searched by the conventional method. In the first approach, the individuals in the GA population consists of binary strings corresponding to all the variables and the fitness value of each individual is evaluated by running the power flow algorithm using the control variables represented by the individual. Then the genetic operators are applied on the GA population. This process is continued until the convergence criterion is satisfied. This is pictorially represented in Figure 4.

In the second approach, only the transformer tap setting which is a discrete quantity alone is coded as the individual of the GA population. In this case each individual is evaluated by running the optimal power flow program using the transformer tap setting represented by the individual. Thus in this approach, the conventional LP- based algorithm searches the continuous variables and GA searches the discrete variables.

Population representation and initialization Fitness evaluation and application of genetic operators need to be addressed while applying GA for the OPF problem

A GA works on chromosomes, which are strings of zeros and ones. Implementation of a problem in a GA starts from the parameter encoding (i.e., the representation of the problem).The encoding must be carefully designed to utilize the GA's ability to efficiently transfer information between chromosome strings and the objective function of problem. Each individual in the population represents a candidate OPF solution. The elements of that solution consist of all the control variables in the system. For the OPF problem under consideration, generator active power P_{gi} and generator terminal node voltages V and transformer tap settings are the control variables.



In the OPF problem under consideration, the objective is to minimize the total fuel cost satisfying the constraints. For each individual, the equality constraints are satisfied by running the Newton Raphson power flow algorithm and the constraints on the state variables are taken into consideration by adding a quadratic penalty function to the objective function. With the introduction of penalty function the new objective function becomes,

$$\text{Min } f = F_r + SP + \sum_{j=1}^{N_l} VP_j + \sum_{i=1}^{N_g} QP_i + \sum_{j=1}^{N_l} LP_j \quad (19)$$

Here SP, VP_j , QP_j and LP_j are the penalty terms for the reference bus generator active power operating limit violation, load bus voltage limit offence; this reactive power generation limit violation and line flow limit violating system respectively. These quantities are defined by the following equations:

$$SP = \begin{cases} K_s (P_s - P_s^{\max})^2 & \text{if } P_s > P_s^{\max} \\ K_s (P_s - P_s^{\min})^2 & \text{if } P_s < P_s^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

$$VP_j = \begin{cases} K_v (V_j - V_j^{\max})^2 & \text{if } V_j > V_j^{\max} \\ K_v (V_j - V_j^{\min})^2 & \text{if } V_j < V_j^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

$$QP_j = \begin{cases} K_q (Q_j - Q_j^{\max})^2 & \text{if } Q_j > Q_j^{\max} \\ K_q (Q_j - Q_j^{\min})^2 & \text{if } Q_j < Q_j^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

$$LP_j = \begin{cases} K_i (S_i - S_i^{\max})^2 & \text{if } S_i > S_i^{\max} \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Where K_s , K_v , K_q and K_l are the penalty factors. The success of the approach lies in the proper choice of these penalty parameters. Using the above penalty function approach, one has to find a correct combination of penalty parameters K_s , K_v , K_q and K_l . However, in order to reduce the number of penalty parameters, often the constraints are normalized and only one penalty factor R is used.

Since GA maximizes the fitness function, the minimization objective function is transformed to a fitness function to be maximized as,

$$Fitness = \frac{k}{f} \quad (24)$$

Where k is a large constant.

VI. Results And Discussions

The first part deals with solving Optimal Power Flow problem for a 30- bus test system without FACTS devices using Genetic Algorithm. The control parameter settings for solving OPF without FACTS devices are number of generations (= 60), Population size (= 50), Cross over rate (= 0.6), Mutation rate (= 0.05).

Table: Control Parameter Settings For Opf For A 30 Bus System

No. of generations	Population size	Crossover rate	Mutation rate
60	50	0.6	0.05

We consider real power of slack bus, reactive power of generator bus, voltage of load bus and power flow through the branches as the constraints. The graph showing generation versus fitness is shown in Fig. 5. The minimum cost of 803.489\$/hr is obtained with the following values in Table 1 for real powers and voltages.

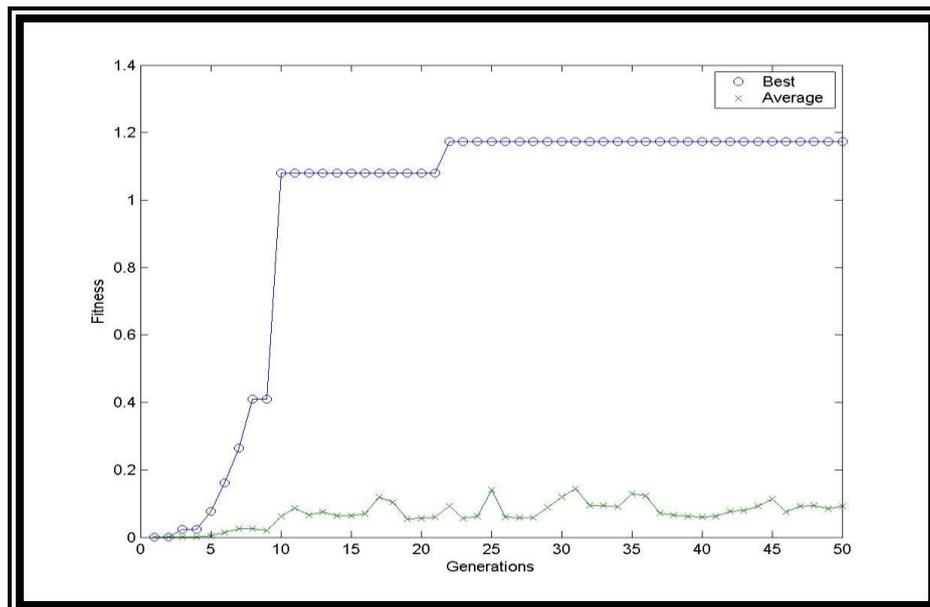


Fig.5.Generation versus fitness for a 30 bus system

Table real power and voltage for generator buses

P ₁	194.353
P ₂	26.86
P ₃	3.22
P ₄	50.76
P ₅	5.16
P ₆	2.66
V ₁	0.9764
V ₂	1.0702
V ₃	0.9728
V ₄	0.9626
V ₅	1.0095
V ₆	1.0694

Table: control parameter settings including tcps

Line outages	No. of generations	Population size	Crossover rate	Mutation rate
1-2	60	50	0.2	0.06
1-3	60	50	0.4	0.02
3-4	60	50	0.1	0.05

It is seen that, on the outage of some of the branches, the power flow through the branches gets violated. We consider outage of branches 1-2, 1-3 and 3-4. In order to relieve these lines from overloading, we place one of the FACTS devices, TCPS in appropriate positions, which are determined by sensitivity analysis. The problem representation then will have an additional constraint which is the phase shifting transformer constraint. The control variables will then be the real power of generator and phase angle of the TCPS[9]. The number of control variables, their range which specifies the minimum limit and maximum limit, number of bits etc are included in the GA coding.

Table: simulation result on including tcps for various line outages

Line outage	1-2	1-3	3-4
P ₁	139.6825	142.8571	161.9048
P ₂	38.0952	38.0952	22.8571
P ₃	15.000	32.2222	32.7778
P ₄	39.6825	40.4762	48.4127
P ₅	18.0952	3.3333	8.0952
P ₆	32.3810	26.0317	8.8889
X _{TCPS1}	-5.2381	9.3651	-9.6825
X _{TCPS2}	-2.0635	2.6984	7.778
X _{TCPS3}	-2.6984	-3.0159	0.1587
X _{TCPS4}	0.1587	-2.3810	-6.5079
SI	0	0	0

Matpower solves power flow and provides real power of slack bus, reactive power of generator bus, voltage of load bus and power flow through the branches. The setting of control parameters for various line outages including TCPS are given in Table 2.

The value of real power, phase angle of TCPS and cost are given in Table 3. It is seen that the line overloads were relieved through the adjustment of phase angle of TCPS.

VII. Conclusion

The minimum cost obtained without including FACTS devices was near to the cost obtained by using Gradient method. We can conclude, the proposed work has given a better global solution. In addition, it saves computational time as well as computer memory and produce most optimal result.

The next part deals with the security enhancement. In the base case, we see that the real power of the slack bus, reactive power of the generator bus, voltage of the load bus and power through the branches are within the limits. However, on line outages, some of the lines get overloaded. FACTS devices are included in order to relieve lines from overloading. By proper positioning of TCPS, the lines get relieved from overloading and hence the severity index is obtained to be zero.

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