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Relative Stability Analysis of Linear Systems Based on Damped Frequency of Oscillation

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Abstract: *Information about the relative stability of a control system is of paramount importance for any design problem. In this paper two algebraic criteria for the relative stability analysis of linear time-invariant systems are formulated. When the relative stability analysis is done based on the damped frequency of oscillation, characteristic equations with complex coefficients arise. These complex coefficients are used in two different ways to form the Modified Routh's tables for the two schemes named as Sign Pair Criterion I and Sign Pair Criterion II .It is found that the proposed algorithms offer computational simplicity compared to other algebraic methods and is illustrated with suitable examples.*
Keywords - *Stability analysis, complex coefficients, Sign pair criterion, Routh's Table, Relative stability*

I. INTRODUCTION

Once the system is found to be stable, it is important to determine how stable it is and this degree of stability is a measure of relative stability. Relative stability analysis can be done based on damping margin and damped frequency of oscillation in time domain analysis as presented by Hwang &Tripathi [1]. In frequency domain, the analysis is done based on gain margin and phase margin as discussed in Nagrath & Gopal [2].When the relative stability analysis is done in time domain, characteristic equations with complex coefficients arise. Then the normal Routh's algorithm cannot be applied.

To analyse the stability of complex polynomials the generalized Routh-Hurwitz method was investigated in [3] - [7]. Frank [3] and Agashe [4] developed a new Routh like algorithm to determine the number of RHP roots in the complex case. Benidir and Picinbono [5] proposed an extended Routh table which considers singular cases of vanishing leading array element. By adding intermediate rows in the Routh array, Shyan and Jason [6] developed a tabular column ,which is also a complicated one. Adel [7] has done the stability analysis of complex polynomials using the J-fraction expansion , Hurwitz Matrix determinant and also generalized Routh's Array.

Hwang & Tripathi [1] suggested an algebraic method using complex conjugates to convert complex coefficient equations to real coefficient equations for relative stability analysis The same method was proposed for the analysis of relative damping margin in the literature [8] which is a complicated one for computation.Here the complex coefficients are used in two different ways to form the Modified Routh's tables for the two schemes named as Sign Pair Criterion I (SPC I) and Sign Pair Criterion II (SPC II). The beauty of the Routh's algorithm lies in finding the aperiodic stability of the system without determining the roots of the system.

In the first approach, formation of Routh's Table is done by retaining the 'j' terms of the complex coefficients and the stability analysis is done using Sign Pair Criterion I (SPC I) . The proof is given in [9]. In the second scheme, a geometrical procedure is presented which is named as Sign Pair Criterion II (SPC II) and is formulated with the help of 'Modified Routh's table' after separating the real and imaginary parts of the characteristic equation by substituting s='jω'. Applying Routh–Hurwitz criterion, the number of the roots of F(s)=0 having positive real part can be revealed. The proof for SPC II is given in [10]. Then by the use of the proposed schemes relative stability is analyzed in a most simple way regardless of the order of the system .The computational simplicity is illustrated with examples.

II. PROPOSED SCHEMES

1.1 Sign Pair Criterion

With all the coefficients positive, the characteristic equation C(s) can be written as,

$$C(s) = s^n + (a_1 + j\,b_1)s^{n-1} + (a_2 + j\,b_2)s^{n-2} + \cdots + (a_k + j\,b_k) = 0 \qquad (1)$$

The first two rows of Routh-like table are written a shown below:

1	jb_1	a_2	jb_3	a_4	. . .
a_1	jb_2	a_3	jb_4	a_5	. . .

Applying the standard Routh multiplication rule [2], the subsequent elements of Routh-like table are computed and the table is computed as given below:

1	jb_1	a_2	jb_3	a_4	. . .
a_1	jb_2	a_3	jb_4	a_5	. . .
r_{31}	r_{32}	r_{34}	r_{35}	$\cdots$	
r_{41}	r_{42}	r_{43}	r_{44}	$\cdots$	
r_{51}	r_{52}	r_{53}	$\cdots$		
r_{61}	r_{62}	r_{63}	$\cdots$		
r_{71}	r_{72}	$\cdots$			
r_{81}	$\cdots$				
.	.	.	.		
.	.	.	.		

Using the first column elements, sign pairs are formed as
$P_1 = (1, a_1)$, $P_2 = (r_{31}, r_{41})$
$P_3 = (r_{51}, r_{61})$) , $P_4 = (r_{71}, r_{81})$ . . .

According to the first scheme SPC I, it is ascertained that each element of all the pairs has to maintain the same sign for the roots of characteristic equation to lie on the left hand side of s-plane for stability. The proof of the criterion is given in [9].

1.2 Sign Pair Criterion II (SPC II)

In this paper, another scheme is proposed for the analysis of stability of a given linear time - invariant system. With the substitution of s = jω , the real and imaginary parts of the characteristic equations , are extracted separately and their coefficients are entered suitably in the first-two rows of Routh-like table to observe the system stability. The formulated stability criterion is termed as 'Sign Pair Criterion-II' (SPC-II). In this procedure, it can be noted that the Routh-like table contains only real elements. Let

$$C(s) = s^n + (a_1 + j\, b_1)s^{n-1} + (a_2 + j\, b_2)s^{n-2} + \cdots + (a_k + j\, b_k) = 0 \qquad (2)$$

Substituting of s = jω ,

$$C(j\omega) = (j\omega)^n + (a_1 + j\, b_1)(j\omega)^{n-1} + (a_2 + j\, b_2)(j\omega)^{n-2} + \ldots + (a_k + jb_k) = 0 \qquad (3)$$

$$C(j\omega) = R(\omega) + jI(\omega) = 0 \qquad (4)$$

Where

$$R(\omega) = (A_0\omega^n + A_1\,\omega^{n-1} + A_2\,\omega^{n-2} + \cdots + A_n) \qquad (5)$$

$$I(\omega) = (B_0\omega^n + B_1\,\omega^{n-1} + B_2\,\omega^{n-2} + \cdots + B_n) \qquad (6)$$

Using the coefficients of above polynomials, the second form of Routh-like table can be formulated as

A_0	A_1	A_2	...	A_n
B_0	B_1	B_2	...	B_n
c_0	c_1	c_2	...	
d_0	d_1	d_2	...	
e_0	e_1	e_2	...	
f_0	f_1	f_2	...	
g_0	g_1	...	...	
.	.	.		
.	.	.		
.	.	.		

2.2.1Algorithm for the proposed approach

1. If the first element in the first row is negative, multiply the full row elements by -1.
2. If the first element in the second row is zero, interchange first and second rows and multiply all elements in the second row by -1.
3. Follow the Common Routh's multiplication rule to get the complete table with '2n+1' rows.
4. If any element of the first column starting from third, comes zero, it is replaced by a small value +0.01.
5. If all the elements in a row become zero, then the auxiliary polynomial is formed using the previous row elements and differentiated once; the coefficients of this modified polynomial are entered instead of zeros and the table is completed by applying the Routh multiplication rule.
6. Get 'n' sign pairs using the first column elements starting from second row.

The sign pairs are developed as

$P_1 = (B_0, c_0)$, $P_2 = (d_0, e_0)$,$P_3 = (f_0, g_0)$ ... .P_n. .

According to the second scheme SPC II, it is ascertained that each element of all the pairs has to maintain the same sign for the roots of characteristic equation to lie on the left hand side of s-plane for stability. The proof of the criterion is given in [10].

III. STABILITY ANALYSIS WITH DAMPING FREQUENCY OF OSCILLATION

The characteristic equation of a given linear time-invariant system with positive real coefficients are used for relative stability analysis. Also all the roots of the characteristic equation lie on the left-half of S-plane with a damped frequency of oscillation. This situation can be shown in the Figure 1. All the roots must lie on the marked region on the left-half of s-plane in Fig.2 for a given damped frequency of oscillation α .To infer this situation, the given real characteristic with positive coefficients is modified into a characteristic equation by substituting s=S+ jα. The modified equation can be tested with the help of SPC–I and SPC-II for further analysis, which are depicted in the following illustrations.

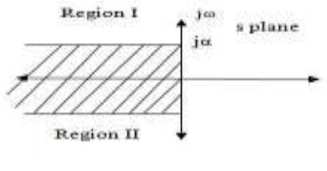

Figure 1. Region of relative stability

3.1 Example

For the equation C(s)=0, given by Hwang & Tripathi [1], determine whether all the roots of this equation have damped frequency less than $\alpha = 1$ rad/sec.

$$C(s) = s^5 + 5s^4 + 15s^3 + 25s^2 + 24s + 10 = 0 \qquad (7)$$

With a substitution of s=S+ j1,

$$C'(s) = S^5 + (5 - j5)S^4 + (-5 - j20)S^3 + (-35 - j5)S^2 + (-16 + j30)S + (9 + j9) = 0 \qquad (8)$$

3.1.1Application of SPC-I

$+1$	$-j5$	-5	$-j5$	-16	$j9$
$+5$	$-j20$	-35	30	9	
$-j1$	2	$-j11$	-17.8	$j9$	
$-j30$	-90	$j119$	54		
$+5$	$-j15$	-19.6	$j9$		
-0.2	1.4	0			
$+j20$	-19.6	$j9$			
$+j1.6$	0.09				
-20.7	$j9$				
-0.6					

The sign pairs are formed as

P_1 = (+1, +5), P_2 = (-j1, -j30) ,P_3 = (+5, -0.2), P_4 = (+j20, +j1.6) and P_5 = (-20.7, -0.6). Since P_3 fails to obey SPC-I, the system is relatively unstable.

3.1.2 Application of SPC-II

$$C'^{(j\omega)} = (5\omega^4 - 20\omega^3 + 35\omega^2 - 30\omega + 9) + j(\omega^5 - 5\omega^4 + 5\omega^3 + 5\omega^2 - 16\omega + 9) = 0 \qquad (9)$$

0	5	-20	35	-30	9
$+1$	-5	5	5	-16	9
$+5$	-20	35	-30	9	
-1	-2	11	-17.8	9	
-30	90	-119	54		
-5	15	-19.6	9		
$+0.2$	-1.4	0			
-20	-19.6	9			
-1.6	0.09				
-20.7	9				
- 0.6					

The sign pairs are formed as P_1 = (+1, +5), P_2 = (-1, -30) ,P_3 = (-5, +0.2), P_4 = (-20, -1.6) and P_5 = (-20.7, -0.6). Since P_3 fails to obey SPC-I, the system is relatively unstable

3.1.3Inference

Since the Region I (shown in figure 4.2)alone is mapped and complex roots exist as conjugate pairs, it is concluded that , if one sign pair (P_3) fails, there exist one conjugate pair of roots (two numbers of roots) on RHS of 'S' plane in SPC I and of 'ω' plane in SPC II. Hence the system represented by equation (4) will be having two roots (one conjugate pair) with damped natural frequencies greater than 1rad/sec. This result is same as that of Hwang & Tripathi (1971).

3.1.4 Verification

The roots of the characteristic equation are found as -1, -1+j1, -1-j1, -2+j2 and -2-j2. The two roots -2+j2 in Region I and -2-j2 in Region II are having damped natural frequency of oscillation of 2 rad/sec which is greater than our reference value 1 rad/sec. Hence the system is found as relatively unstable.

IV. CONCLUSION

In this paper, the relative stability analysis of a time invariant continuous systems represented in the form of their respective characteristic equations have been performed with the help of the proposed SPC-I and SPC-II. The transient response behavior of a linear time-invariant continuous system having absolute stability has been carried-out with the help of damped frequency of oscillations . For the analysis of stability, the proposed Routh-like tables are formed and ascertained with the application of SPC-I and SPC-II .The proposed algebraic criteria are simple and direct in application compared to other schemes .

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