Performance of Channel Estimation in OFDM System for Different PSK Modulations

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ABSTRACT: Channel Estimation is very important in any wireless communication. In this paper the performance evaluation of different types of PSK modulations with three different channel estimation methods in OFDM system for wireless communication for slow fading channel is compared. Comb type channel estimation is used. Channel is estimated at pilots position which are all one’s for all OFDM block and complete channel is estimated with linear interpolation. Performance of estimated channel is compared with result of known channel at the receiver.

Keywords: OFDM, Comb type Channel estimation, LS, RLS, QRD, PSK Modulation, interpolation

I. INTRODUCTION

OFDM has attracted considerable attention in the last decade due to its desirable properties such as its high data rate transmission capability with high bandwidth efficiency as well as its robustness to multipath delay spread. It has been adopted as a standard for wire and wireless communication such as Digital Subscriber Line (DSL), European digital Audio and Video Broadcasting (DAB/DVB), American IEEE 802.11(a) and European HiperLAN/2 [1]. However, accurate channel state estimation is important in communication systems and the performance of these systems is their reliance upon the availability of accurate channel state information (CSI) at the receiver. When the channel varies from block to block adaptive filtering techniques are suitable for estimation of such time variant CSI. Matrix inversion is an essential computation for various algorithms which are employed in wireless communication systems. Matrix inversion increases the computation complexity. There are many adaptive filter algorithms that are widely used for channel estimation, but they either have high mean-square error with slow convergence rate like LMS or a high computation complexity with fast convergence rate and low mean square error like Recursive Least Square (RLS) algorithm. Thus, mean–square error (MSE), convergence rate and computation and complexity are three main parameters that should be considered in selection of the adaptive algorithms for channel estimation.

In this paper, a comparative study of different types of phase shift keying (PSK) modulations with least square (LS), Recursive least square (RLS) and QR decomposition Recursive least square (QRD-RLS) channel estimation techniques applied to OFDM systems for the purpose of detecting the received signal, improving the throughput of orthogonal frequency-division multiple – access (OFDMA) systems.

The main contribution of this work are as follows:
First we propose novel adaptive approach for estimation of OFDM channel using pilot signals
Develop LS, RLS algorithm for OFMD systems.
Evaluate the performance of system with BPSK, QPSK and 16 PSK

II. SYSTEM MODEL

A block diagram of a System model of OFDM system is shown in Figure 1. The binary information is first grouped, coded, and mapped according to the modulation in a “signal mapper.” After the guard band is inserted, an N-point inverse discrete-time Fourier transform (IDFTN) block transforms the data sequence into time domain (note that N is typically 256 or larger). Following the IDFT block, a cyclic extension of time length TG, chosen to be larger than the expected delay spread, s inserted to avoid intersymbol and intercarrier interferences. The D/A converter contain low-pass filters with bandwidth 1/TS, where TS is the sampling interval. The channel is modeled as an impulse response h(t) followed by the complex additive white Gaussian noise (AWGN) w(n) [5].
A multipath fading channel is modeled as FIR filter. The impulse response of channel is modeled as Rayleigh fading (Clarke’s Model – sum of sinusoids method) \[4\]

\[ h_i(nT_s) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} \cos(2\pi f_d nT_s \cdot \cos(\alpha(m)) + \phi(y)) \]

\[ h_q(nT_s) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} \cos(2\pi f_d nT_s \cdot \cos(\alpha(m)) + \phi(y)) \]

\[ h(nT_s) = h_i(nT_s) + h_q(n) \]

where \( \alpha(m) = (2m - 1)\pi + \theta \)/4N

\( \phi(y) \) are uniformly distributed over \([0, 2\pi]\)

\( F_d = \) maximum Doppler spread

\( T_s = \) sampling period

\( N = \) sample index

At the receiver, after passing through the analog-to-digital converter (ADC) and removing the CP, the DFT is used to transform the data back to frequency domain. Lastly, the binary information data is obtained back after the demodulation and channel decoding. The received signal can be written as

\[ Y = XH + V \]

Where \( Y \) is received signal matrix and \( V \) is noise signal matrix. \( H \) is channel frequency response matrix and \( X \) is transmitted signal.

III. CHANNEL ESTIMATION TYPE

The optimal channel estimator in terms of mean-square error is based on 2D Wiener filter interpolation. Unfortunately, such a 2D estimator structure is too complex for practical implementation. The combination of high data rates and low bit error rates in OFDM systems necessitates the use of estimators that have both low complexity and high accuracy, where the two constraints work against each other and a good trade-off is needed. The one-dimensional (1D) channel estimations are usually adopted in OFDM systems to accomplish the trade-off between complexity and accuracy. In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both, can be used for channel estimation. In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account. \[5\]

A. Pilot Structure:

Depending on the arrangement of pilots, there are three different types of pilot structures. The one-dimensional (1D) channel estimations are usually adopted in OFDM systems to accomplish the trade-off between complexity and accuracy. The two basic 1D channel estimations are block-type pilot channel estimation and comb-type pilot channel estimation, lattice type channel estimation.

B. Block Type:

A block type of pilot arrangement is depicted in Figure 2. In this type, OFDM symbols with pilots at all subcarriers (referred to as pilot symbols herein) are transmitted periodically for channel estimation. Using these pilots, a time-domain interpolation is performed to estimate the channel along the time axis.
Let $S_t$ denote the period of pilot symbols in time. In order to keep track of the time-varying channel characteristics, the pilot symbols must be placed as frequently as the coherence time is. As the coherence time is given in an inverse form of the Doppler frequency $f_{\text{Doppler}}$ in the channel, the pilot symbol period must satisfy the following inequality:

$$S \leq \frac{1}{f_{\text{Doppler}}} \times 2.5$$

Since pilot tones are inserted into all subcarriers of pilot symbols with a period in time, the block-type pilot arrangement is suitable for frequency-selective channels. For the fast-fading channels, however, it might incur too much overhead to track the channel variation by reducing the pilot symbol period. Block type estimation can be based on Least square (LS), minimum mean-square error (MMSE), and modified MMSE. [5]

### C. Comb Type:

Comb-type pilot arrangement is depicted in Figure 2. In this type, every OFDM symbol has pilot tones at the periodically-located subcarriers, which are used for a frequency-domain interpolation to estimate the channel along the frequency axis.

Let $S_f$ be the period of pilot tones in frequency. In order to keep track of the frequency-selective channel characteristics, the pilot symbols must be placed as frequently as coherent bandwidth is. As the coherence bandwidth is determined by an inverse of the maximum delay spread $\sigma_{\text{max}}$, the pilot symbol period must satisfy the following inequality:

$$S_f \leq \frac{1}{\sigma_{\text{max}}}$$  \hspace{1cm} (3)

As opposed to the block-type pilot arrangement, the comb-type pilot arrangement is suitable for fast-fading channels, but not for frequency-selective channels.

In comb-type pilot based channel estimation, as shown in Figure 3.2, for each transmitted symbol, $nu$ number of pilot signals $p_{\text{data}}$ are uniformly inserted into $X$ with $N_{ps}$ subcarriers apart from each other, where $N_{ps} = N/nu$. And $N$ is OFDM block length

The receiver knows the pilot locations $N_{ps}$ at subcarrier position, the pilot values $p_{\text{data}}$ and the received signal $Y$. The LS estimates to the channel conditions at the pilot subcarriers $H_{LS}$ are calculated by Equation.

$$H_{LS} = \frac{Y}{p_{\text{data}}}$$  \hspace{1cm} (4)

The task here is to estimate the channel conditions at the data subcarriers (specified by $H$ with length $N$), given the LS estimates at pilot subcarriers, received signals $Y$, and maybe certain additional knowledge of the channel statistics. The solutions include LS estimator with 1D interpolation, the maximum likelihood (ML) estimator, and the parametric channel modeling-based (PCMB) estimator. [5]

### IV. ADAPTIVE CHANNEL ESTIMATION

An adaptive algorithm is a process that changes its parameters as it gain more information of its possibly changing environment. The channel estimation update parameters of the channel continuously, form the transmitted signal, so that knowledge of channel and noise statistics are not required. This is done by transmitting a training sequence that is known to the receiver. In this paper, Comb type channel estimation is used. Channel coefficients are estimated at pilot subcarriers and one dimensional interpolation is used to estimate the channel at data subcarriers. Here we developed QRD RLS algorithm and result are compared with known channel at receiver. QPSK modulation is used.
The LS estimator is the basic algorithm and gives regular results. The LS estimates the channel coefficient at the pilot subcarriers as given in [6] [7].

\[ H_{LS} = \hat{\chi} \]  

The LMS and LMMS algorithm are less complex and requires no matrix inversion but it requires initial knowledge of auto covariance matrix of channel estimates and noise variance. The estimate \( H_{MMSE} \) is given by

\[ H_{MMSE} = H_{LS} \cdot G_{MMSE} \]  

Where \( G_{MMSE} \) is autocorrelation matrix of Channel estimate. The RLS approach is based on obtaining directly the inverse of the autocorrelation matrix, recursively for the filter coefficient update. RLS algorithm used matrix inversion leema [10] to find the matrix inversion which increases the complexity of algorithm [7][8][9]

V. QRD RLS ALGORITHM

To reduce the complexity of matrix inversion, and the number of computations to invert the matrix in less time, we develop an architecture for matrix inversion by generalizing the QR decomposition based RLS algorithm (QRD-RLS) [2][10][11]. QR decomposition of a real square matrix A is a decomposition of A as A = QxR, where Q is an orthogonal matrix (\( Q^T \cdot Q = I \)) and R is an upper triangular matrix [13]. There are different methods which can be used to compute QR decomposition. The techniques for QR decomposition are Gram-Schmidt orthonormalization method, Householder reflections, and the Givens rotations. In our paper we used Givens rotation. [12]

Let input signal matrix be given by

\[ X_T(k) = [x(k) \sqrt{\lambda_2} x(k-1) \ldots \sqrt{\lambda_{k/2}} x(0)] \]  

and desired signal vector

\[ D(k) = [d(k) \sqrt{\lambda_2} d(k-1) \ldots \sqrt{\lambda_{k/2}} d(0)]^T \]  

The above matrix and vector are related to the corresponding quantities at the previous time instant as

\[ X(k) = \begin{bmatrix} x(k) \\ \sqrt{\lambda_2} x(k-1) \end{bmatrix} \]  

\[ D(k) = \begin{bmatrix} d(k) \\ \sqrt{\lambda_2} d(k-1) \end{bmatrix} \]  

The exponentially-weighted least-squares deterministic autocorrelation matrix and cross correlation vector can be found from \( X(n) \) and \( D(n) \) as

\[ Z_{ad} = X_T(k) \cdot D(k) \]  

The deterministic cost function \( J(n) \) of a RLS channel estimation algorithm is defined as an exponential weighted sum of errors squares:

\[ J(n) = \sum_{i=1}^{N} \frac{1}{\lambda} [d(i) - x^H(i) \cdot w(k)]^2 \]  

\[ = \| D(k) - X(k) \cdot w(k) \|^2 \]  

Where \( \| \cdot \|^2 \) denotes Euclidean norm of the vector. Rather than directly finding the solution for \( w(n) \), here error vector is transformed by multiplying with orthogonal matrix \( Q(n) \) k x k such that \( Q^T(k) \cdot Q(k) = I \)

\[ J(k) = \| Q(k) \cdot D(k) - Q(k) \cdot X(k) \cdot w(k) \|^2 \]  

\( Q(n) \) is an unitary matrix selected such that

\[ Q(k) = \begin{bmatrix} \mathcal{R}(k) \\ 0 \end{bmatrix} \]  

where \( \mathcal{R}(n) \) is an N by N - element upper triangular matrix and 0 is a matrix with all zero elements and appropriate dimensions. The above decomposition of \( X(n) \) into an orthogonal matrix multiplied by an upper triangular matrix is known as the QR-decomposition of \( X(n) \).

To find optimal Coefficient Vector that minimises \( J(k) \) let

\[ Q(k) \cdot D(k) = \begin{bmatrix} U(k) \\ V(k) \end{bmatrix} \]
Then as per equation of cost function given in 13 namely

\[ J(k) = ||Q(k)D(k) - Q(k)X(k)w(k)||^2 \]  

\[ J(k) = ||U(k)^T - R(k)w(k)||^2 \]  

\[ J(k) = ||U(k) - R(k)w(k)||^2 + ||V(k)||^2 \]  

(17)

Since V(k) do not depend on coefficient vector the cost function can be minimize by equating only the first term of right hand side to zero. Thus, the solution to the least-squares estimation problem is given by

\[ w(k) = R^{-1}(k)U(k) \]  

(18)

Since R(k) in upper triangular matrix of X(k), w(k) can be found by back substitution. The Optimal solution to LS problem at a given instant of time k is found by solving the equation.

\[ X^T(k)X(k)w(k) = x^T(k)d(k) \]  

(19)

However, solving this equation by using conventional RLS algorithm can be a problem when the matrix \( R(k) = X^T(k)X(k) \) and its correspondent inverse estimate become ill-conditioned due to loss of persistence of excitation of the input signal. The QR decomposition approach solves the above problem and avoids inaccurate solution to the RLS problem.

The back substitution algorithm given in Eq. 20 is use to find Channel coefficient by using triangularization of input data matrix X.

\[ w_{ch}(k) = \sum_{i=1}^{\infty} \frac{r(j)^{\tau(i)} - L^2}{r(i)} \]  

(20)

VI. RESULT

We consider an SISO-OFDM system with one OFDM frame of 64 subcarriers corresponding to the BPSK, QPSK, and 16 PSK input signal. The frequency selective channel has \( \sigma = 40 \) zero-mean uncorrelated complex Gaussian random taps and the number of pilot symbols is Np=16. The forgetting factor are set equal to \( \lambda = 0.99 \) for RLS and QRD-RLS algorithms. The maximum signal to noise ratio (SNR) is equal to 50 dB. Performance of different algorithm can be compared with signal estimated with know channel. It is seen that QRD RLS result is more close to known channel. More ever, the QRD method can be implemented in a highly parallel systolic array structure which makes it desirable for realtime implementations. The result of simulation are shown below.

BER Vs SNR

![Figure 3. Performance comparison of BPSK bit error rate.](image-url)
VII. CONCLUSION

In this paper, we investigated LS, RLS and QRD RLS algorithm for comb type channel estimation. Result of BER vs SNR are compared with result of known channel. Result shows improvement of BER with QRD RLS algorithm over normal RLS algorithm. It is seen that the performance of channel estimation is improved with QRD RLS channel estimation algorithm. Result of simulation of channel estimation for different PSK modulation is shown below Application of QRD also greatly reduces the complexity of hardware implementation by using systolic algorithm.

REFERENCES

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