

Blind Channel Estimation Using Maximum Likelihood In OFDM Systems

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ABSTRACT : A new blind channel estimation scheme for OFDM systems is based on the maximum likelihood (ML) principle. The approach combines different modulation schemes on adjacent subcarriers, such as BPSK and QPSK, to resolve phase ambiguity. By avoiding the use of second- and higher-order statistics, a very fast convergence rate is achieved. In this paper the maximum likelihood (ML) is used for symbol-time and carrier-frequency offset estimator in orthogonal frequency-division multiplexing (OFDM) systems. It also shows the results for bit error rate and signal to noise ratio. In the uplink of such systems users must be aligned in time and frequency to maintain the orthogonality of the subcarriers. Redundant information contained within the cyclic prefix enables this estimation without additional pilots. Simulations show that the frequency estimator may be used in a tracking mode and the time estimator in an acquisition mode. It is important to note that probability of error (POE) is proportional to E_b/N_0 , which is a form of signal-to-noise ratio.

Keywords - Blind channel estimation, ML estimation and OFDM system.

I. Introduction

In multiuser OFDM the orthogonality of the subcarriers facilitates a subcarrier- division of different users, where one OFDM symbol contains many users. In the uplink of such systems users must be aligned in time and frequency to maintain the orthogonality of the subcarriers. Orthogonal frequency division multiplexing (OFDM) systems have recently gained increased interest. OFDM is used in the European digital broadcast radio system and is being investigated for other wireless applications such as digital broadcast television and mobile communication systems, as well as for broadband digital communication on existing copper networks [1] [2]. We address two problems in OFDM receivers. One problem is the unknown OFDM symbol arrival time. Sensitivity to a time offset is higher in multi-carrier systems than in single-carrier systems and has been discussed in [3] [4]. A second problem is the mismatch of the oscillators in the transmitter and receiver. The demodulation of a signal with an offset in the carrier frequency can cause a high bit error rate and may degrade the performance of a symbol synchronizer [3] [5]. A symbol clock and a frequency offset estimate may be generated at the receiver with the aid of pilot symbols known to the receiver [6] [7] [8] by maximizing the average log-likelihood function. Redundancy in the transmitted OFDM signals also offers the opportunity for synchronization. Such an approach is found in [7] [9] for a time offset and in for a frequency offset [10] [11]. This paper present and evaluate the maximum likelihood (ML) estimation of the time and carrier-frequency offset in OFDM systems. The key element that will rule the discussion is that the OFDM data symbols already contain sufficient information to perform synchronization. Our novel algorithm exploits the cyclic prefix preceding the OFDM symbols, thus reducing the need for pilots.

II. The OFDM system model

Figure 1 illustrates the baseband, discrete-time OFDM system model we investigate. The complex data symbols are modulated by means of an inverse discrete Fourier transform (IDFT/IFFT) on N parallel subcarriers. The resulting OFDM symbol is serially transmitted over a discrete-time channel, whose impulse response we assume is shorter than L samples. At the receiver, the data are retrieved by means of a discrete Fourier transform (DFT/FFT).

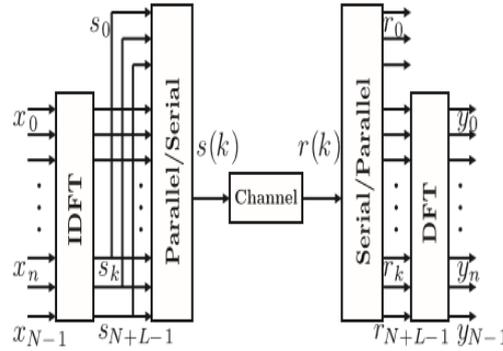


Figure 1: The OFDM system, transmitting subsequent blocks of N complex data.

An accepted means of avoiding inter-symbol interference (ISI) and preserving orthogonality between subcarriers is to copy the last L samples of the body of the OFDM symbol (N samples long) and append them as a preamble(cyclic prefix) to form the complete OFDM symbol [1] [2]. The effective length of the OFDM symbol as transmitted is this cyclic prefix plus the body (L + N samples long). The insertion of a cyclic prefix can be shown to result in an equivalent parallel orthogonal channel structure that allows for simple channel estimation and equalization. In spite of the loss of transmission power and bandwidth associated with the cyclic prefix, these properties generally motivate its use [1] [2]. In the following analysis we assume that the channel is non-dispersive and that the transmitted signal $s(k)$ is only affected by complex additive white Gaussian noise (AWGN) $n(k)$. We will, however, evaluate our estimator’s performance for both the AWGN channel and a time-dispersive channel. Consider two uncertainties in the receiver of this OFDM symbol: the uncertainty in the arrival time of the OFDM symbol (such ambiguity gives rise to a rotation of the data symbols) and the uncertainty in carrier frequency (a difference in the local oscillators in the transmitter and receiver gives rise to a shift of all the subcarriers). The first uncertainty is modeled as a delay in the channel impulse response $\delta(k - \theta)$, where θ is the integer-valued unknown arrival time of a symbol. The latter is modeled as a complex multiplicative distortion of the received data in the time domain $e^{j2\pi\epsilon k/N}$, where ϵ denotes the difference in the transmitter and receiver oscillators as a fraction of the inter-carrier spacing (1/N in normalized frequency). Notice that all subcarriers experience the same shift ϵ . These two uncertainties and the AWGN thus yield the received signal,

$$r(k)=s(k-\theta)\expj2\pi\epsilon k/N + n(k). \tag{1}$$

Two other synchronization parameters are not accounted for in this paper. First, an offset in the carrier phase may affect the symbol error rate in coherent modulation. If the data is differentially encoded, however, this effect is eliminated. An offset in the sampling frequency will also affect the system performance. We assume that such an offset is negligible. Now consider the transmitted signal $s(k)$. This is the DFT of the data symbols $x(k)$ which we assume are independent. Hence, $s(k)$ is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, we know from the central limit theorem that $s(k)$ approximates a complex Gaussian process whose real and imaginary parts are independent. This process, however, is not white, since the appearance of a cyclic prefix yields a correlation between some pairs of samples that are spaced N samples apart. Hence, $r(k)$ is not a white process, either, but because of its probabilistic structure, it contains information about the time offset θ and carrier frequency offset ϵ . This is the crucial observation that offers the opportunity for estimation of these parameters based on $r(k)$. A synchronizer cannot distinguish between phase shifts introduced by the channel and those introduced by symbol time delays [4]. Time error requirements may range from the order of one sample (wireless applications, where the channel phase is tracked and corrected by the channel equalizer) to a fraction of a sample (in, e.g., high bit-rate digital subscriber lines, where the channel is static and essentially estimated only during startup). Without a frequency offset, the frequency response of each sub-channel is zero at all other subcarrier frequencies, i.e., the sub-channels do not interfere with one other [2]. The effect of a frequency offset is a loss of orthogonality between the tones. The resulting inter-carrier interference (ICI) has been investigated in [11]. The effective signal-to-noise ratio (SNRe) due to both additive noise and ICI is shown to be lower bounded.

$$SNRe(\epsilon) \geq \frac{SNR}{1+0.5947 SNR \sin^2 \pi\epsilon} \left(\frac{\sin \pi\epsilon}{\pi\epsilon}\right)^2 \tag{2}$$

Where

$$\text{SNR} = \sigma_s^2 / \sigma_n^2, \sigma_s^2 \triangleq E \{ |s(k)|^2 \} \quad (3)$$

And

$$\sigma_n^2 \triangleq E \{ |n(k)|^2 \} \quad (4)$$

The difference between the SNR and the SNRe is a measure of the sensitivity to a frequency offset ϵ . Notice that in the absence of additive noise the frequency offset must satisfy $|\epsilon| \leq 1$, in order to obtain an SNRe of 30 dB or higher. This result agrees well with the analysis of multiuser OFDM systems in [3], which states that a frequency accuracy of 1–2% of the inter-carrier spacing is necessary.

III. ML estimation

Assume that we observe $2N+L$ consecutive samples of $r(k)$. Figure 2, and that these samples contain one complete $(N + L)$ sample OFDM symbol. The position of this symbol within the observed block of samples, however, is unknown because the channel delay θ is unknown to the receiver. Define the index sets

$$I \triangleq \{\theta, \dots, \theta + L - 1\} \text{ and } I' \triangleq \{\theta + N, \dots, \theta + N + L - 1\}$$

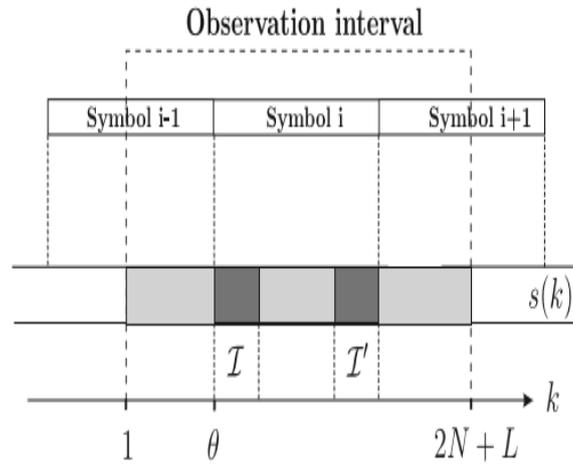


Figure 2: Structure of OFDM signal with cyclically extended symbols, $s(k)$. The set I contains the cyclic prefix, i.e. the copies of the L data samples in I' .

The set I' thus contains the indices of the data samples that are copied into the cyclic prefix, and the set I contains the indices of this prefix. Collect the observed samples in the $(2N + L) \times 1$ -vector $r = [r(1) \dots r(2N + L)]^T$. The likelihood function for θ and ϵ , $\Lambda(\theta, \epsilon)$, is the logarithm of the probability density function $f(r|\theta, \epsilon)$ of the $2N + L$ observed samples in r given the arrival time θ and the carrier frequency offset ϵ . In the following, we will drop all additive and positive multiplicative constants that show up in the expression of the likelihood function, since they do not affect the maximizing argument. Moreover, we drop the conditioning on (θ, ϵ) for notational clarity. Since the ML estimation of θ and ϵ is the argument maximizing $\Lambda(\theta, \epsilon)$, we may omit this factor. Under the assumption that r is a jointly Gaussian vector, to be

$$\Lambda(\theta, \epsilon) = |\gamma(\theta)| \cos(2\pi\epsilon + 6\gamma(\theta)) - \rho\Phi(\theta) \quad (5)$$

The argument of a complex number is the magnitude of the correlation coefficient between $r(k)$ and $r(k+N)$. The first term is the weighted magnitude of $\gamma(\theta)$, a sum of L consecutive correlations between pairs of samples spaced N samples apart. The weighting factor depends on the frequency offset. The term $\Phi(\theta)$ is an energy term, independent of the frequency offset ϵ . Notice that its contribution depends on the SNR (by the

weighting-factor ρ). The maximum with respect to the frequency offset ϵ is obtained when the cosine term in equation equals to one. This yields the ML estimation of

$$\hat{\epsilon}^{ML}(\theta) = -1/2\pi < \gamma(\theta) + n \tag{6}$$

Where n is an integer. A similar frequency offset estimator can derive under different assumptions. Notice that by the periodicity of the cosine function, several maxima are found. We assume that an acquisition, or rough estimate, of the frequency offset has been performed and that $|\epsilon| < 1/2$; thus $n = 0$. Since $\cos(2\pi \hat{\epsilon}^{ML}(\theta) + \gamma(\theta)) = 1$, the log-likelihood function of θ (which is the compressed log-likelihood function with respect to ϵ) becomes $\Lambda(\theta, \hat{\epsilon}^{ML}(\theta)) = |\gamma(\theta)| - \rho\Phi(\theta)$ and the joint ML estimation of θ and ϵ becomes

$$\hat{\theta}^{ML} = \arg \max_{\theta} \{|\gamma(\theta)| - \rho\Phi(\theta)\}, \quad \hat{\epsilon}^{ML} = -1/2\pi < \gamma(\hat{\theta}^{ML}) \tag{7}$$

IV. Experimental Results

In the basic OFDM system model data to be transmitted is first modulated by using QPSK modulation scheme. Then it is converted to the OFDM signal which is in time domain. At the receiver side it is demodulated to obtain the original data.

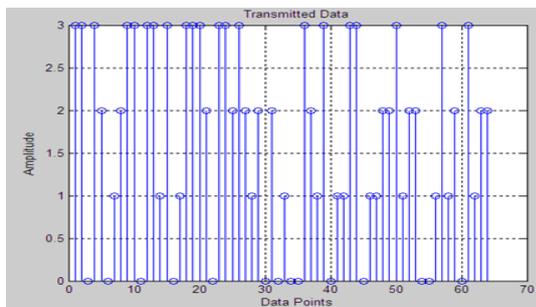


Figure 3: Data to be transmitted

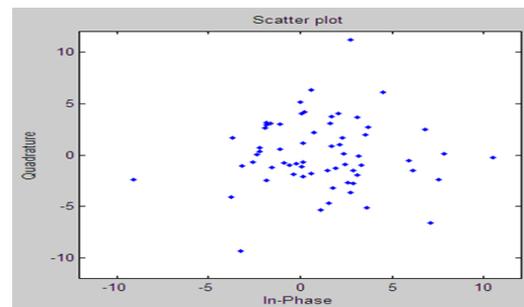


Figure 6: Demodulated data at the receiver.

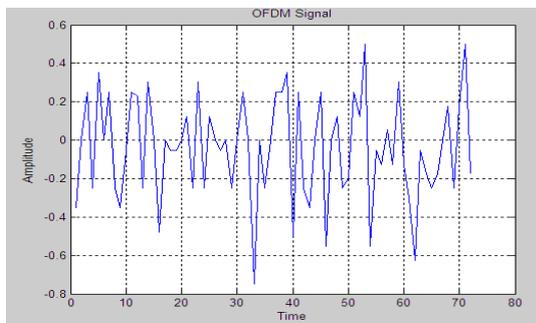


Figure 5: OFDM signal

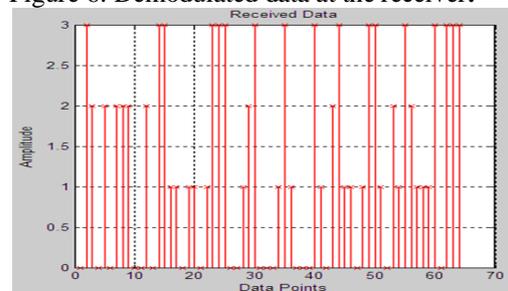


Figure 7: Received data.

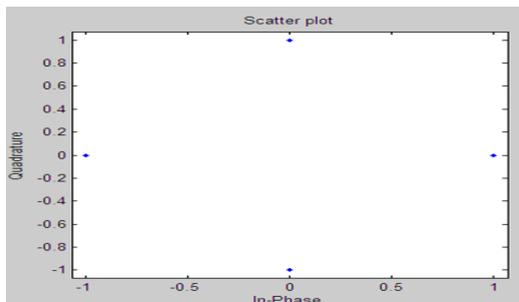


Figure 4: Modulated data

Timing offset, frequency offset, carrier phase jitter and sampling clock frequency offset are the main synchronization errors encountered by OFDM systems. As many literatures have proved, OFDM systems are very sensitive to frequency errors & time errors.

Frequency offsets are generally caused by unmatched local oscillators at the two ends of the communication links, Doppler shifts or phase noise introduced by non-linear channel. In fact, the FFT filter's frequency response extending over the whole frequency range and the very narrow spacing between sub-carriers all contribute to the sensitivity of OFDM systems to frequency synchronization errors. The effects of frequency errors are mainly twofold: the reduction of signal amplitude and the introduction of ICI from other sub-carriers.

In this paper, we estimated the integer part of frequency offset and timing offset. Simulation results show its robustness to mitigate the multi-path effect and its enlarged estimation range for frequency offset and timing offset in figure 8 & figure 9.

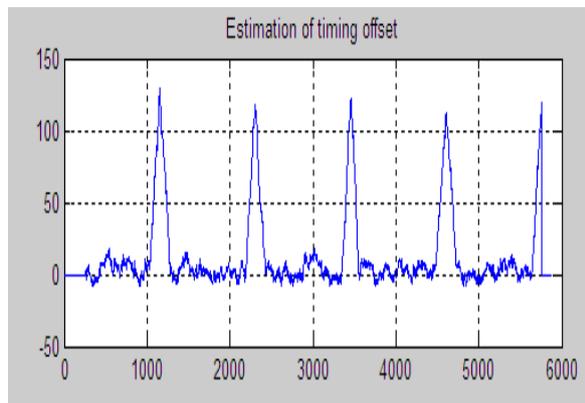


Figure 8: The signals that generate the ML-estimates ($N = 1024$, $L = 128$, $\epsilon = 0.25$ and $\text{SNR} = 15$ dB) for timing offset.

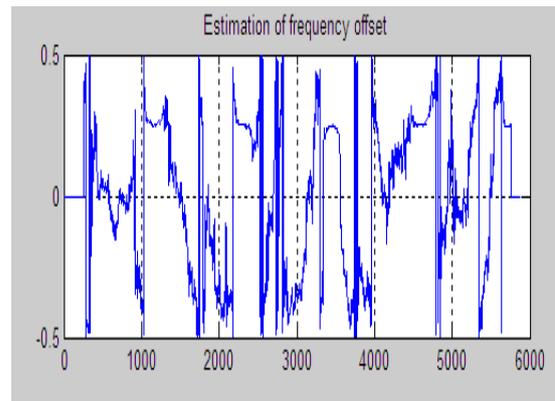


Figure 9: The signals that generate the ML-estimates ($N = 1024$, $L = 128$, $\epsilon = 0.25$ and $\text{SNR} = 15$ dB) for frequency offset.

With a strong signal and an unperturbed signal path, bit error rate (BER) so small as to be insignificant. It becomes significant when we wish to maintain a sufficient signal-to-noise ratio in the presence of imperfect transmission through electronic circuitry (amplifiers, filters, mixers, and digital/analog converters) and the propagation medium (e.g. the radio path or optical fiber).

BER can also be defined in terms of the probability of error (POE),

$$POE = \frac{1}{2}(1 - \text{erf})\sqrt{E_b / N_o} \quad (8)$$

Where *erf* is the error function, *E_b* is the energy in one bit and *N_o* is the noise power spectral density (noise power in a 1 Hz bandwidth). The error function is different for each of the various modulation methods. What is more important to note is that POE is proportional to *E_b/N_o*, which is a form of signal-to-noise ratio. This theoretical value & simulation values are shown in the figure 10 below.

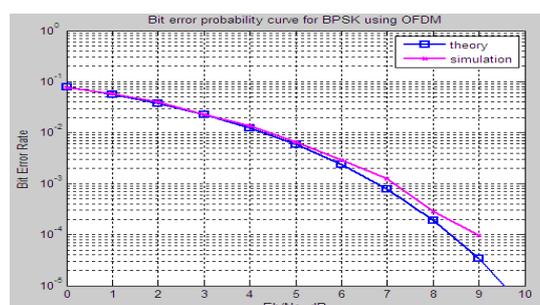


Figure 10: The signals that show BER Vs signal to noise ratio with theoretical value & simulation value.

V. Conclusion

The redundant information contained within the cyclic prefix enables this estimation without additional pilots. This paper presents the ML estimator for time and frequency offset in OFDM systems. It is derived under the assumption that the channel distortion only consists of additive noise, but simulations show that it can perform well even in a dispersive channel. The frequency estimator performs better than the time estimator because of its implicit averaging. Without additive noise $n(k)$, each term $r(k)r^*(k+N)$ has the same argument. Hence, they contribute coherently to the sum, while the additive noise contributes incoherently. This explains why the performance will improve as the size of the cyclic prefix increases. Also ML estimation finds particular parametric values for BER and signal to noise ratio that makes the observed results most probable.

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