# The Concept of the Golden Proportion in Dentistry 

Abu-Hussein Muhamad *, Abdulgani Azzaldeen**<br>*Practice limited to Children's Dentistry, Aesthetics Dental Clinic and Athens, Greece<br>** Istituto Stomatologico Toscano, University Guglielmo Marconi of ROME, Italy, Al-Quds University, School of Dentistry, Jerusalem, Palestine


#### Abstract

The evolution of Dentistry is dependant on the development and refinement of the principles we rely on for the understanding and application of current dental techniques. The Golden Proportion, a mathematical ratio, represents beauty, harmony and balance in physical form. Over the centuries, this geometric constant has influenced architecture, biological systems, mathematics and art. This ratio is believed to hold the key to the secret of beauty and finds its representation in innumerous natural and manmade masterpieces. The paper discusses various aspects of this ratio and their relevance in human aesthetics


Key Words: tooth proportion, dimension, size, gauge, biometric, aesthetics, maxillary anterior

## I. Introduction

The Golden Proportion is based on a very specific mathematical proportion that is acquired from mathematics and nature. [1]This proportion, also referred to as the Divine Proportion or Magic ratio, describes a number often encountered when taking the ratio of distance in simple geometric figures such as the pentagram, decagon, and dodecagon. [2]To understand the Golden Proportion from a mathematical standpoint, consider a line segment with a length of one. Divide this segment into two segments, a shorter segment, $x$, and a longer segment, 1-x. [3]The Golden Proportion is one where the ratio of the shorter segment to the longer segment ( $\mathrm{x}: 1-\mathrm{x}$ ) is equal to the ratio of the longer segment to the whole segment(1-x:1).[4] (Figure 1)


Figure 1. A line segment AC is divided in the golden section point B

The mathematical equation that needs to be solved for x is $\mathrm{x} /(1-\mathrm{x})=(1-\mathrm{x}) / 1$. Solving this equation results in a shorter segment length of 0.382 and the longer segment length at 0.618 . [5]This is the only ratio that solves the equation. (Figure 2)

$1 / \Phi=0.6180339887 . .$.
Figure 2; The inverse of $\varphi$

This proportion can be found throughout the universe; from the spirals of galaxies to the spiral of a Nautilus seashell; from the harmony of music to the beauty in art.[7]A botanist will find it in the growth patterns of flowers and plants, while a zoologist sees it in the breeding of rabbits. [8] (Figure 2)
The ancient Greeks have often been credited with defining the Golden Proportion. [9]Pythagorus and Phidius are believed to have identified it. [10] The Golden Proportion was


Figure 3; The Greeks knew it as the Golden Section and used it for beauty and balance in the design of architecture
closely studied by the Greek sculptor Phidias, hence the designated mathematical symbol for the Golden Proportion ratio is phi. [11]The Greeks studied phi through their mathematics and used it in their architecture. (Figure 3)The Parthenon at Athens is a classic example of the use of the Golden Proportion. [12]The initial discovery of the Golden Proportion by the Greeks has been challenged by architects and archeologists.[13] Badawy [15]claims that the use of proportions was initiated by the ancient Egyptians, based on his study of Egyptian buildings, sculptures, and paintings. The ancient Egyptians used it in the construction of the great pyramids and in the design of hieroglyphs found on tomb walls.Separately, the ancients of Mexico applied the Golden Proportion while building the Sun pyramid at Teotihuacan. ${ }^{8}$ Although debate remains as to the origin of this proportion, it is clearly evident in Egyptian, Greek, Roman, Romanesque, Gothic, and Renaissance culture. [16,17] (Figure 4)


Figure 4; Great pyramids of Giza
The Golden Proportion is also evident in the arts.Renowned artists such as Michelangelo, Raphael, and Leonardo da Vinci made use of this concept. Leonardo da Vinci drew the "ideal man" using the Golden Proportion, and the head of Mona Lisa was drawn using this relationship. [18]Evidence suggests that classical music composed by Mozart, Beethoven, and Bach incorporated the Golden Proportion. Whether the use of the Golden Proportion by these artists and musicians was by design, intuition, or accident is not known. [19,20]

In dentistry, the Golden Proportion has been suggested as one possible mathematical approach to development of ideal size and shape relationships for maxillary teeth.[21]These principles can be used to determine the width of the teeth as they relate to each other.[22] There are many important design considerations that accompany application of the Golden Proportion.[23] Only after the incisal edge position, incisal plane, gingival plane, and central incisor length have been determined, can the Golden Proportion be applied. As applied to the maxillary teeth, the Golden Proportion requires a $62 \%$ reduction in the viewing width of each tooth, beginning with the central incisor, and proceeding posteriorly.[24]

It is important to distinguish between the viewing width and the clinical width of a tooth. Viewing width is measured on a straight-on frontal photograph of the patient's smile. Anatomical measurements taken in a patient's mouth or on a model will not provide this relationship. To apply the concept of the Golden Proportion, a lateral incisor can be arbitrarily assigned a viewing width of 1 , in which case the central incisor
width should be 1.6 , the cuspid 0.62 , the first premolar 0.38 , and the second premolar 0.23 . This allows for a smile dominated by the central incisors, with the other teeth becoming progressively smaller.[25]

A variation of the Golden Proportion was suggested by Snow. [16]He proposed that the Golden Proportion could be used to develop symmetry, dominance, and proportion for aesthetically pleasing smiles. To simplify these proportions, the assigned values for the six anterior teeth were added (assigning 1.6 to the central, 1 to the lateral, and 0.62 to the cuspid). Then the assigned width of each tooth was divided by the total to determine the relative percentage. [26]The resulting percentages were: central incisor $25 \%$, lateral incisor $16 \%$, and cuspid $9 \%$. These percentages illustrate the dominance of the central incisors, which are $50 \%$ of the cuspid-to-cuspid width.[27]


Figure 5; . Two different types of golden triangle ABC
Egyptians used the Golden Proportion, phi, in the construction of the great pyramids and in the design of hieroglyphs found on tomb walls. The ancient inhabitants of Mexico embraced phi in the Sun Pyramid at Teotihuacan. The Greeks used phi in their architecture. The Parthenon at Athens, Greece, was a classic example of the use of the Golden Rectangles throughout this structure, which had thus become to be known as the «perfect building» and was perhaps the best example of a mathematical approach to art.[28] These unusual mathematics properties were key reasons for a building's aesthetic beauty and the attention received for ages. Plato, in his Timaeus, considered the Golden Proportion the most binding of all mathematical relations, and made it the key to the physics of the cosmos. [29]Renowned artists such as Michelangelo, Raphael, and Leonardo da Vinci made use of it for they knew of its appealing qualities. Throughout history, the ratio of $1.6180339887 . .$. , for the length-width of rectangles, has been considered the most pleasing to the eye. The Golden Section is a special ratio that is also called the Golden Ratio, the Divine Proportion or the Golden Rectangle. Euclid's visually pleasing geometric proportion, regarded as the formation of accepted beauty, is the ratio between the two dimensions of a plane figure or the two divisions of a line, such that the smaller element is to the larger as the larger is to the whole: the Golden Proportion. The Golden Section - a precise way of dividing a line, music or anything else - goes back at least as far as 300 BC , when Euclid described it in his major work the Elements, and to around 500 BC , when Pythagoras ( $560-480 \mathrm{BC}$ ) proved that the Golden Section was the basis for the proportions of the human figure and that the human body was built with each part in a definite Golden Proportion to all the other parts.[2-5] (Figure 5) Pythagoras' proportions of the human figure had a tremendous effect on Greek art. In order to describe the golden section, a line that is one unit long is divided in two unequal segments, such that the shorter one equals $x$, the longer one equals $(1-x)$ and the ratio of the shorter segment to the longer one equals the ratio of the longer segment to the overall line; that is, $\mathrm{x} /(1-$ $x)=(1-x) / 1$. This equality leads to a quadratic equation that can be used to solve for $x$, and substituting that value back into the equality yields a common ratio of approximately 0.618 . The Greek letter phi is used for


Figure 6.; Golden rectangle ABCD and golden spiral
this Golden Proportion and, also, Phi is used for the closely related value 1.6180339887 ... The Golden Section relationship asserts a natural balance, a dynamic symmetry and the Golden Mean is present in the proportions in nature, specifically the human body.[7] For example, the length of the first finger joint to the length of the next two joints is equal to the length of the two joints to the length of the whole finger. Similarly, the length of the middle finger to the length of the palm is equal to the length of the palm to the length of the whole hand. [5]Also, the length of the hand to the length of the forearm is equal to the length of the forearm to the length of the finger to the elbow. This amazing discovery is the main reason for the beauty of the human figure and the patterns commonly found in nature, art, and architecture. [18,19] (Figure 6)


Figure 7; THE UNIQUE PROPERTIES OF PHI (Ф)
The Golden Section, also known as Phi, is manifested in the structure of the human body. The human face abounds with examples of the Golden Section. The head forms a Golden Rectangle with the eyes at its midpoint. The mouth and nose are each placed at golden sections of the distance between the eyes and the bottom of the chin. Phi defines the dimensions of the human profile. Even when viewed from the side, the human head illustrates the Golden Proportion. The golden section is a proportion that occurs when something (e.g. a line) is divided into two unequal parts such that the smaller ( m ) is to the larger ( M ) as the larger is to the sum of the two $((\mathrm{m} / \mathrm{M}=\mathrm{M} /(\mathrm{m}+M)=0.618)$; Dalzell's theorem means that the use of $M /(M+m)$ ratios leads to results that are more supportive of the golden section hypothesis than does the use of $\mathrm{m} / \mathrm{M}$ ratios.[30] Measurements of the human face have been performed since the Greek era, and many aspects of ancient measurements can be found in modern clinical anthropometry.[31] (Figure 7)

The Golden Number, what the ancient Greeks called the «Divine Proportion», has a value of 1.618; this number is found in numerous natural phenomena, geometrical propositions, and human architectural constructions; these proportions are worth comparing to those of the human face; this Golden Proportion is found in many cephalometric measurements and in various stages of facial growth .[32,33] The golden proportion mathematically states that the ratio of a smaller length to a larger length is equal to the ratio of the larger length to the whole length, which is equal to 'phi' that is equal to 1.618 . The concept of the golden proportion was used .[34] The present study was carried out to determine whether or not the length of the face divided

$$
\varphi=\frac{1+\sqrt{5}}{2}=1.6180339887 \ldots \ldots
$$

Figure 8; The value of Phi
by bizygomatic width is 1.618 . If that value was for normal face, any deviation from that value produced a long or a short face and hence such an individual could be at risk of developing maxillofacial or respiratory or jaw disorders.[1-7] (Figure 8)
Following is a clinical case in which the concept of the Golden Proportion is applied.

Case presentation


Figure 9. Preoperative view of patient presenting with aesthetic concerns.

The patient, a 42-year-old female, was unhappy with the appearance of her maxillary anterior teeth (Figure 9). She had four veneers that had been placed approximately 15 years ago. One had debonded and the other three had marginal leakage. She was unhappy with her smile because of the color and shape/size discrepancies of the maxillary anterior teeth. The patient did not display the mandibular teeth at full smile or during conversation, and therefore was not concerned about the appearance of these teeth.
The clinical examination revealed leaking porcelain veneers on the central incisors and the left lateral incisor. The veneer on the maxillary right cuspid had debonded. The tissue heights were slightly uneven, resulting in disproportional sizes of the teeth. The general periodontal health, however, was good. The patient's occlusion exhibited loss of anterior guidance, which created group function. The patient did not report any tooth pain, and there was no history of TMJ symptoms.
The patient sought a smile of uniform symmetry, appropriate tooth shape, and lighter color. Tissue recontouring was discussed to allow for development of proper proportions of the new restorations. To evaluate the position of the incisal edge of the teeth, a direct mock-up was prepared. Although this can be accomplished with a waxup, a direct mock-up not only allows the patient to preview and approve the length of the veneers, but serves as a preparation guide for the clinician.
Determining the location of the incisal edge of the central incisors is one of the most important factors for a successful reconstruction. The three determinants to be considered are occlusion, phonetics, and aesthetics. The occlusion should be characterized by anterior guidance, cuspid disclusion, lack of slide between centric relation and centric occlusion, absence of balancing or working side interferences, and equal and simultaneous force on each tooth when the teeth are in contact, with no deflection when additional force is exerted. Phonetics is also used as a guide to determine the position of the incisal edge. During the production of " F " or " V " sounds, the maxillary anterior teeth should lightly touch the vermilion border of the lower lip. Finally, both the clinician and the patient should evaluate the aesthetics of the proposed incisal edge position. (Figure 10)


Figure 10. Tracing from preoperative Figure 11. Proposed incisal edge photograph to apply smile design principles.


Figure 12. New central incisor
position and gingival heights are marked.


Figure 13; A 75\% width-to-height
length is measured using a ruler. ratio is determined and marked.


Figure 14; The shapes of the central incisors are drawn using the length and width measurements.

After determining the incisal edge position, the length and tissue heights of the central incisors are established. From this point a trace drawing can be used to apply design concepts. It is important to remember that the Golden Proportion is applied to the viewing width of teeth, using a frontal photograph. Using tracing paper, the patient's teeth are outlined (Figure 10and 11). The new incisal edge position and the gingival position are placed on the working outline (Figure 12). To give the central incisors a distinct rectangular shape, the width-to-height ratio should be approximately $75 \%$. The length of the central incisors is measured on the photograph and multiplied by 0.75 to give the correct photographic width for these teeth (Figures 12 and 13). After marking the width measurement, the proposed shape of the new central incisors can be drawn (Figure 14). At this time, the width of the other anterior teeth can be established using the Golden Proportion.


Figure 15. The width of the central incisors is reduced by $62 \%$ to give the width of the lateral incisors.


Figure 16. The shapes of the lateral incisors are drawn.


Figure 17. A 62\% reduction in width of the lateral incisors gives the profile width of the cuspids.

The width of the central incisors is multiplied by 0.62 , thereby establishing the width of the lateral incisors. This new width of the lateral incisors is marked, and the shapes are drawn (Figures 15and 16). The same technique is utilized for the profile view of the cuspids and premolars (Figure 17). It is important to note that the Golden Proportion only approximates the width of each tooth, not the length.

This tracing helps to define the treatment plan. The tracing shows where tissue recontouring is required and helps in preparation design. The tracing demonstrates whether the widths of the teeth must be altered in order to design a proportional and symmetrical smile. To do this may require aggressive preparation of the teeth to open the contact areas, so the laboratory technician can establish the new proportions.

Prior to prescribing the final veneers, the provisional restorations are fabricated to the guidelines set forth in the smile design process. This gives the practitioner and patient an opportunity to evaluate the proposed aesthetics and occlusion. If necessary, changes can be made in the provisional restorations and relayed to the laboratory technician.

After patient approval, the final pressed ceramic restorations were placed using a resin cement. The patient's occlusion and aesthetics were reevaluated 1 week after cementation. The patient reported no sensitivity or complications. Occlusion and gingival health were excellent. The patient was given a mandibular bleaching tray to achieve a shade match between the mandibular and maxillary teeth. The completed case is shown in Figure 18.

## II. Discussion

The application of a system of aesthetic predictions to dental aesthetics was facilitated by a dental grid for the anterior aesthetic


Figure 18. The final smile designed using the concept of the Golden Proportion. segment . [35]The principle of Golden Proportion (1.6:1:0.6) can be achieved among the centrals, laterals and cuspids .[36] The Golden Proportion (1.618) is found in many cephalometric measurements and in several stages of facial growth.[32] The mean intercanthal distance and maxillary central incisor width were significantly higher for male subjects, and inner canthal distance, when multiplied by a decreasing function value of the geometric progression term (0.618) and then divided by 2 , is a reliable predictor of maxillary central incisor width . [37]The maxillary central incisor and canine dimensions of the men were significantly greater than those of the women, and bizygomatic width and interalar width could serve as reference for establishing the ideal width of the maxillary anterior teeth, particularly in women . [38]The influence of the Greek proportional sciences on modern facial anthropometry, the golden proportion, and the canons of important Renaissance artists, physical anthropology and cephalometry were discussed and for reconstructive and cosmetic surgery, realistic sizes and proportions have been assessed using anthropometric techniques and used as guidelines to correct deformities or disproportions. [29]Apparent individual tooth width was considered as a percentage of the total apparent width of the anterior segment, and the concept of the golden percentage has been a more useful application in diagnosing and developing symmetry, dominance, and proportion for aesthetically pleasing smiles .[16]The vertical craniofacial dimension was a more accurate measure of facial proportion; alterations in the vertical dimension of occlusion could dramatically affect the aesthetics of the soft facial tissue; faces with deficiencies in lower facial balance (brachyfacial) often exhibited insufficient height of the occlusal plane; facial balance and location of the occlusal planes are the primary determinants for establishing an appropriate vertical dimension of occlusion .[31] The Golden Proportion was closer to the proportions of the serious and relaxed faces than of the smiling ones.[39] Golden Proportion has been considered useful for maxillofacial surgery .[40] (Figure 18)

It is not necessary to achieve the width of the anterior teeth exactly as described by the Golden Proportion when designing an anterior reconstruction. In fact, these exact proportions rarely occur in the natural dentition. The Golden Proportion is just one of many factors involved in smile design. The value of the Golden Proportion is as a diagnostic tool in evaluating a smile, and as a guide to veneer preparation and fabrication.[19,20,23,25,27,19]

In fact, several studies have indicated that wide variation exists for patients and dentists regarding ideal anterior tooth proportions.In a study by Rosenstiel et al,549 dentists evaluated computer images of the same six maxillary anterior teeth. Dentists preferred $80 \%$ proportions when viewing short or very short teeth, and the Golden Proportion (62\%) for very tall teeth. [41]There was no identifiable preference for teeth of normal length or tall teeth, and choices could not be predicted based on gender, specialist training, experience, or patient volume. The results of a similar study by Kokich et aldemonstrated that orthodontists, general dentists, and lay people detect specific aesthetic discrepancies at different levels of change. In the case of the Golden Proportion, lay people did not discern a lateral incisor narrowing until the deviation reached 4 mm .[42]

Human faces are also comprised of this ratio within the relationships between the eyes, ears, mouth and nose $[17,22]$. The human being is the most beautiful and most perfect instauration of Allah. Allah said in the Holly Quran;


Figure19; [ Surah At-Tin (95), Verse 8 ]. Meaning: We have indeed created human in the best of modules It is known that the Arabic text direction is from right to left. There are exactly 26 characters in this sentence and the word "Insana"
word "Insana" comes at the golden mean of the sentence. (Figure 19)The 26 characters can be divided into two sections p and q . The left section p has 16 characters started from the word "Insana" and q has 10 characters from the beginning to that word. The ratio between p and q is 1.6 which is closer to the golden number 1.618... Indeed, Quran is a miracle for its eloquence in language, for applicability and relevance of its verses in all spheres of human life, for its inimitable verses and in many other aspects. It does not need to prove as scientific, rather we need to guide by it.[43]

The patient who presents for cosmetic rehabilitation will probably not be comfortable with some of the wide deviations identified in the previous studies. Nevertheless, these studies demonstrate that no single rule or formula can be used to generalize across a population.

## III. Conclusion

The design elements presented here do not represent a complete discussion of the available principles and techniques of smile design. In addition, patients and dentists will vary in their preferences. The principles described here offer one set of guidelines for aesthetic dentistry. In the case of the Golden Proportion, the exact proportions are not as important as are the concepts of symmetry and the use of a logical approach to aesthetic restoration of the maxillary anterior teeth.

## References

[1]. Moskowitz ME, Nayyar A(1995). Determinants of dental esthetics: a rational for smile analysis and treatment. Compend Contin Educ Dent.;16:1164-1166.
[2]. Eubank J. (2001)Macroesthetic elements of smile design. J Am Dent Assoc.;132:39-45.
[3]. Morely J.(1997) Smile design: specific considerations. J Calif Dent Assoc. ;25:633-637.
[4]. Preston JD. (1993)The Golden Proportion revisited. J Esthet Dent.;5:247-251.
[5]. Gillen RJ, Schwartz RS, Hilton TJ, et al. (1994)An analysis of selected normative tooth proportions. Int J Prosthodont.; 7:410-417.
[6]. Herz-Fischler R. A Mathematical History of the Golden Number. New York, NY: Dover Publications Inc; 1998.
[7]. Ghyka MC. The Geometry of Art and Life. New York, NY: Dover Publications Inc; 1977.
[8]. Williams R. The Golden Proportion. The Geometrical Foundation of Natural Structure: Source Book of Design. New York, NY: Dover Publications Inc; 1979.
[9]. Huntley HE. The Divine Proportion: A Study In Mathematical Beauty. New York, NY: Dover Publications Inc; 1970.
[10]. Fischler R. (1981)How to find the golden number without really trying. Fibonacci Quart.:19:406-410.
[11]. Boyer CB. History of Mathematics. New York, NY: Wiley \& Sons; 1968.
[12]. Robins G, Shute C.(1985) Mathematical bases of ancient Egyptian architecture and graphic art. Historia Math.;12:107-122.
[13]. Fields M.(1933) Practical mathematics of roman times. Mathematics Teacher. ;26:77-84.
[14]. Kappraff J. Connections: The Geometric Bridge Between Art and Science. New York, NY: McGraw-Hill, Inc; 1991.
[15]. Badawy A. A History of Egyptian Architecture. 3 volumes. New York, NY: University of California Press; 1966-68
[16]. 16.. Snow SR. (1999)Esthetic smile analysis of maxillary anterior tooth width: the golden percentage. J Esthet Dent. 11:177-184.
[17]. K. Elam. Geometry of Design, Studies in Proportion and Composition. Princeton Architectural Press, 37 East 7th Street, New York 10003
[18]. .Muhamad Abu-Hussein, Nezar Watted, Azzaldeen Abdulgani (2015) Gummy Smile and Optimization of Dentofacial Esthetics. Journal of Dental and Medical Sciences 14(4): 24-28.
[19]. Abu-Hussein Muhamad(2015): RESTORATION OF ESTHETICS USING CERAMICS LAMINATE VENEER- CLINICAL REVIEW -A CASE REPORT. Journal of Advanced Medical and Dental Sciences Research 03
[20]. Abu-Hussein Muhamad(2015): DIASTEMA CLOSURE WITH DIRECT COMPOSITE- ARCHITECTURAL GINGIVAL CONTOURING. Journal of Advanced Medical and Dental Sciences Research 02
[21]. M. Abu-Hussein , Aspasia Sarafianou;(2012) MATHEMATICAL ANALYSIS OF DENTAL ARCH OF CHILDREN IN NORMAL OCCLUSION: A LITERATURE REVIEW. International Journal of Medical Dentistry, 16, 1, 33-40
[22]. Abu-Hussein Muhamad, Abdulgani Azzaldeen, Abdulgani Mai (2016) Esthetics of Class IV Restorations with Composite Resins. Journal of Dental and Medical Sciences 15(1): 61-66.
[23]. Abu-Hussein Muhamad, Watted Nezar, Abdulgani Azzaldeen. (2015)The Curve of Dental Arch in Normal Occlusion. Open Science Journal of Clinical Medicine. 3, 2, 47-54.
[24]. Abu-Hussein Muhamad et al.(2019), Restoring Fractured Anterior Tooth Using Direct Composite Restoration: A Case Report. Global Journal of Dental Sciences1:1
[25]. Abu-Hussein Muhamad, et al. (2019)Anterior Esthetic Restorations Using Direct Composite Restoration; a Case Report. Dentistry \& Dent Pract J , 2(1): 180008.
[26]. Abdulgani A, Watted N, Abu-Hussein M.(2014) Direct bonding in diastema closure high drama, immediate resolution: a case report. Ijdhs. 1(4): 430-435
[27]. Muhamad Abu-Hussein, Nezar Watted(2015); Maxillary Midline Diastema - Aetiology And Orthodontic Treatment- Clinical Review, Journal of Dental and Medical SciencesVolume 15, Issue 6 Ver. II ,116-130, DOI: 10.9790/0853-150602116130
[28]. Abu-Hussein M, Watted N, Abdulgani A. (2015)An Interdisciplinary Approach for Improved Esthetic Results in the Anterior Maxilla Diastema Journal of Dental and Medical Sciences. 14(12): 96-101.
[29]. Muhamad Abu-Hussein et al. (2016 ), Congenitally Missing Lateral Incisors, Orthodontic, Restorative, and Implant Approaches. Int J Dent and Oral Heal. 2:3, 24-33. DOI: 10.25141/2471-657X-2016-3.0071
[30]. BENJAFIELD JG (2000). Dalzell's theorem and the analysis of proportions: a methodological note. Br J Psychol, 91: 287-291. BLITZ N (1996). Direct bonding in diastema closure - high drama, immediate resolution. Oral Health, 86: 23-26.
[31]. Mack MR. (1996)Perspective of facial esthetics in dental treatment planning. J Prosthet Dent ;75:169-76
[32]. AMORIC M (1989). The golden number. Applications to architectural and structural cranio-facial analysis. Actual Odontostomatol (Paris), 42: 205-219.
[33]. AMORIC M (1995). The golden number: applications to craniofacial evaluation. Funct Orthod, 12:18-21, 24-25.
[34]. JAVAHERI DS and SHAMAVAZ S (2002). Utilizing the concept of the golden proportion. Dent Today, 21: 96-101
[35]. E. I. Levin, B. Ch. D.(1978) Dental aesthetics and the golden proportion. The journal of prosthetic Dentistry, removable prosthodontics. Vol. 40, Number 3, 244-252
[36]. Blitz N.: Diagnosis and Treatment Evaluation in Cosmetic Dentistry-a guide to accreditation criteria. American Academy of Cosmetic Dentistry Madison, WI: AACD 2001
[37]. ABDULLA MA (2002). Inner canthal distance and geometric progression as a predictor of maxillary central incisor width. J Prosthetic Dent, 88: 16-20.
[38]. HASANREISOGLU U, BERKSUN S, ARAS K, and ARSLAN I (2005). An analysis of maxillary anterior teeth: facial and dental proportions. J Prosthet Dent, 94: 530-538
[39]. HARALABAKIS NB, LAGONDAKIS M and SPANOUDAKIS E (1989). A study of esthetic harmony and balance of the facial soft tissue. Orthod Epitheorese, 1: 175-189.
[40]. KAWAKAMI S, TSUKADA S, HAYASHI H, TAKADA Y and KOUBAYASHI S (1989). Golden proportion for maxillofacial surgery in Orientals. Ann Plast Surg, 23: 417-425.
[41]. Rosenstiel SF, Ward DH, Rashid RG.(2000) Dentists' preferences of anterior tooth proportion: a web-based study. J Prosthodont. 9:123-136.
[42]. Kokich VO Jr, Kiyak HA, Shapiro PA. (1999)Comparing the perception of dentists and lay people to altered dental esthetics. J Esthet Dent.;11:311-324.
[43]. Holly Quran, Surah At-Tin (95), Verse 8

