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Stochastic Analysis with Simulation Studies Of Time to Hospitalization and Hospitalization Time for Diabetes When One Organ Is Defective and another Organ Is Exposed To Damage Process with Prophylactic Treatment

Rajkumar.A[1] , Gajivaradhan.P[2], Ramanarayanan.R[3]
[1]*Assistant Professor of Mathematics, SriRam Engineering College, Veppampattu, Thiruvallur, 602024, India.*
[2]*Principal & Head Department of Mathematics, Pachaiyappas College, Chennai, 600030, India.*
[3]*Professor of Mathematics Retired, VelTech University, Chennai, India*

Abstract: *This paper assumes that one organ A of a diabetic person is exposed to organ failure due to a two phase risk process and another organ B is exposed to a damage process. Prophylactic treatment starts after an exponential time. In Model 1, his hospitalization for diabetes starts when one of the organs A or B fails or when the prophylactic treatment starts. In Model 2, his hospitalization starts when both the organs A and B are in failed state or when the prophylactic treatment starts. The joint transform of the distribution of time to hospitalization for treatment and hospital treatment time are presented along with their expectations. The expected time to hospitalization for treatment and expected treatment time are obtained for numerical Studies. Simulation studies are under taken using linear congruential generators. Erlang and phase distributions are considered for hospital stage treatment times. Random values of all the variables are generated to present simulated values of time to hospitalization and hospitalization times for various parameter values of time to prophylactic treatment.*

Keywords: *Diabetic mellitus, Prophylactic treatment, PH phase2 distribution, Linear congruential generator.*

I. Introduction

Risk factors of diabetes mellitus have been presented by Bhattacharya, Biswas, Ghosh and Banerjee in [1]. Foster, Fauci , Braunward, Isselbacher, Wilson, Mortin and Kasper have treated diabetes mellitus in [2]. Kannell and McGee [3] have analyzed Diabetes and Cardiovascular Risk Factors. King, Aubert and Herman in [4] have listed the global burden of diabetes during the period 1995-2025. King and Rewers [5] have estimates for the prevalence of diabetes mellitus and impaired glucose tolerance in adults. Usha and Eswariprem in [6] have focused their discussions on the models with metabolic disorder. Eswariprem, Ramanarayanan and Usha [7] have analyzed such models with prophylactic treatment to avoid the disease. Mathematical models and assumptions play a great and distinctive role in this area. Any study with prophylactic treatment will be very much beneficial to the society. Moreover cure from the disease after treatment is time consuming and above all is seldom achieved in many cases. This paper concentrates on situations of prophylactic treatment to prevent the disease when one organ A of a person is exposed to failure due to a two phase failure process and another organ B is exposed a damage process. Rajkumar, Gajivaradhan and Ramanarayanan [8] have treated recently a diabetic model where the patient is admitted for prophylactic treatment after a random time which has exponential distribution. They have also discussed the effect of prophylactic treatment. But so far a damage model approach has not been attempted so far. Recent advancements in Probability, Operations Research and Simulation methods are utilized for the presentation of the results here. Analyzing real life stochastic models researchers collect data directly from the source/ hospitals (primary data) or use secondary data from research organizations or use simulated data for studies. Simulation studies are more suitable in this area since in most of the cases in general, hospitable real life data may not be sufficiently available and at times they may heavily depend on the biased nature of the data collectors. They may vary hospital to hospital and they may not be genuine enough for the study since many other factors such as the quality of nursing and medical treatments provided to the patients by hospitals are involved. These are necessary to generate perfect and genuine data. Since the reputation of many connected organizations are involved, there may not be anybody to take the responsibility of the perfectness of the data provided. In this area not much of significant simulation studies are available or taken up so far. For the simulation analysis here, Martin Haugh [9] results and Law and Kelton methods using Hull and Dobell results [10] are utilized to generate uniform random values and all other random values required. In the model treated here, a person with two defective organs A and B is considered. In Model 1, he is provided hospital treatment when any one of the organs fails or when he is admitted for prophylactic treatment. In Model 2, he is provided hospital treatment when both the organs are in failed state or when he is admitted for prophylactic treatment. The joint Laplace-Stieltjes transform of distribution of time to hospitalization for treatment and hospital treatment time, the expected time to hospitalization for treatment and expected treatment time for the models are derived. Numerical examples are presented. Simulation studies are provided considering the sum of Erlang and phase-two hospital treatment times for the two cases. Varying the parameter of time to prophylactic treatment several simulated values are generated for time to hospitalization and hospitalization times.

II. Model 1: One Organ Failure And Prophylactic Treatment

2.1. Assumptions

(1) A patient has two defective organs A and B.

(2) Defective organ A producing insulin functions in two phases of damaged levels namely damaged level 1 (phase 1) and damaged level 2 (phase 2) where level 1 is considered to be a better level of the two with lesser failure rate. Due to negligence and carelessness, the organ A may move to level 2 from level 1 and due to pre-hospital medication the organ A may move to level 1 from level 2. The failed level of the organ A is level 3. The transition rates of the organ A to the failed level 3 from level 1 and from level 2 are respectively λ_1 and λ_2 with $\lambda_2 > \lambda_1$. The transition rates from level 1 to level 2 is μ_1 and from level 2 to level 1 is μ_2. At time 0 the organ A level is 1 and let F_A denote the life time (time to failure) of organ A.

(3) At time 0 the organ B is damage free but defective. It is exposed to a damage process with exponential inter occurrence time distribution whose parameter is θ. The organ B fails on the k-th damage with probability $p_k \geq 0$ and survives the k-th damage with probability P_k for $k \geq 1$ where $\sum_1^\infty p_k = 1$ and $P_k = 1 - \sum_1^k p_k$ with $p_0 = 0$ and $P_0 = 1$. Let their generating functions be $\phi(r) = \sum_0^\infty p_k r^k$ and $\Phi(r) = \sum_0^\infty P_k r^k$.

(4) Irrespective of the status of the organs the patient is observed for a random time F_P which has exponential distribution with parameter α after which prophylactic treatment starts.

(5) The hospital treatment begins when organ A or organ B fails or when the prophylactic treatment starts whichever occurs first.

(6) The hospital treatment times for the organs A and B are random variables H_1 and H_2 with (Cumulative distribution functions) Cdfs $H_1(x)$ and $H_2(x)$ and (probability density functions) pdfs $h_1(x)$ and $h_2(x)$. The prophylactic treatment time in the hospital is a random variable H_3 with Cdf $H_3(x)$ and pdf $h_3(x)$.

2.2. Analysis

To study the above model probability distribution, $F_A(x)$ of the time to failure of the organ A from the level 1 at time 0 is required. Levels 1 and 2 of the organ A are considered as phases 1 and 2 respectively of PH phase 2 distribution. Considering the failed state as absorbing state 3, the infinitesimal generator describing the transitions is given by

$$Q=\begin{bmatrix} -(\lambda_1+\mu_1) & \mu_1 & \lambda_1 \\ \mu_2 & -(\lambda_2+\mu_2) & \lambda_2 \\ 0 & 0 & 0 \end{bmatrix} . \quad (1)$$

The pdf $f_A(t)$ and the Cdf $F_A(t)$ of the time to absorption starting from state 1 at time zero (the time to failure of organ A) have been derived in Rajkumar, Gajivaradhan and Ramanarayanan [8]. For easy reference they are given below. Using the results of [8], it can be seen that

$$f_A(t) = k_1(a - b)\, e^{-(a-b)t} - k_2(a + b)\, e^{-(a+b)t} \quad (2)$$

Here $a = (\frac{1}{2})(\lambda_1+\lambda_2+\mu_1+\mu_2)$; $b = (\frac{1}{2})\sqrt{(\lambda_1-\lambda_2+\mu_1-\mu_2)^2+4\mu_1\mu_2}$; $k_1 = \frac{a+b-\lambda_1}{2b}$ and $k_2 = \frac{a-b-\lambda_1}{2b}$.(3)

and its Cdf is $F_A(t) = \int_0^t f_A(u)du = 1 - k_1\, e^{-(a-b)t} + k_2\, e^{-(a+b)t}$. (4)

To study the model, the joint pdf of two variables (T, H) where T is the time to hospitalization and H is the hospitalization time is required. Here variable T = Minimum $\{$ the life time of the organ A, the life time of B, the time to prophylactic treatment$\}$ and variable H is the hospitalization time = H_1 or H_2 or H_3 according as the hospitalization begins when the organ A fails or the organ B fails or the patient is admitted for prophylactic treatment respectively. The joint pdf of (T, H) is

$$f(x, y) = f_A(x)\left(\sum_0^\infty P_k\, e^{-\theta x}\frac{(\theta x)^k}{k!}\right)e^{-\alpha x}\, h_1(y) + \overline{F_A}(x)\left(\sum_1^\infty p_k\, \theta e^{-\theta x}\frac{(\theta x)^{k-1}}{(k-1)!}\right)e^{-\alpha x}\, h_2(y)$$
$$+ \overline{F_A}(x)\left(\sum_0^\infty P_k\, e^{-\theta x}\frac{(\theta x)^k}{k!}\right)\alpha e^{-\alpha x}\, h_3(y) . \quad (5)$$

The first term of the RHS of (5) is the pdf-part that the organ A fails at time x, organ B does not fail up to x, the patient is not admitted for prophylactic treatment till time x and the hospitalization is provided for the failure of the organ A. The second term is the pdf-part that the organ A does not fail up to time x, organ B fails at time x, the patient is not admitted for prophylactic treatment till time x and the hospitalization is provided for the failure of the organ B. The third term is the pdf part that the patient is admitted for prophylactic treatment at time x before the failure of any of the two organs A or B and the hospitalization for the prophylactic treatment is provided. The double Laplace transform of the pdf of (T, H) is $f^*(\xi, \eta) = \int_0^\infty \int_0^\infty e^{-\xi x-\eta y}\ f(x, y)\ dx\ dy$. (6)

The equation (6) using the structure of the equation (5) becomes a single integral.

$$f^*(\xi, \eta) = \int_0^\infty e^{-\xi x} \{f_A(x) \left(\sum_0^\infty P_k\, e^{-\theta x} \frac{(\theta x)^k}{k!}\right) e^{-\alpha x}\, h_1^*(\eta) + \overline{F_A}(x) \left(\sum_1^\infty p_k\, \theta e^{-\theta x} \frac{(\theta x)^{k-1}}{(k-1)!}\right) e^{-\alpha x}\, h_2^*(\eta)$$
$$+ \overline{F_A}(x) \left(\sum_0^\infty P_k\, e^{-\theta x} \frac{(\theta x)^k}{k!}\right) \alpha e^{-\alpha x}\, h_3^*(\eta)\}\ dx. \quad (7)$$

Using (2), (4), $\Phi(r) = \frac{1-\phi(r)}{1-r}$ and $\int_0^\infty e^{-\xi x}\ e^{-\theta x} \frac{(\theta x)^k}{k!}\ dx = \frac{1}{\theta}\left(\frac{\theta}{\xi+\theta}\right)^{k+1}$ equation (7) becomes

$$f^*(\xi, \eta) = [k_1(a - b) \left(\frac{1}{\xi+\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\xi+\theta+a-b+\alpha}\right) - k_2(a + b) \left(\frac{1}{\xi+\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\xi+\theta+a+b+\alpha}\right)]\, h_1^*(\eta)$$
$$+ [k_1\, \phi\left(\frac{\theta}{\xi+\theta+a-b+\alpha}\right) - k_2\, \phi\left(\frac{\theta}{\xi+\theta+a+b+\alpha}\right)]\, h_2^*(\eta)$$
$$+ [k_1 \left(\frac{1}{\xi+\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\xi+\theta+a-b+\alpha}\right) - k_2 \left(\frac{1}{\xi+\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\xi+\theta+a+b+\alpha}\right)]\alpha\, h_3^*(\eta) \quad (8)$$

The Laplace transform of the pdf of the time to hospitalization T may be obtained after simplification by taking $\eta = 0$ in equation (8) and using the relation between $\Phi(r)$ and $\phi(r)$ as follows.

$$f^*(\xi, 0) = k_1 \left(\frac{a-b+\alpha}{\xi+a-b+\alpha}\right) - k_2 \left(\frac{a+b+\alpha}{\xi+a+b+\alpha}\right) + k_1 \left(\frac{\xi}{\xi+a-b+\alpha}\right) \phi\left(\frac{\theta}{\xi+\theta+a-b+\alpha}\right) - k_2 \left(\frac{\xi}{\xi+a+b+\alpha}\right) \phi\left(\frac{\theta}{\xi+\theta+a+b+\alpha}\right). \quad (9)$$

Now $E(T) = -\frac{d}{d\xi} f^*(\xi, 0)\, |_{\xi=0}$ gives $E(T) = \left(\frac{k_1}{a-b+\alpha}\right) [1 - \phi\left(\frac{\theta}{\theta+a-b+\alpha}\right)] - \left(\frac{k_2}{a+b+\alpha}\right) [1 - \phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]$ (10)

Using equation (8) the Laplace transform of the pdf of the hospitalization time H may be obtained by taking $\xi = 0$.

$$f^*(0, \eta) = [k_1(a - b) \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2(a + b) \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, h_1^*(\eta)$$
$$+ [k_1\, \phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2\, \phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, h_2^*(\eta)$$
$$+ [k_1 \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2 \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\alpha\, h_3^*(\eta) \quad (11)$$

Since $E(H) = -\frac{d}{d\eta} f^*(0, \eta)\, |_{\eta=0}$,

$$E(H) = [k_1(a - b) \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2(a + b) \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, E(H_1)$$
$$+ [k_1\, \phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2\, \phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, E(H_2)$$
$$+ [k_1 \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2 \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\alpha\, E(H_3) \quad (12)$$

Inversion of Laplace transform of (9) and (11) are straight forward. Noting $P(T \le t) = L^{-1}\left(\frac{f^*(\xi,0)}{\xi}\right)$, the Cdf of the time to hospitalization T is

$$P(T \le t) = F_T(t) = 1 - k_1 e^{-(a-b+\alpha)t} + k_2 e^{-(a+b+\alpha)t} + [k_1 e^{-(a-b+\alpha)t} - k_2 e^{-(a+b+\alpha)t}][\textstyle\sum_1^\infty p_k\, \Gamma_{(k,\theta)}(t)] \quad (13)$$

where $\Gamma_{(k,\theta)}(t)$ is the Cdf of Erlang with k phases and θ as parameter.

The Cdf of the hospitalization time H is given below.

$$P(H \le t) = F_H(t) = [k_1(a - b) \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2(a + b) \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, H_1(t)$$
$$+ [k_1\, \phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2\, \phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\, H_2(t)$$
$$+ [k_1 \left(\frac{1}{\theta+a-b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a-b+\alpha}\right) - k_2 \left(\frac{1}{\theta+a+b+\alpha}\right) \Phi\left(\frac{\theta}{\theta+a+b+\alpha}\right)]\alpha\, H_3(t). \quad (14)$$

III. Model 2: Both Organs Failure And Prophylactic Treatment

The assumptions of the model studied here are given below. In real life situations and in many cases it may be seen that due to ignorance and negligence the patient may not be sent for hospitalization unless both organs become failed. It is very common in many cases that the pancreas failed patients may not be aware of their lower insulin levels / higher sugar levels until their another organ also fails.

3.1. Assumptions

(1) A patient has two defective organs A and B.

(2) Defective organ A producing insulin functions in two phases of damaged levels namely damaged level 1 (phase 1) and damaged level 2 (phase 2) where level 1 is considered to be a better level of the two with

lesser failure rate. Due to negligence and carelessness, the organ A may move to level 2 from level 1 and due to pre-hospital medication the organ A may move to level 1 from level 2. The failed level of the organ A is level 3. The transition rates of the organ A to the failed level 3 from level 1 and from level 2 are respectively $\lambda_1 and\ \lambda_2\ with\ \lambda_2 > \lambda_1$.The transition rates from level 1 to level 2 is μ_1 and from level 2 to level 1 is μ_2. At time 0 the organ A level is 1 and let F_A denote the life time (time to failure) of organ A.

(3) At time 0 the organ B is damage free but defective. It is exposed to a damage process with exponential inter occurrence time distribution whose parameter is θ. The organ B fails on the k-th damage with probability $p_k \geq 0$ and survives the k-th damage with probability P_k for $k \geq 1$where $\sum_1^\infty p_k = 1$ and $P_k = 1 - \sum_1^k p_k$ with $p_0 = 0$ and $P_0 = 1$. Let their generating functions be $\phi(r) = \sum_0^\infty p_k r^k$ and $\Phi(r) = \sum_0^\infty P_k r^k$.

(4) Irrespective of the status of the organs the patient is observed for a random time F_P with exponential distribution with parameter α after which prophylactic treatment starts.

(5) The hospital treatment begins when both organs A and B are in failed state or when the prophylactic treatment starts whichever occurs first.

(6) The hospital treatment times for the failure of the two organs A and B is random variables H_4 with Cdf $H_4(x)$ and pdf $h_4(x)$. The prophylactic treatment time in the hospital is a random variable H_5 with Cdf $H_5(x)$ and pdf $h_5(x)$.

3.2. Analysis

The Cdf and the pdf of organ A life time $F_A(x)$ and $f_A(x)$ are presented in (4) and (2).

To study the model, the joint pdf of two variables (T, H) where T is the time to hospitalization and H is the hospitalization time is required.

Here variable T = Minimum $\{$ Maximum (the life time of the organ A, the life time of B), the time to prophylactic treatment$\}$ and variable H is the hospitalization time = $H_4\ or\ H_5$ according as the hospitalization begins when the two organs are in failed state or the patient is admitted for prophylactic treatment respectively.

The joint pdf of (T, H) is

$$f(x, y) = [\, f_A(x) \sum_1^\infty p_k\, \Gamma_{(k,\theta)}(x) + F_A(x)\, (\sum_1^\infty p_k\, \theta e^{-\theta x} \frac{(\theta x)^{k-1}}{(k-1)!})]\, e^{-\alpha x}\, h_4(y) + [1 - F_A(x)\, (\sum_1^\infty p_k\, \Gamma_{(k,\theta)}(x))\,]\, \alpha e^{-\alpha x}\, h_5(y). \quad (15)$$

The first term of the RHS of (15) has a square bracket. Its first term is the pdf-part that the organ A fails at time x when organ B is already in failed state at x and its second term is the pdf- part that organ B fails at time x when organ A is already in failed state at x. The multiplier of the square bracket is the probability that the patient is not admitted for prophylactic treatment up to time x and the pdf part of treatment time H_4 for the failure of the two organs. The second term is the probability that both the organs are not in failed state up to time x (at least one is working) and pdf-part of the completion of the time to prophylactic treatment at time x and the hospitalization H_5 is provided for the prophylactic treatment. The double Laplace transform of the pdf of (T, H) is

$$f^*(\xi, \eta) = \int_0^\infty \int_0^\infty e^{-\xi x - \eta y}\, f(x, y)\, dx\, dy. \quad (16)$$

The equation (16) using the structure of the equation (15) becomes a single integral.

$$f^*(\xi, \eta) = \int_0^\infty e^{-\xi x} \{[\, f_A(x) \sum_1^\infty p_k\, \Gamma_{(k,\theta)}(x) + F_A(x)\, (\sum_1^\infty p_k\, \theta e^{-\theta x} \frac{(\theta x)^{k-1}}{(k-1)!})]\, e^{-\alpha x}\, h_4^*(\eta) + [1 - F_A(x)\, (\sum_1^\infty p_k\, \Gamma_{(k,\theta)}(x))\,]\, \alpha e^{-\alpha x}\, h_5^*(\eta)\, \}dx. \quad (17)$$

Using (2) , (4) and $\int_0^\infty e^{-\xi x}\, e^{-\theta x} \frac{(\theta x)^k}{k!}\, dx = \frac{1}{\theta} (\frac{\theta}{\xi+\theta})^{k+1}$ equation (17) becomes

$$f^*(\xi, \eta) = [-\, (\frac{k_1(\xi + \alpha)}{\xi + a - b + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + a - b + \alpha}) + (\frac{k_2(\xi + \alpha)}{\xi + a + b + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + a + b + \alpha}) + \phi\, (\frac{\theta}{\xi + \theta + \alpha})]\, h_4^*(\eta) + [\, \frac{1}{\xi + \alpha} - (\frac{1}{\xi + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + \alpha}) + (\frac{k_1}{\xi + a - b + \alpha})\phi\, (\frac{\theta}{\xi + \theta + a - b + \alpha}) - \frac{k_2}{\xi + a + b + \alpha}\, \phi\, (\frac{\theta}{\xi + \theta + a + b + \alpha})\,]\alpha\, h_5^*(\eta). \quad (18)$$

The Laplace transform of the pdf of the time to hospitalization T may be obtained after simplification by taking η = 0 in equation (18).

$$f^*(\xi, 0) = \frac{\alpha}{\xi + \alpha} + (\frac{\xi}{\xi + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + \alpha}) - (\frac{k_1 \xi}{\xi + a - b + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + a - b + \alpha}) + (\frac{k_2 \xi}{\xi + a + b + \alpha})\, \phi\, (\frac{\theta}{\xi + \theta + a + b + \alpha}) \quad (19)$$

Now $E(T) = -\frac{d}{d\xi} f^*(\xi, 0)\, |_{\xi=0}$ gives

E (T) = $\frac{1}{\alpha}$ − ($\frac{1}{\alpha}$) ϕ ($\frac{\theta}{\theta+\alpha}$) + ($\frac{k_1}{a-b+\alpha}$) ϕ ($\frac{\theta}{\theta+a-b+\alpha}$) −($\frac{k_2}{a+b+\alpha}$) ϕ ($\frac{\theta}{\theta+a+b+\alpha}$) (20)

Using equation (18) the Laplace transform of the pdf of the hospitalization time H may be obtained by taking ξ = 0.

f*(0, η) =[$h_4^*(\eta)$ - $h_5^*(\eta)$] [ϕ ($\frac{\theta}{\theta+\alpha}$) − ($\frac{k_1\,\alpha}{a-b+\alpha}$) ϕ ($\frac{\theta}{\theta+a-b+\alpha}$) + ($\frac{k_2\,\alpha}{a+b+\alpha}$) ϕ ($\frac{\theta}{\theta+a+b+\alpha}$)] + $h_5^*(\eta)$. (21)

Since E (H) = $-\frac{d}{d\eta}$ f*(0, η) $|_{\eta=0}$,

E (H) = [E(H_4) - E(H_5)] [ϕ ($\frac{\theta}{\theta+\alpha}$) − ($\frac{k_1\,\alpha}{a-b+\alpha}$) ϕ ($\frac{\theta}{\theta+a-b+\alpha}$) + ($\frac{k_2\,\alpha}{a+b+\alpha}$) ϕ ($\frac{\theta}{\theta+a+b+\alpha}$)] + E(H_5). (22)

Inversion of Laplace transform of (19) and (21) are straight forward. The Cdf of the time to hospitalization T is

P (T ≤ t) = F_T (t) =1 - $e^{-\alpha t}$ + $e^{-\alpha t}$ [1 - k_1 $e^{-(a-b)t}$ + k_2 $e^{-(a+b)t}$] [$\sum_1^\infty p_k\, \Gamma_{(k,\theta)}(t)$] (23)

The Cdf of the hospitalization time H is given below.

P (H ≤ t) = F_H(t) = [H_4(t) - H_5(t)] [ϕ ($\frac{\theta}{\theta+\alpha}$) − ($\frac{k_1\,\alpha}{a-b+\alpha}$) ϕ ($\frac{\theta}{\theta+a-b+\alpha}$) + ($\frac{k_2\,\alpha}{a+b+\alpha}$) ϕ ($\frac{\theta}{\theta+a+b+\alpha}$)] + H_5(t). (24)

IV. Numerical And Simulation Studies

As an application of the results obtained numerical and simulation studies are taken up. In the mathematical model the organ B is assumed to survive any number of damages n with probability P_n. But in real life situations within a finite number of damages the organ may fail. Such situations are taken up for studies in the two models for numerical examples and simulation analysis.

4.1. Numerical Studies:

Let $\lambda_1 = .3$, $\lambda_2 = .35$, $\mu_1 = .4$, $\mu_2 = .5$. From equation (1), a=0.775; b=0.45345893; a + b = 1.228458929; a - b = 0.321541071 which gives the life time of organ A has Cdf, F_A(t) =1 - 1.02375195 $e^{-(0.321541071\,)\,t}$+ 0.02375195$e^{-(1.228458929\,)t}$ with pdf f_A (t)= (0.3291783001) $e^{-(0.3215410713\,)\,t}$- (0.291783001) $e^{-(1.228458929\,)t}$.

Let the Cdf of the inter occurrence times of damages to organ B be F_B (t) = 1-$e^{-\theta t}$. The value for θ is fixed as 0.4. Let the generating function of probabilities of failure of B on the occurrences of damages be ϕ(r) = $\sum_0^4 p_k r^k$ where p_1= 0.3, p_2= 0.4, p_3 = 0.2, p_4 =0.1 and p_n = 0 for n ≥ 5.

This gives ϕ(r) = 0.3r +0.4 r^2+ 0.2 r^3 + 0.1 r^4. (25)

Let the parameter of the exponential time to prophylactic treatment for the two models be with parameters α = .1, .2, .3 and .4. For writing E(H), the first moment of H_j for j =1, 2, and 3 are required. It is assumed that for Model 1 E(H_1)=0.06; E(H_2)=0.03 and E(H_3)=0.02. Let for Model 2, E(H_4)=0.2 and E(H_5)=0.06. For the two models E (T) and E (H) values are presented using equations (10), (12), (20) and (22) for various values of α in the tables 1 and 2.

Table 1: E(T) fixing θ and varying α for Model 1 Table 2: E(T) fixing θ and varying α for Model 2

(Model 1),θ=0.4	E(T)	E(H)	**(Model 2),θ=0.4**	E(T)	E(H)
α = 0.1	1.757648786	0.045855781	α = 0.1	4.2594655	0.127783184
α = 0.2	1.505147075	0.042131159	α = 0.2	3.157211032	0.100216589
α = 0.3	1.314691533	0.039327622	α = 0.3	2.483484666	0.087538852
α = 0.4	1.166199562	0.03714511	α = 0.4	2.034930757	0.080850934

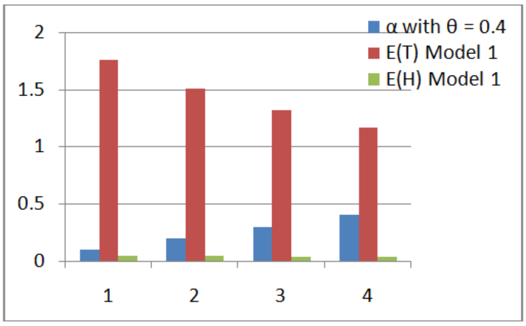

Figure 1. Expectations Varying α Model 1.

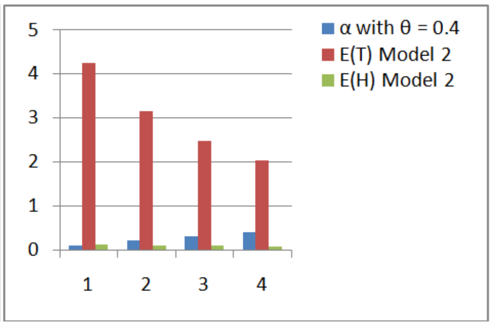

Figure 2. Expectations Varying α Model 2

Figures 1 and 2 present the expected values E(T) and E(H) graphically of values listed in tables 1 and 2. The variations of them on varying the parameters are clearly exhibited. The parameter values are substituted to obtain the statistical estimates. They present exact values of the estimates but they do not present any sample-runs or real life-like situations. Although α causes variations in the values of the estimates as seen above, the effects of the minimum or maximum of life times of organs A , B and the time to prophylactic treatment and the effects of various treatments provided in the hospital can be studied only by simulating the situations and analyzing them. In simulation studies one may find real life like situations.

4.2 Simulation Studies:

The simulated values of the life times of organ A, organ B, the time to prophylactic treatment and the three hospital treatment times H_i, i = 1, 2 and 3 are required for the study. They are generated using the methods presented by Martin Haugh [9] by generating uniform random values u using Linear Congruential Generator (LCG).

4.2.1. Organ A

As in the numerical case-study presented above, the same values for the organ A transition rates are assumed with $\lambda_1 = 0.3, \lambda_2 = 0.35,\ \mu_1 = 0.4,\ \mu_2 = 0.5$. Then from (1), a = 0.775; b = 0.453458929; a + b = 1.228458929; a - b = 0.3215410713 and p' = 0.755791591 which gives the life time of organ A has Coxian-2 Cdf, F_A (t) =1 - 1.02375195 $e^{-(0.3215410713\ )\,t}$+ 0.02375195$e^{-(1.228458929\ )t}$ with pdf f_A (t)= (0.3291783) $e^{-(0.3215410713\ )\ t}$ - (0.0291783) $e^{-(1.228458929\ )t}$. The simulated values of exponential random variable with parameter (a + b), p' and of exponential random variable with parameter (a - b) respectively are presented in table 5 using LCG (5, 1, 16, 1), LCG(9, 1,16, 2) and LCG(13, 1,16, 3) to generate random uniform values to simulate them. They are presented below where exponential simulated values of columns 2 and 4 are added when simulated value u for p' is less than 0.755791591. In the tables Sim means simulation.

Table 5: Life Time of Organ A

Organ A	a + b = 1.228458929	U for p'= 0.755791591	a - b= 0.3215410713	Life time of organ A
Sim 1	2.256964931	0.125	5.206104548	7.463069479
Sim 2	0.798422503	0.1875	2.1557034	2.954125903
Sim 3	0.052536165	0.75	1.789395496	1.841931662
Sim 4	0.23418127	0.8125	3.050401148	0.23418127
Sim 5	0.169024263	0.375	0.20071626	0.369740522
Sim 6	1.692723698	0.4375	4.311406799	6.004130498
Sim 7	0.30501097	0.0625	3.617425311	3.922436281
Sim 8	0.564241233	0.625	6.467110199	7.031351432
Sim 9	0.468362541	0.6875	1.165305097	1.633667638
Sim 10	0.108698296	0.25	8.622813599	8.731511895
Sim 11	0.672939529	0.3125	0.415285649	1.088225179
Sim 12	1.128482466	0.875	2.570989049	1.128482466
Sim13	0.94683736	0.9375	0.894697748	0.94683736
Sim14	0.382596128	0.5	0.645763118	1.028359246
Sim 15	1.362663736	0.5625	1.461721911	2.824385647

4.2.2. Organ B

As in the numerical case let the Cdf of the inter occurrence times of damages to organ B be assumed as F_B (t) = 1-$e^{-\theta t}$ with θ = 0.4. Similarly the generating function of probabilities p_k of k damages required to cause failure of B be ϕ(r) = $\sum_0^4 p_k r^k$ where p_1= 0.3, p_2= 0.4, p_3 = 0.2, p_4 =0.1 and p_n = 0 for n ≥ 5 and p_0 = 0 which gives ϕ(r) = 0.3r +0.4 r^2+ 0.2 r^3 + 0.1 r^4. For organ B, corresponding to the parameter value of θ = 0.4 and for the four inter occurrence times of damages four LCGs are used, namely, LCG(1, 3,16, 4), LCG(5, 3,16, 5), LCG(9, 3,16, 6), and LCG(13, 3,16, 7). For simulating the number of damages to cause failure of B, the LCG(9, 15, 16, 15) is used where the u values in the inequalities **0 < u ≤ 0.3 =** $\boldsymbol{p_1}$; **0.3 < u ≤ 0.7 =** $\boldsymbol{p_1 + p_2}$; **0.7 < u ≤ 0.9 =** $\boldsymbol{p_1 + p_2 + p_3}$ and **0.9 < u ≤ 1 =** $\boldsymbol{p_1 + p_2 + p_3 + p_4}$ indicate, respectively one, two, three and four damages are required for the failure of organ B which are shown by **black**, **green**, **purple** and **red** colors in the column 6 table 6. For example, in simulation 1 corresponding to column 6 u value is u = **0.9375** in **red** color and the exponential simulated times for simulation 1 presented in columns 2, 3, 4 and 5 are added and presented in column 7. Similarly when u value is in **0.7 < u ≤ 0.9,** with **purple** color the exponential times in columns 2, 3 and 4 are added and presented in column 7. Similarly for **green** color the exponential times in columns 2 and 3 are added and presented in column 7 and for **black** color the exponential time in column 2 is presented in column 7. The minimum of life times of A and B are presented in column 8 which is required for Model 1. The maximum of life times of A and B are indicated in column 9 for Model 2. The **red** and **green** colors in the column 8 indicate the minimum of life time is of the life time of A and the life time of B respectively. Similarly the **red** and **green** colors in the column 9 indicate the maximum of life time is of the life time of A and the life time of B respectively.

Table 6: Life Time of Organ B and minimum and maximum of life times of A and B

θ =0.4	First Exp(.4)	Second Exp(.4)	Third Exp(.4)	Fourth Exp(.4)	u for failure	Life time of B	MIN(A, B)	MAX(A,B)
Sim 1	3.465735903	2.907877025	2.452073133	2.066696433	0.9375	10.89238249	7.463069479	10.89238249
Sim 2	2.066696433	0.719205181	1.438410362	0.333828482	0.375	2.785901614	2.785901614	2.954125903
Sim 3	1.175009073	0.161346303	3.465735903	1.438410362	0.3125	1.336355376	1.336355376	1.841931662
Sim 4	0.519098412	0.333828482	2.066696433	1.732867951	0.75	2.919623326	0.23418127	2.919623326
Sim 5	4.184941084	1.438410362	5.198603854	0.936733624	0.6875	5.623351446	0.369740522	5.623351446
Sim6	2.452073133	4.184941084	2.907877025	5.198603854	0.125	2.452073133	2.452073133	6.004130498
Sim 7	1.438410362	5.198603854	4.184941084	0.519098412	0.0625	1.438410362	1.438410362	3.922436281
Sim 8	0.719205181	0.519098412	0.333828482	0.719205181	0.5	1.238303593	1.238303593	7.031351432
Sim 9	0.161346303	3.465735903	6.931471806	0.161346303	0.4375	3.627082206	1.633667638	3.627082206
Sim 10	5.198603854	2.066696433	0.719205181	2.452073133	0.875	7.984505468	7.984505468	8.731511895
Sim 11	2.907877025	2.452073133	0.161346303	6.931471806	0.8125	5.52129646	1.088225179	5.52129646
Sim 12	1.732867951	6.931471806	1.175009073	4.184941084	0.25	1.732867951	1.128482466	1.732867951
Sim 13	0.936733624	1.732867951	0.519098412	1.175009073	0.1875	0.936733624	0.936733624	0.94683736
Sim 14	0.333828482	0.936733624	1.732867951	2.907877025	0.625	1.270562105	1.028359246	1.270562105
Sim 15	6.931471806	1.175009073	0.936733624	3.465735903	0.5625	8.106480879	2.824385647	8.106480879

4.2.3. Time to Prophylactic Treatment

Let the parameter of the exponential time to prophylactic treatment for the two models be with parameters α = .1, .2, .3 and .4 as assumed in the numerical case and let they be simulated for the four α values by LCGs, namely, LCG(13, 15, 16, 1), LCG(5, 15,16, 12), LCG(9, 15, 16, 13) and LCG(13, 15, 16, 14) respectively by generating uniform values. Table 7 presents them in **purple** color.

Table 7: Time to Prophylactic Treatment

	Prophy α=0.1	Prophy α=0.2	Prophy α=0.3	Prophy α=0.4
Sim 1	27.72588722	1.438410362	0.692131216	0.333828482
Sim 2	2.876820725	1.873467247	4.620981204	2.907877025
Sim 3	3.746934494	4.904146265	5.579921445	0.161346303
Sim 4	1.335313926	1.038196824	1.566678764	5.198603854
Sim 5	11.6315081	0.322692606	1.917880483	1.438410362
Sim 6	0.645385211	2.350018146	0.215128404	3.465735903
Sim 7	20.79441542	13.86294361	3.269430843	4.184941084
Sim 8	5.753641449	6.931471806	3.877169366	2.452073133
Sim 9	13.86294361	8.369882168	0.958940242	0.519098412
Sim 10	16.73976434	0.667656963	1.248978165	1.732867951
Sim 11	9.80829253	5.815754049	6.931471806	2.066696433
Sim 12	2.076393648	3.465735903	9.241962407	1.175009073
Sim 13	6.931471806	4.133392866	2.310490602	6.931471806
Sim 14	8.266785732	10.39720771	2.755595244	0.719205181
Sim 15	4.700036292	2.876820725	0.445104642	0.936733624

4.2.4. Time to Hospitalization

The simulated time to hospitalization for treatment T is min { x_A, x_B, x_P} for the Model 1 and for Model 2 the simulated time to hospitalization for treatment T is min {max{x_A, x_B}, x_P} where x_A, x_B, x_P are respectively the simulated life time of organ A, simulated life time of organ B and time to admit the patient for prophylactic treatment respectively. They are exhibited in tables 8 and 9 for Models 1 and 2 respectively. In table 8 the **red** color indicates the failure of organ A; the **green** color indicates the failure of organ B and the

purple color indicates the time at which prophylactic treatment starts. In table 9 the **red** color indicates the time at which both organs A and B are in failed state and the **purple** color indicates the time at which the prophylactic treatment starts.

Table 8: Model 1 Time to Hospitalization T= Min(A,B,P)

Model 1 T	T for α=0.1	T for α=0.2	T for α=0.3	T for α=0.4
Sim 1	7.463069479	1.438410362	0.692131216	0.333828482
Sim 2	2.785901614	1.873467247	2.785901614	2.785901614
Sim 3	1.336355376	1.336355376	1.336355376	0.161346303
Sim 4	0.23418127	0.23418127	0.23418127	0.23418127
Sim 5	0.369740522	0.322692606	0.369740522	0.369740522
Sim 6	0.645385211	2.350018146	0.215128404	2.452073133
Sim 7	1.438410362	1.438410362	1.438410362	1.438410362
Sim 8	1.238303593	1.238303593	1.238303593	1.238303593
Sim 9	1.633667638	1.633667638	0.958940242	0.519098412
Sim 10	7.984505468	0.667656963	1.248978165	1.732867951
Sim 11	1.088225179	1.088225179	1.088225179	1.088225179
Sim 12	1.128482466	1.128482466	1.128482466	1.128482466
Sim 13	0.936733624	0.936733624	0.936733624	0.936733624
Sim 14	1.028359246	1.028359246	1.028359246	0.719205181
Sim 15	2.824385647	2.824385647	0.445104642	0.936733624

Table 9: Model 2 Time to Hospitalization T= Min(Max(A,B), P)

Model 2 T	T for α=0.1	T for α=0.2	T for α=0.3	T for α=0.4
Sim 1	10.89238249	1.438410362	0.692131216	0.333828482
Sim 2	2.876820725	1.873467247	2.954125903	2.907877025
Sim 3	1.841931662	1.841931662	1.841931662	0.161346303
Sim 4	1.335313926	1.038196824	1.566678764	2.919623326
Sim 5	5.623351446	0.322692606	1.917880483	1.438410362
Sim 6	0.645385211	2.350018146	0.215128404	3.465735903
Sim 7	3.922436281	3.922436281	3.269430843	3.922436281
Sim 8	5.753641449	6.931471806	3.877169366	2.452073133
Sim 9	3.627082206	3.627082206	0.958940242	0.519098412
Sim 10	8.731511895	0.667656963	1.248978165	1.732867951
Sim 11	5.52129646	5.52129646	5.52129646	2.066696433
Sim 12	1.732867951	1.732867951	1.732867951	1.175009073
Sim 13	0.94683736	0.94683736	0.94683736	0.94683736
Sim 14	1.270562105	1.270562105	1.270562105	0.719205181
Sim 15	4.700036292	2.876820725	0.445104642	0.936733624

4.2.5.Hospital Treatment Time

The three hospital treatment times H_i, i = 1, 2 and 3 with five, three and two stages of treatments one by one respectively are assumed as follows. One may refer Mark Fackrell [11], for similar cases.

- Treatment H_1 : Emergency Department (ED) → Operation Theatre (OPT) → Intensive Care Unit (ICU) ↔ High Dependency Ward (HDW) → Ward (W) → Discharge
- Treatment H_2 : ED → ICU → W → Discharge and
- Treatment H_3 : ED → W → Discharge.

In the treatment H_1 , loop like treatment, namely, ICU → HDW → ICU is also assumed to study the repetition of treatment at a stage. Let transitions from ED→OPT and OPT→ICU occur following exponential distributions with rate 60. From ICU let the patient move to HDW in an exponential time with rate 60. From HDW let the patient move to W in an exponential time with rate 60 or let the patient move back to ICU in an exponential time with rate 40 so that the total holding time at HDW be exponential with rate 100. Let the holding time at W be exponential with rate 60 for the discharge of the patient. The infinitesimal generator which is a 6 by 6 matrix with states for H_1 is presented below.

$$\begin{bmatrix} States & ED & OPT & ICU & HDW & W & D \\ ED & -60 & 60 & 0 & 0 & 0 & 0 \\ OPT & 0 & -60 & 60 & 0 & 0 & 0 \\ ICU & 0 & 0 & -60 & 60 & 0 & 0 \\ HDW & 0 & 0 & 40 & -100 & 60 & 0 \\ W & 0 & 0 & 0 & 0 & -60 & 60 \\ D & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

. Using (1), (2), (3) and (4), the presented infinitesimal generator with cyclic part can be seen equivalent to acyclic type (27) by considering the parameters of (1), $\lambda_1 = 0$, $\lambda_2 = 60$, $\mu1=60$, $\mu2=40$, a = (12) ($\lambda1+\lambda2+\mu1+\mu2$) = 80 and b = (12) ($\lambda1-\lambda2+\mu1-\mu2)2+4\mu1\mu2$ = 52.9150262213, a + b =132.9150262213 = c (say), a - b = 27.0849737787 = d (say) (26) and replacing the cyclic sub matrix $\begin{bmatrix} -60 & 60 & 0 \\ 40 & -100 & 60 \end{bmatrix}$ by acyclic sub matrix $\begin{bmatrix} -c & c & 0 \\ 0 & -d & d \end{bmatrix}$. The acyclic infinitesimal generator obtained for treatment H_1 is presented below.

$$\begin{bmatrix} \text{States} & \text{ED} & \text{OPT} & \text{ICU} & \text{HDW} & \text{W} & \text{D} \\ \text{ED} & -60 & 60 & 0 & 0 & 0 & 0 \\ \text{OPT} & 0 & -60 & 60 & 0 & 0 & 0 \\ \text{ICU} & 0 & 0 & -c & c & 0 & 0 \\ \text{HDW} & 0 & 0 & 0 & -d & d & 0 \\ \text{W} & 0 & 0 & 0 & 0 & -60 & 60 \\ \text{D} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (27)$$

Here c and d are as given in (26).The random hospitalization times H_2 and H_3 are assumed to have Erlang distributions E(3, 60) and E(2, 60) respectively. The simulated treatment times generated using the LCG method are presented below in tables 10 and 11. In table 10, for the first and second exponential times of hospitalization time of H_1, LCG(13,9,16,4) and LCG(1,11,16, 5) are used; for the acyclic first and second exponentials parts LCG(5,11,16,6) and LCG(9,11,16,7) are used and for the last exponential time LCG(13,11,16,8) is used. In table 11, all the row wise column values of table 10 are added to get H_1 treatment time in column 2. For the Erlang phase 3 hospitalization time of H_2, LCG(1,13,16,9), LCG(5,13,16,10), and LCG(9,13,16,11) are used and simulated values of H_2 are presented in column 3 table 11. For the Erlang phase 2 hospitalization time of H_3, LCG(13, 13,16,12) and LCG(1,15,16,13) are used and simulated values of H_3 are presented in column 4 table 11

.

Table 10: Hospital Treatment Times for Stage Treatments Type H_1

	H1 Erlang (2 , 60)	H1 exp(c)	H1 exp(d)	H1 exp(60)
Sim1	0.042490753	0.007379371	0.030521668	0.011552453
Sim 2	0.009705547	0.004328812	0.017352929	0.027899607
Sim 3	0.051004513	0.005214965	0.042944506	0.034657359
Sim 4	0.074109419	0.012594336	0.025591577	0.019385847
Sim 5	0.014384104	0.003536121	0.061804617	0.004794701
Sim 6	0.016003499	0.001562196	0.036213041	0.013777976
Sim 7	0.035733001	0.002164406	0.102366306	0.016347154
Sim 8	0.008255357	0.006219602	0.051183153	0.009589402
Sim 9	0.0309383	0.001004637	0.002382816	0.006244891
Sim10	0.035733001	0.020859859	0.07677473	0.007833394
Sim11	0.008470414	0.002819045	0.00766622	0.003460656
Sim12	0.021141855	0.015644894	0.013833997	0.023104906
Sim13	0.069314718	0.008751086	0.004930091	0.001075642
Sim14	0.017422796	0.01042993	0.021242928	0.002225523
Sim15	0.02161137	0.000485562	0.010621464	0.046209812

Table 11: Hospital Treatment Times for Types H_1, H_2 and H_3

	H1Treatment time	H2 Erlang (3,60)	H3 Erlang (2,60)
Sim1	0.091944244	0.023667687	0.008255357
Sim 2	0.059286895	0.020883452	0.014384104
Sim 3	0.133821343	0.074109419	0.04090225
Sim 4	0.131681178	0.023922145	0.02161137
Sim 5	0.084519543	0.047285454	0.021141855
Sim 6	0.067556712	0.066232679	0.0309383
Sim 7	0.156610868	0.062556966	0.016003499
Sim 8	0.075247515	0.120319231	0.044246761
Sim 9	0.040570644	0.048435335	0.042490753
Sim10	0.141200984	0.039408714	0.069314718
Sim11	0.022416335	0.022846503	0.035733001
Sim12	0.073725653	0.035389346	0.035733001
Sim13	0.084071537	0.067351667	0.049670468
Sim14	0.051321177	0.015459746	0.017422796
Sim15	0.078928208	0.016609627	0.008470414

For Model 1, H_j for j = 1, 2, 3 are assumed respectively as treatment times for organ A failure, organ B failure and for prophylactic treatment. For Model 2, let H_4 be same as H_1 and H_5 be same as H_2 respectively as treatment times for organs A and B failure and for prophylactic treatment. In table 12, Model 1 treatment times are listed using table 8 and table 11. For example in table 8 simulation 1 corresponding to α =0.1, organ A fails at time **7.463069479** and the patient is admitted for treatment for organ A in Model 1. In table 11 simulation 1 for α = 0.1, the required treatment time H_1 is **0.091944244**. This is listed in table 12 simulation 1 for α = 0.1. Similarly various treatment times for different values of α for 15 simulations are listed in table 12 with **red**, **green** and **purple** colors to indicate organ A, organ B and prophylactic treatments.

Table 12: The Treatment Times H for Model 1

Model 1	H for α=0.1	H for α=0.2	H for α=0.3	H for α=0.4
Sim 1	0.091944244	0.008255357	0.008255357	0.008255357
Sim 2	0.020883452	0.014384104	0.020883452	0.020883452
Sim 3	0.074109419	0.074109419	0.074109419	0.040902250
Sim 4	0.131681178	0.131681178	0.131681178	0.131681178
Sim 5	0.084519543	0.021141855	0.084519543	0.084519543
Sim 6	0.030938300	0.030938300	0.030938300	0.066232679
Sim 7	0.062556966	0.062556966	0.062556966	0.062556966
Sim 8	0.120319231	0.120319231	0.120319231	0.120319231
Sim 9	0.040570644	0.040570644	0.042490753	0.042490753
Sim 10	0.039408714	0.069314718	0.069314718	0.069314718
Sim 11	0.022416335	0.022416335	0.022416335	0.022416335
Sim 12	0.073725653	0.073725653	0.073725653	0.073725653
Sim 13	0.067351667	0.067351667	0.067351667	0.067351667
Sim 14	0.051321177	0.051321177	0.051321177	0.017422796
Sim 15	0.078928208	0.078928208	0.008470414	0.008470414

In Model 2, H_1 is assumed as the treatment time when both organs A and B are in failed state and H_2 is assumed as the treatment time for prophylactic treatment. Table 9 and table11 values are matched for **red** color of table 9 with **red** color values of table 11 and **purple** color values of table 9 are matched with **green** color H_2values of table 11. They are presented in table 13 with **red** and **purple** colors to indicate the treatment times of both the organs and of the prophylactic treatment as the case may be.

Table 13: The Treatment Times H for Model 2

Model 2	H for α=0.1	H for α=0.2	H for α=0.3	H for α=0.4
Sim 1	0.091944244	0.023667687	0.023667687	0.023667687
Sim 2	0.020883452	0.020883452	0.059286895	0.020883452
Sim 3	0.133821343	0.133821343	0.133821343	0.074109419
Sim 4	0.023922145	0.023922145	0.023922145	0.131681178
Sim 5	0.084519543	0.047285454	0.047285454	0.047285454
Sim 6	0.066232679	0.066232679	0.066232679	0.066232679
Sim 7	0.156610868	0.156610868	0.062556966	0.156610868
Sim 8	0.120319231	0.120319231	0.120319231	0.120319231
Sim 9	0.040570644	0.040570644	0.048435335	0.048435335
Sim 10	0.141200984	0.039408714	0.039408714	0.039408714
Sim 11	0.022416335	0.022416335	0.022416335	0.022846503
Sim 12	0.073725653	0.073725653	0.073725653	0.035389346
Sim 13	0.084071537	0.084071537	0.084071537	0.084071537
Sim 14	0.051321177	0.051321177	0.051321177	0.015459746
Sim 15	0.016609627	0.016609627	0.016609627	0.016609627

The average values of T and H for Models 1 and 2 simulated in tables (8), (9), (12) and (13) are presented in table (14) for values of α, the parameter of exponential random time to admit the patient for prophylactic treatment.

Table 14: Averages of T and H for Models 1 and 2 for Values of α

Estimates	α = 0.1	α = 0.2	α = 0.3	α = 0.4
Average T Model 1	2.142380446	1.302623315	1.009665061	1.071675448
Average T Model 2	3.961430498	2.42411658	1.897270904	1.713185257
Average H Model 1	0.066044982	0.057800988	0.057890278	0.055769533
Average H Model 2	0.075211297	0.061391103	0.058205385	0.060200718

The following figures 3 and 4 present the graphical representation of table 14. Figures 3 and 4 present the effect of α on the averages of simulated values of T and H for the two Models. The variations of average of simulated values are comparatively high when α is small and increases when α increases.
The results obtained for θ = 0.4 for various values of α =0.1, 0.2, 0.3 and 0.4 in table 1 and 2 for E(T) for the Models 1 and 2 corresponding to numerical studies are comparable with the simulated averages of T for the two models since the parameters used to study them are same. It may be noted that for hospital treatments different assumptions are made in numerical and simulation studies. Only 15 simulated values are generated here in this paper for various random values. The similarities of the structures are exhibited in figure 5 among E(T) and average SIM(T) of the two models. Even for fifteen simulations they almost look alike. The usefulness of the approach is thus established.

Table 15:Comparison of E(T) and Simulated averages of T for Models 1 and 2

	E(T) Model 1	SIM(T) Model 1	E(T) Model 2	SIM(T) Model 2
α=0.1	1.757648786	2.142380446	4.2594655	3.961430498
α=0.2	1.505147075	1.302623315	3.157211032	2.42411658
α=0.3	1.314691533	1.009665061	2.483484666	1.897270904
α=0.4	1.166199562	1.071675448	2.034930757	1.713185257

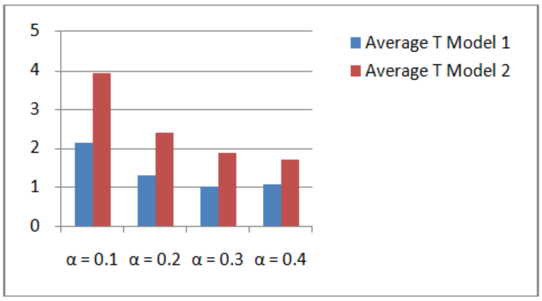

Figure 3: T Averages Models 1 and 2 for Values of α.

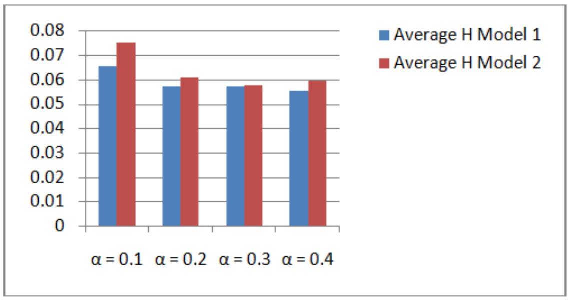

Figure 4: H Averages Models 1 and 2 for Values of α

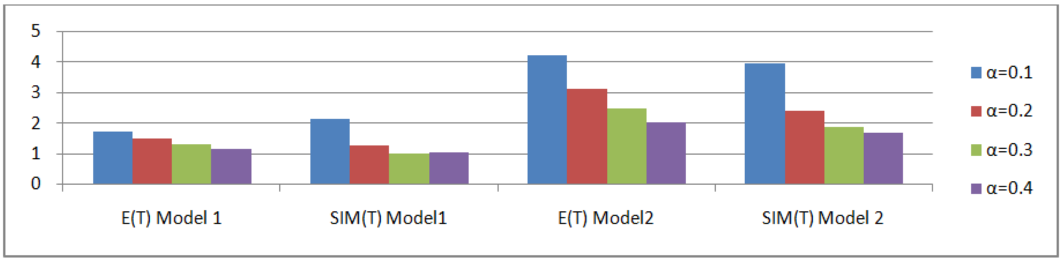

Figure 5: Comparison E(T) values with Average Simulated T Values of Models 1 and 2 for Values of α.

V. Conclusion

Diabetic models with two organs are considered. The organ A of the patient has two phase PH life distribution and his organ B is exposed to a damage process. The time to prophylactic treatment has exponential distribution. In Model 1, the patient is sent for treatment when an organ fails or on the completion of time to admit him for prophylactic treatment whichever occurs first. In Model 2, the patient is sent for treatment when both organs are in failed state or on the completion of time to admit him for prophylactic treatment whichever occurs first. The hospital treatment times for the organs A and B and for prophylactic treatments have distinct distributions respectively for Model 1. In Model 2, organs A and B are treated together or prophylactic treatment is provided as per the case. The joint Laplace Stieltjes transform of the joint distribution of time to hospitalization and hospitalization times have been obtained. Individual distributions are also presented. The expected time to hospitalization and the expected hospitalization times are derived. Numerical studies are presented. Simulation study has been taken up in this area by considering a set of parameter values for two phase life distribution of the organ A; by considering parameter values of damage process causing failure of organ B; by considering different parameter values of exponential time to prophylactic treatment and by considering various Erlang and cyclic phase type distributions for hospitalization times for the two Models. Cyclic type treatment in the hospital has been identified as acyclic type using equality of distributions so that simulation can be performed easily. All results are tabulated and graphical presentation where ever required are presented. Since not much of simulation analysis are available in literature for diabetic models, this study opens up a real life like study in this area. Various other distributions if used for simulation studies also may produce more interesting results.

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