

Dynamic logistics model for projecting water consumption in México city

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Abstract: This paper proposes a dynamic model based on a nonlinear logistic differential equation for a fixed approximate quantity of potable water, intended to supply Mexico City under various scenarios for the year 2026. The mathematical formulation describes the temporal evolution of the water volume, considering constraints associated with the system's maximum capacity. Three water consumption scenarios were analyzed: 100, 300, and 380 liters per capita per day (L/person/day), based on WHO and SACMEX regulatory criteria. The analytical solution and its numerical approximations were determined using the fourth-order Runge-Kutta (RK4) method. Comparing both results, they show a nonlinear increase in the system's saturation curve as per capita consumption increases, indicating a high sensitivity to increased demand. The projections obtained suggest scenarios of increasing water pressure in the medium term, highlighting the need to implement strategies for situations where the available resource is compromised in the medium term. This underscores the urgent need to implement strategies aimed at reducing consumption. The strategy focuses on mitigating water demand and promoting the efficient use of the resource. These strategies are geared towards reducing consumption, modernizing hydraulic infrastructure, and utilizing the resource efficiently.

Background: Mexico City (CDMX) faces a structural problem related to the availability and sustainable management of water resources. The limited capacity of water sources, coupled with the deterioration and lag in the hydraulic infrastructure, represents a significant risk to water security for CDMX. In the absence of strategic investments aimed at modernizing, expanding, and improving the efficiency of water collection, treatment, and distribution systems, a progressive increase in water stress is expected. Additionally, demographic and urban growth will continue to increase the demand for drinking water, which could exceed the operational capacity of the supply and exacerbate shortages in various areas of the city.

Materials and Methods: A logistic model was used to project the evolution of drinking water supply and evaluate the behavior of water shortages toward 2026. The Mathematical formulation considered a maximum supply capacity and was solved using analytical and numerical methods. The study compiled three per capita consumption scenarios to analyze their influence on the future availability of water resources.

Results: The logistic model showed that the scenario of 100 L/capita/day, recommended by the OMS, significantly prolongs the availability of water resources compared to the scenarios of 100 and 380 L/capita/day. The results demonstrate that higher levels of per capita consumption accelerate water shortages, reducing the sustainability of the drinking water supply.

Conclusion The model simulation shows the need to restructure the drinking water reduction system, promoting consumption patterns aligned with OMS recommendations. Likewise, sustained investment in water infrastructure is required, including the modernization of distribution networks and the expansion of treatment and purification plants, in order to guarantee the sustainability and security of the population's water supply.

Key Word: Logistic model, water consumption, nonlinear dynamics, fourth-order Runge-Kutta, water sustainability

I. Introduction

Water availability in large urban centers is one of the most critical environmental challenges for contemporary public management. In Mexico City, this problem demands the urgent adoption of preservation strategies, given that rapid population growth exerts significant anthropogenic pressure on the distribution infrastructure (National Institute of Statistics and Geography [INEGI], 2020). Globally, water security faces systematic risks exacerbated by climate change and deficiencies in urban planning (Muñoz Padilla, 2020). Consequently, macroeconomic and sectoral indicators warn that global consumption patterns are moving toward unsustainability. Similarly, international indicators show that global patterns are approaching unsustainable conditions (Worldometer, n.d.). Water sustainability in Mexico City demands optimizing distribution systems and mitigating losses caused by leaks and inadequate infrastructure (Mexico City Congress, 2022; Rosales, 2024; Rosales, 2024). This scenario of urban water supply losses is exacerbated and reaches a critical state due to dependence on external sources (Oviedo and Sarmiento-Pastor, 2024). Within this context of vulnerability, the public debate has incorporated the concept of "day zero," understood as the point of collapse at which the water system would be unable to meet the minimum demand of the population (Spinetto, 2024). This problem is intensified by the recurring decline in storage levels of the strategic dams of the Cutzamala System (Montoya, 2026). Therefore, it is essential to develop dynamic mathematical models of consumption and evaluate urban water sustainability scenarios.

II. Material And Methods

The evolution of water volume was modeled using nonlinear logistic ordinary differential equations (ODEs), employing a hybrid approach that integrates the analytical solution and a fourth-order Runge-Kutta (RK4) algorithm implemented in C++ (Abbas and Ali, 2024). The model is evaluated under three scenarios of daily per capita consumption: (100, 300, and 380 L/person/day), based on the WHO recommendation (2019s), urban studies (López-Pérez & Rodríguez-Vázquez, 2020), and SACMEX averages (2024).

Mathematical Model

The model used is based on the Verhulst logistic equation, widely employed to describe dynamic processes with limited capacity (Bacaer, 2011). This formulation allows for the representation of systems subject to constraints associated with resource availability (Koné et al., 2012). The logistic differential equation used in this work is expressed as:

$$\frac{dA(t)}{dt} = rA(t) \left(1 - \frac{A(t)}{K}\right) \quad (1)$$

Where:

$A(t)$ is the cumulative volume of water demand or consumption pressure exerted on the system at time t .

- t is the time in days.
- r is the effective consumption rate of the system.
- K is the maximum sustainable supply capacity.
- A_0 is the initial volume given by each scenario.

The initial condition of the model is given by:

$$A(0) = A_0 \quad (2)$$

Where A_0 represents the initial volume associated with each daily consumption scenario multiplied by the number of people in Mexico City.

.2 Fourth-Order Runge-Kutta Method

(Abbas & Ali, 2024) demonstrated that the RK4 method exhibits high accuracy and numerical stability for nonlinear ordinary differential equations, reporting an associated global error of order $O(h^4)$. The iterative formulation used is described by:

$$A_{n+1} = A_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (3)$$

Where:

$$k_1 = h f(t_n, A_n) \quad (4)$$

$$k_2 = h f\left(t_n + \frac{h}{2}, A_n + \frac{k_1}{2}\right) \quad (5)$$

$$k_3 = h f\left(t_n + \frac{h}{2}, A_n + \frac{k_2}{2}\right) \quad (6)$$

$$k_4 = h f(t_n + h, A_n + k_3) \quad (7)$$

Analytical Solution

Considering the initial condition of the system, the analytical solution of the differential equation was derived using the method of separation of variables complemented with a convenient substitution:

$$A(t) = \frac{K}{1 + \left(\frac{K - A_0}{A_0}\right) e^{-rt}} \quad (8)$$

The expression models the temporal behavior under capacity constraints, where the effective consumption rate r governs the rate of approach to the critical threshold. The analytical result provides the formal validation criterion required to test the numerical approximation derived from the RK4 scheme.

Model parameters

The system calibration was based on a base population (P) of 9,209,944 inhabitants (NEGI, 2020) and a maximum water carrying capacity (K) of 1×10^{11} million m^3 . Per capita consumption (C_i) was evaluated at three discrete levels ($C_1 = 100$, $C_2 = 300$, $C_3 = 380$ L/hab/day) to establish the initial condition (A_0). Finally, the consumption (r_i) was dynamically adjusted according to water demand using the scalar equation:

$$r_i = \alpha \left(\frac{PC_i}{K}\right) \quad (9)$$

Where $\alpha = 5.89038 \times 10^{-11}$ represents the dimensionless scaling factor of the structural resilience of the distribution network. From this relationship, the effective saturation rates were calculated: $r_1 = 0.05425$, $r_2 = 0.16275$ and $r_3 = 0.20615$. This forecasting scheme is for the year 2026. The scheme formalizes that the increase in per capita consumption non-linearly accelerates the convergence of the system towards its critical collapse limit.

III. Result

Scenario of 100 liters per inhabitant per day consumption

Figure 1 shows the convergence between the exact model solution and the numerical solution obtained using the (RK4) method for a water supply of 100 L/inhabitant/day. Under this condition, the model projects a water saturation time of 2.679 days.

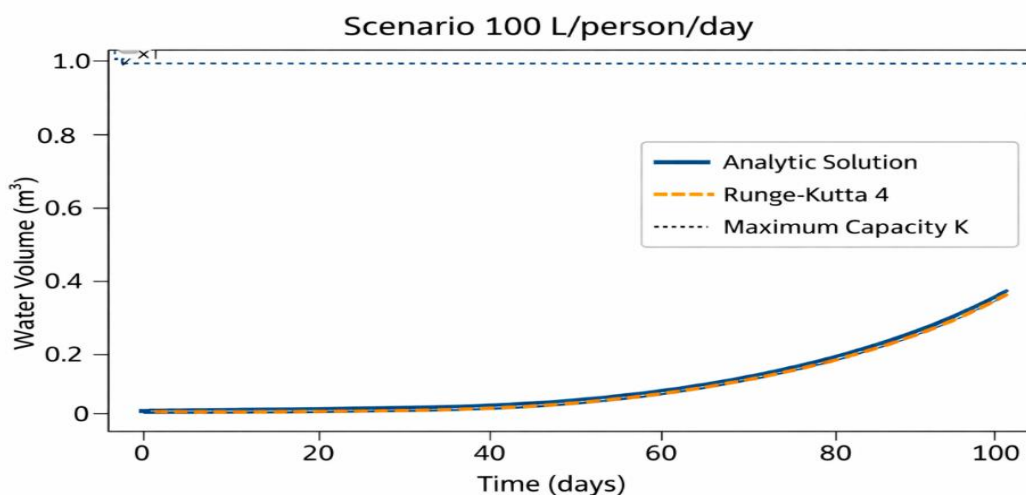


Figure 1. The comparison of the analytical and numerical solutions shows a high degree of agreement and adequate accuracy of the simulation method implemented in the simulation of the system's dynamic behavior.

Scenario of 300 liters per capita per day consumption

When demand increases to an intermediate level of 300 liters per capita per day (Figure 2), an acceleration in the saturation process of the urban water system and a significant reduction in the available resource are observed. In this scenario, the estimated time for depletion of the operating margin decreases drastically to 540.9 days, demonstrating the immediate impact of growth in dense residential demand.

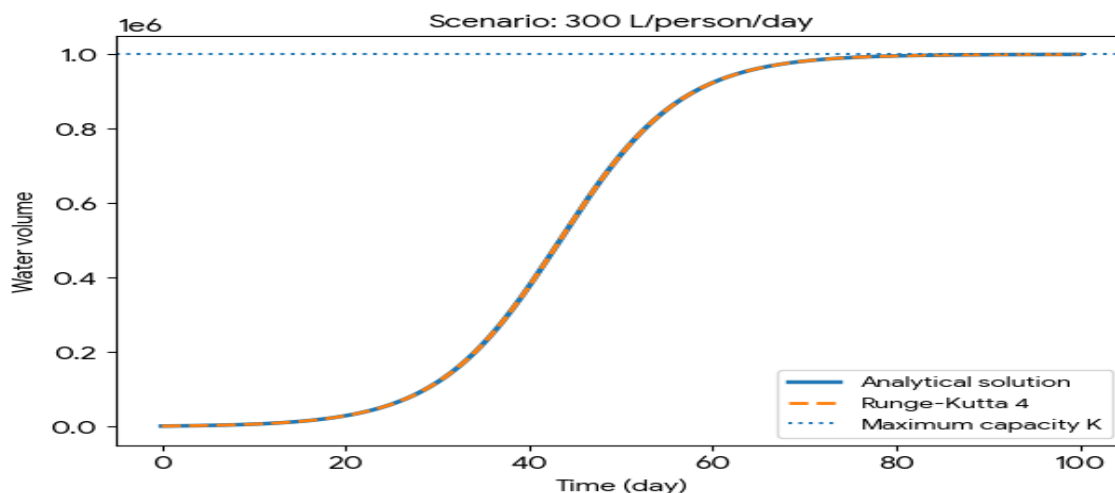
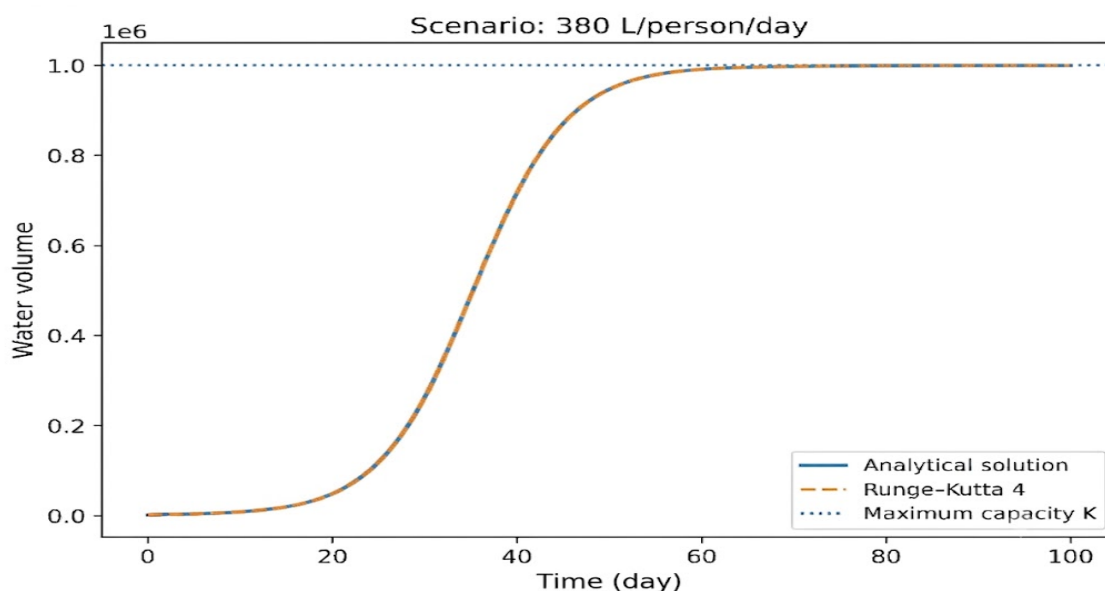


Figure 2. Comparison between the analytical solution and the one approximated by (RK4) for a consumption of 300 L/hab/day

Consumption Scenario of 380 Liters per Inhabitant per Day

Figure 3 shows that a water supply of 380 L/inhabitant/day non-linearly exacerbates the pressure on the system. The model's dynamic response reveals a critical reduction in the stability horizon, shortening the estimated saturation time to only 429.35 days due to the rapid approach to the collapse limit of the hybrid infrastructure.



IV. Discussion

The systematic comparison between the analytical and numerical solutions showed a virtually exact match, validating the accuracy of the RK4 method for simulating nonlinear urban water dynamics. The results indicate a critical reduction in the resource's availability time as per capita consumption increases.

The model demonstrates that the recommended baseline scenario of 100 L/capita/day (WHO, 2019) allows for a longer period of system stabilization. In contrast, the scenario of 380 L/capita/day, representative of the current conditions reported by SACMEX (2024), considerably increases the pressure on urban water

infrastructure. These findings are consistent with previous studies related to residential consumption patterns and the unsustainability of water resources in high-density metropolitan areas (López-Pérez and Rodríguez, 2020; Rondinel-Oviedo and Sarmiento-Pastor, 2020).

V. Conclusion

The agreement between the analytical and numerical approximation solutions using RK4 validates the model's accuracy. The results show that increased per capita consumption accelerates system saturation, consistent with previous urban studies (López-Pérez and Rodríguez; Rondinel-Oviedo and Sarmiento-Pastor, 2020).

The findings demonstrate that reducing residential consumption, modernizing water infrastructure, and controlling leaks are priority actions for strengthening the sustainability and resilience of the water supply system (Rosales, 2024). These actions are essential for improving water sustainability and the resilience of the supply system in the medium and long term.

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