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Electrohydrodynamic Stability of Poorly Conducting Homogeneous Fluid Saturated Porous Layer in the Presence of Combined Transverse Electric and Magnetic Fields

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Abstract:- *The study of electrohydrodynamic (EHD) stability in a poorly conducting fluid-saturated porous layer under the influence of combined transverse electric and magnetic fields is of significant interest in engineering, geophysics, and materials science. This research examines the interplay between fluid flow, electric fields, and magnetic fields within a porous medium, employing the Darcy-Lapwood-Brinkmann model to analyze stability conditions. By incorporating the effects of induced and applied electric fields, the study explores the emergence of self-generating electric forces that contribute to sensing and actuation in smart materials. The formulation of stability equations and the derivation of a modified Orr-Sommerfeld equation offer new insights into the fundamental mechanisms governing electromagnetohydrodynamic stability (EMHDS).*

The findings have direct implications for industrial applications, including the design of smart materials, the development of more efficient filtration systems, and the improvement of fluid transport in petroleum reservoirs. Additionally, the study sheds light on the impact of electric and magnetic fields in controlling flow stability, a concept applicable in nuclear reactor cooling, chemical processing, and biomechanical systems. By establishing sufficient conditions for stability, this work enhances the understanding of fluid dynamics in porous media, paving the way for advanced engineering applications and optimization of electrohydrodynamic processes.

Keywords: *Electrohydrodynamic Stability, Porous Media, Magnetic Fields, Smart Materials, Orr-Sommerfeld Equation*

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I. Introduction

Flow through and past porous media has attracted considerable interest in recent years because of their natural occurrence and also of its potential applications in soil physics, geophysics, filtration of solids from liquids, chemical engineering and in biomechanics. Much interest has also been evinced in wetting and dewetting of solids by liquids, which involve the movement of fluid over the solid particles. The stability of this movement is of immense use in many industrial problems particularly in the petroleum industry. This is because the behavior of fluid in petroleum reservoir rock depends to a large extent on the property of the rock and the stability of the fluid that yields new or additional information on the characteristic of the rock that would contribute to a better understanding of petroleum reservoir performance. Furthermore, the resistance offered by the solid particles of the porous media to the fluid may reduce its drag ensuring laminar flow. It is known that in the stability of ordinary parallel flow it is necessary to use extremely thin fluid to ensure laminar flow but in porous media due to large resistance the depth of the flow can be greatly increased to maintain laminar flow.

The stability of parallel flow in ordinary fluid mechanics that is in the absence of porous media is a classical problem associated with the name stability of Poiseuille flow. Copius literature on the stability of Poiseuille flow and its extension is available. [see C.C. Lin (1957), Stuart (1960)]. The literature on the corresponding stability of parallel flow in porous media is very sparse in spite of its importance in problems cited above. To our knowledge only the work of Rudraiah (1980) on stability of parallel flow through porous media using Brinkmann Model is available.

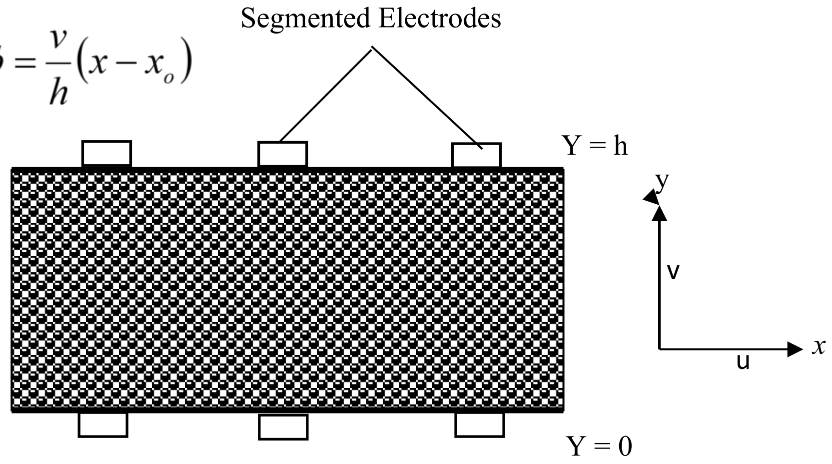
Recently, smart materials have attracted much attention because of its importance in many engineering, medicine and other fields for effective performance of corresponding models. Smart materials consist of elements, which serve as sensors and actuators. Among various materials as sensors and actuators in smart material, piezoelectric materials have attracted considerable interest because of their effective coupling between elastic and electric fields. [see Ishihara and Noda (2003)] . Recently Rudraiah has proposed the use of poorly conducting material like Nickel-Titanium as a shape memory alloys to synthesize smart materials instead of piezoelectric materials. The physical mechanism that provides sensing and actuating properties in a poorly conducting alloy is as follows. In a poorly conducting alloy the electrical conductivity increases with depth which may be due to either variation of temperature or due to variation of concentration of species. This variation of electrical conductivity induces charges which in turn produce and electric filed called self-generating electric field which in turn produces the true current. There may be an applied electric field generated due to segmented electrodes at the boundaries. The total electric field namely, the combination of induced and applied electric field produces considerable true current which acts as sensing. The total electric field along with the distribution of charges produces and force . This force acts as actuators. The literature on smart materials made up of poorly conducting alloy is very sparse [see Rudraiah 2003]. The manufacturing process of smart materials using poorly conducting fluid leads to mushy layer which is a mixture of dendrites of nano- structure in a fluid. This can be regarded as a fluid saturated porous layer [see Rudraiah 2003, 2003a]. The study of stability of such mushy layer in a poorly conducting parallel flow in the presence of an electric and magnetic fields is called electromagnetohydrodynamic stability of parallel flow through porous media which is important not only to manufacture smart materials free from impurities but also in cleaning of electronic equipments, design of nuclear reactors, fix bed chemical reactors and so on. In spite of these applications this problem has not been given much attention to our knowledge and the study of this is the main objective of this chapter.

II. Objectives

Basic Equations and the Boundary Conditions

The physical configuration considered here is a homogeneous, incompressible viscous fluid saturated porous layer in the presence of an electric field and magnetic field. The porous media is sparsely packed. In this case we use Darcy-Lapwood-Brinkmann equation governing a poorly conducting fluid in the presence of electric and magnetic fields. For this physical configuration, the required basic equations i.e. conservation of mass and electric charges, the conservation of momentum, considering the combined effect of electric and magnetic fields in a porous medium assuming Brinkmann viscosity, the viscosity of fluid, is given by

$$\phi = \frac{v}{h}\left(x - x_o\right)$$



$$\varphi = \frac{v}{h}x$$

Figure 1. Physical Configuration

$$\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_e}{\rho_0}\vec{E} + \frac{\mu}{\rho_0}\nabla^2\vec{q} + \frac{1}{\rho_0}\vec{J}\times\vec{B} - \frac{\mu}{k}\vec{q} \qquad (1)$$

Equation (2.1) using $\vec{J} = \sigma\vec{E} + \mu_m\sigma\,\vec{q}\times\vec{H}$ can be written as

$$\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_e}{\rho_0}\vec{E} + \frac{\mu}{\rho_0}\nabla^2\vec{q} + \frac{\mu_m\sigma}{\rho_0}\vec{E}\times\vec{H} + \frac{\mu_m^2\sigma}{\rho_0}\vec{q}\times\vec{H}\times\vec{H} - \frac{\mu}{k}\vec{q} \qquad (2)$$

We assume two-dimensional flow Under these approximations the basic equations along with Maxwell`s equation, for a poorly electrically conducting, viscous, incompressible two-dimensional homogeneous fluid saturated porous layer in the presence of electric and magnetic fields. After making them dimensionless, Conservation of Momentum takes the form

$$\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + W_1 \rho_e E_x + \nabla^2 u - W_3 \sigma E_y - M^2 u \sigma - A\sigma_p^2 u \tag{3}$$

$$3\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + W_1\left(\rho_e E_y\right) + \nabla^2 v - W_3 \sigma E_x - M^2 v \sigma - A\sigma_p^2 v \tag{4}$$

where $W_1 = \frac{\varepsilon_0 v^2}{\rho_0 U^2 h^2}$ is the Electric Number which has the same meaning as explained earlier.

$M^2 = \frac{\mu_m^2 \sigma h^2 H_0^2}{\rho_0 U^2 h^2}$ is the Hartmann number which is a measure of the ratio of Lorentz force to Viscous force.

$W_3 = \frac{v}{\mu H_0 U h}$ is the ratio of energies. $A.\sigma_p^2 = \frac{\mu}{\rho_0 \upsilon}.\frac{h^2}{k}$ is the porous parameter.

The boundary conditions, in dimensionless form, are

$(u, v) = 0 \quad at \quad y = 0$ and 1

along with

$$\varphi = x \qquad at \quad y = 0$$
$$\varphi = x - x_0 \quad at \quad y = 1$$

1. Basic State

Substituting the basic state into (3), (4) and to Conservation of electric charge equation and then simplifying we get,

$$\frac{d^2 u_b}{dy^2} - M^2 u_b \sigma_b = \frac{\partial p_b}{\partial x} + W_1 \rho_{e_b} E_{bx} - W_3 \sigma_b E_{by} - A\sigma_p^2 u \tag{5}$$

$$\frac{\partial^2 \phi_b}{\partial y^2} + \alpha \frac{\partial \phi_b}{\partial y} = 0 \tag{6}$$

where $\rho_{eb} = -\frac{\partial^2 \phi_b}{\partial y^2}, W_1 = \frac{\varepsilon_0 v^2}{\rho_0 U^2 h^2}$ is the electric number which physically represents the ratio of electric energy to kinetic energy. Solution of (6) using the boundary conditions is,

$$\phi_b = x - \frac{x_0\left(1 - e^{-\alpha y}\right)}{\left(1 - e^{-\alpha}\right)} \tag{7}$$

Then the solution of (5) using (7) and the no-slip boundary conditions is

$$u_b = -\frac{P}{M^2 - A\sigma_p^2}y + \frac{x_0 W_1}{M^2 - A\sigma_p^2} + Ae^{My}\left[1 + \frac{y^2}{4}\left(M - \frac{1}{y}\right)\right] + Be^{-My}\left[1 - \frac{y}{4}\left(M - \frac{1}{y}\right)\right] \tag{8}$$

where $P = \frac{\partial p_b}{\partial x}$

$$A = \frac{4e^{-M}}{M+3}\left[\frac{e^{-M}(5-M)}{4}\left\{\frac{-4x_0W_1\{e^M(M+3)-1\}+P}{M^2\{e^M(M+3)+e^{-M}(5-M)\}}\right\} - \frac{x_0W_1}{[M^2 - A\sigma_p^2]} + \frac{P}{[M^2 - A\sigma_p^2]}\right]$$

$$B = \frac{-4x_0W_1[e^M(M+3)-1]+P}{[M^2 - A\sigma_p^2]\{e^M(M+3)+e^{-M}(5-M)\}}.$$

2. Stability Equations

To study the linear stability of the basic state we superimpose infinitesimal symmetrical disturbances substituting these into (3) and (4) and linearizing them by neglecting the product of higher order terms in perturbed quantities and for simplicity neglecting the primes we get,

$$\left[\frac{\partial u}{\partial t} + u_b\frac{\partial u}{\partial x} + v\frac{\partial u_b}{\partial x}\right] = -\frac{\partial p}{\partial x} + \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + W_1\rho_{e_b}E_x + W_3E_y - M^2u - A\sigma_p^2 u \tag{9}$$

$$\left[\frac{\partial v}{\partial t} + u_b\frac{\partial v}{\partial x}\right] = -\frac{\partial p}{\partial y} + \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + W_1\left(\rho_{e_b}E_y + \rho_e E_{b_y}\right) - W_3E_x + M^2v - A\sigma_p^2 v \tag{10}$$

Eliminating the pressure between the eqns. (9) and (10) and using the stream function and using the normal mode solution of the form

$$f'(x,y,t) = f(y)e^{il(x-Ct)}$$

where *l* is the wave number and *C = n/l*, the wave velocity, *n* is the frequency, and after simplification we get the stability equation,

$$\begin{aligned}(D^2-l^2)^2\psi - il(u_b - C)(D^2-l^2)\psi + ilD^2u_b\psi + ilW_1x_0(D^2-l^2)\phi \\ -W_3(D^2-l^2)\phi - M^2(D^2-l^2)\psi + A\sigma_p^2(D^2-l^2)\psi = 0\end{aligned} \tag{11}$$

Similarly, the equation of continuity of charges takes the form

$$il(u_b - C)(D^2-l^2)\phi = -\alpha^2 x_0\psi \tag{12}$$

Substituting $(D^2-l^2)\phi$ from (12) into (11) and after simplification we get

$$\left(D^2-l^2\right)^2\psi-il\left(u_b-C\right)\left(D^2-l^2\right)\psi+ilD^2u_b\psi-il\frac{W_1x_0^2\alpha^2}{\left(u_b-C\right)}\psi+\frac{W_3x_0\alpha^2\psi}{\left(u_b-C\right)}$$
$$-M^2\left(D^2-l^2\right)\psi+A\sigma_p^2\left(D^2-l^2\right)\psi=0 \tag{13}$$

Equation (13) is a modified form of Orr-Sommerfeld Equation, modified in the sense of incorporating the contribution from the electric force, $\rho_e\,\vec{E}$ and the effect of magnetic field.

3. Stability Analysis

A sufficient condition for stability is established in the following theorem:

Theorem:

A sufficient condition for Electromagnetohydrodynamic stability [EMHDS] of viscous homogeneous poorly conducting fluid saturated porous medium in the presence of electric field and magnetic field is

$$A\le\frac{\left[I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2\right]-lqI_0I_1}{I^2}$$

together with

$$lq<\frac{\left[I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2\right]}{I_0I_1}$$

Proof:

To study the stability of a poorly conducting parallel flow we use the complex conjugate method. Consider the modified Orr-Sommerfeld eqn.

$$\left(D^2-l^2\right)^2\psi=il\left[\left(u_b-C\right)\left(D^2-l^2\right)\psi-D^2u_b\psi+\frac{W_1x_0^2\alpha^2}{\left(u_b-C\right)}\psi\right]-\frac{W_3x_0\alpha^2\psi}{\left(u_b-C\right)}$$
$$+M^2\left(D^2-l^2\right)\psi-A\sigma_p^2\left(D^2-l^2\right)\psi=0 \tag{14}$$

Multiplying (14) by ψ^*, the complex conjugate of ψ and integrating from 0 to 1 with respect to *y* using the boundary condition we get,

$$I_2^2+\left(2l^2+M^2+A\sigma_p^2\right)I_1^2+\left(l^4-M^2l^2-A\sigma_p^2l^2\right)I_0^2=-ilQ+ilC\left[I_1^2+l^2I_0^2\right] \tag{15}$$

$$I_n^2=\int_0^1\left|D^n\psi\right|^2dy\qquad\left(n=0,1,2\right)$$

where

$$Q=\int_0^1\left\{u_b\left|D\psi\right|^2+\left(l^2u_b+u_b''\right)\left|\psi\right|^2\right\}dy+\int_0^1u_b'\left(D\psi\right)\psi^*dy-W_1x_0^2\alpha^2\int_0^1\frac{\left(u_b-C_r\right)+iC_i}{u_b-C}\left|\psi\right|^2dy$$
$$-W_3x_0\alpha^2\int_0^1\frac{\left(u_b-C_r\right)+iC_i}{u_b-C}\left|\psi\right|^2dy \tag{16}$$

From this equation we have $Re(Q)$ and $Im(Q)$ of the form

$$Re(Q) = \int_0^1 \left\{ u_b |D\psi|^2 + \left(l^2 u_b + u_b'' \right) |\psi|^2 \right\} dy - \int_0^1 \frac{(W_1 x_0 + W_3) x_0 \alpha^2 (u_b - C_r)}{|u_b - C|^2} |\psi|^2 \, dy \tag{17}$$

and

$$Im(Q) = \frac{1}{2} i \int_0^1 u_b' \left\{ \psi (D\psi^*) - \psi^* (D\psi) \right\} dy + \int_0^1 \frac{(W_1 x_0 + W_3) x_0 \alpha^2 C_i}{|u_b - C|^2} \, |\psi|^2 \, dy \tag{18}$$

Equating the real parts and imaginary parts of the eqn. (15) using eqns. (18) and (19) and $C = C_r + iC_i$ we get, from the real parts,

$$C_i = \frac{l \int_0^1 u_b' \left\{ \psi (D\psi^*) - \psi^* (D\psi) \right\} dy - \left[I_2^2 + (2l^2 + M^2 + A\sigma_p^2) I_1^2 + (l^4 - M^2 l^2 - A\sigma_p^2 l^2) I_0^2 \right]}{l \left[I_1^2 + l^2 I_0^2 \right] + A_1 I^2} \tag{19}$$

where $A_1 = (W_1 x_0 + W_3) x_0 \alpha^2$ and $I^2 = \int_0^1 \frac{|\psi|^2}{|u_b - C|^2} dy$

This eqn. (19) can be rewritten in the form

$$C_i = \frac{l |Im(Q)| - \left[I_2^2 + (2l^2 + M^2 + A\sigma_p^2) I_1^2 + (l^4 - M^2 l^2 - A\sigma_p^2 l^2) I_0^2 \right]}{l \left[I_1^2 + l^2 I_0^2 \right] + A_1 I^2} \tag{20}$$

By equating the imaginary parts we get,

$$C_r = \frac{\int_0^1 \left\{ u_b |D\psi|^2 + \left(\left(l^2 - \frac{(W_1 x_0 + W_3) x_0 \alpha^2}{|u_b - C|^2} \right) u_b + u_b'' \right) |\psi|^2 \right\} dy}{I_1^2 + l^2 I_0^2 - A_1 I^2} \tag{21}$$

This eqn. can be rewritten in the form

$$C_r = \frac{Re(Q)}{I_1^2 + l^2 I_0^2 - A_1 I^2} \tag{22}$$

Eqn. (20) is the growth rate of the perturbations and physically it represents the energy equation for two-dimensional disturbances propagating in the direction of the basic flow. Similarly, C_r given in eqn. (22) represents the phase velocity of the disturbances.

$$|Im(Q)| \leq \int_0^1 |u_b'| \, . |\psi| \, . |D\psi| dy + \int_0^1 A_1 |\psi|^2 \tag{23}$$

Hence, by Schwarz's inequality,

$$\left|Im(Q)\right| \le qI_0 I_1 + A_1 I^2 \tag{24}$$

where $q = \max\limits_{0 \le y \le 1} \left|u_b'\right|$

This gives an upper bound for C_i

$$C_i \le \frac{lqI_0 I_1 + A_1 I^2 - \left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{l\left[I_1^2 + l^2 I_0^2\right] + A_1 I^2} \tag{25}$$

from which it follows that a sufficient condition for stability is

$$A_1 \le \frac{\left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right] - lqI_0 I_1}{I^2} \tag{26}$$

together with

$$lq < \frac{\left[I_2^2 + \left(2l^2 + M^2 + A\sigma_p^2\right)I_1^2 + \left(l^4 - M^2 l^2 - A\sigma_p^2 l^2\right)I_0^2\right]}{I_0 I_1} \tag{27}$$

Hence the theorem.

Simulation and Analysis with Machine Learning:

The equations 26 and 27 are simulated for their veracity with python for handling continuous variables. The results are shown in figures 2 and 3. The 4th A value plotted against I shows clearly that, the electromagnetic system is stable as long as the upper bound for A is governed by Eqn 26.

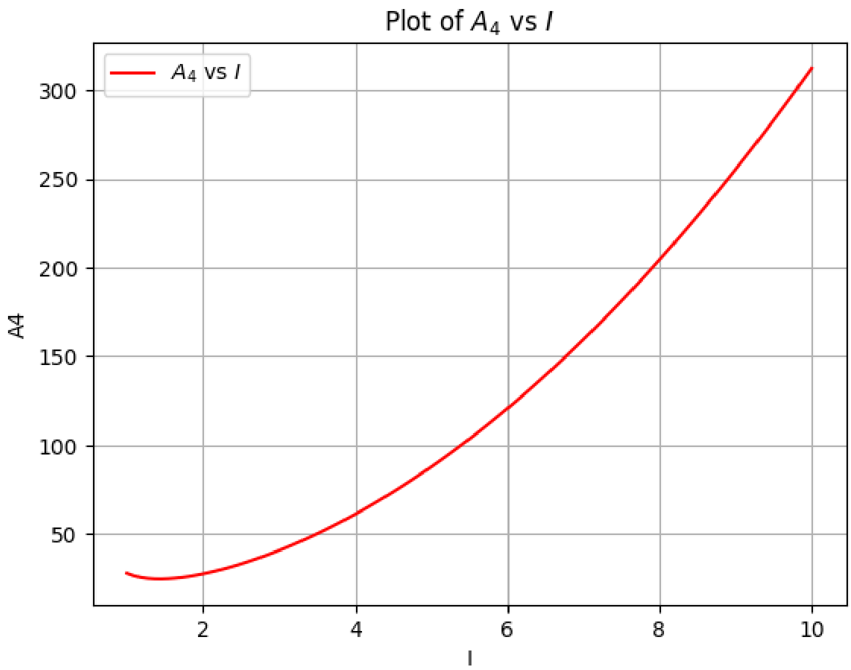


Figure 2. Graph showing stability criterion where A_i gives the lower bound

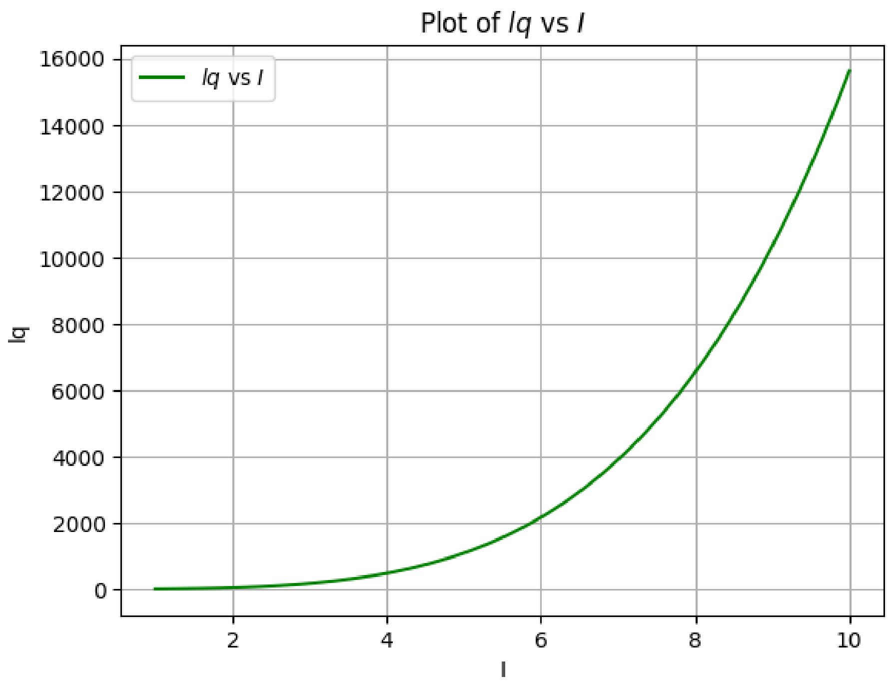


Figure 3. Graph showing the increase in I_q against I, which provides the stability condition for C_i

Figure 3, depicts the exponential rise of I_q against I, showing the second stability condition for C_i, where, the two-dimensional perturbations in the fluid grow exponentially with linearly increasing number of incidents.

III. Conclusion

This study provides a comprehensive analysis of electrohydrodynamic stability in fluid-saturated porous media influenced by electric and magnetic fields. By deriving and analyzing the stability equations, it demonstrates how the interaction of electric charges, fluid motion, and magnetic forces can be harnessed to control flow stability and enhance the design of smart materials. The results highlight the role of self-induced electric fields in shaping fluid behavior and offer a theoretical foundation for optimizing industrial processes where porous media play a crucial role.

Future research can extend these findings by incorporating non-linear stability analysis, experimental validation, and the study of different fluid compositions. The applications of this research are far-reaching, including advancements in nanotechnology, efficient energy systems, and biomedical engineering. By bridging the gap between theory and real-world applications, this work contributes to a deeper understanding of fluid mechanics and its relevance in emerging technologies.

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