IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 27, Issue 5, Ser. 5 (Sept. – Oct. 2025), PP 01-06
www.iosrjournals.org

Pythagorean Fuzzy Soft Ideals And Their Compatibility With Soft Topological Structures

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Abstract
As a generalization of intuitionistic fuzzy set, the Pythagorean fuzzy set is interesting and very useful in modeling uncertain information. The concept of Pythagorean fuzzy soft ideal and pythagorean fuzzy soft topological space in Pythagorean fuzzy soft set (PyFSS) theory is presented in this study. Additionally, we provide the idea of a PyFS local function. The goal of this discussion is to identify a new PyFS topology that differs from the original. This study also examines the fundamental structure, particularly a foundation for such created PyFS topology. Here, the concept of PyFS ideal compatibility with PyFS topology is presented, and some important related results are investigated.
Keywords: *Intuitionistic fuzzy sets, Pythagorean fuzzy sets, Soft set, Topology.*

Date of Submission: 12-10-2025 Date of Acceptance: 22-10-2025

I. Introduction

Understanding that decision-making is inherently imprecise, Zadeh [22] developed the idea of fuzzy sets, which are defined by a membership function ρ. This function determines the degree of membership in the associated set for each element of the universe of discourse by assigning it a value from the unit interval $[0,1]$. ρ measures how much an element in a fuzzy set belongs to a certain class: No membership is indicated by $\rho = 0$, complete membership is shown by $\rho = 1$, and partial membership is captured by intermediate values.

Despite being fundamental, the classical theory of fuzzy sets has certain drawbacks, chief among them being the lack of a non-membership function and the formal handling of reluctance. In order to get beyond these restrictions, Atanassov [7] introduced the idea of intuitionistic fuzzy sets (IFSs), which are characterised by a hesitation margin π (which stands for indeterminacy), a membership function ρ, and a non-membership function σ. The criteria $\rho + \sigma \leq 1$ and $\rho + \sigma + \pi = 1$ are satisfied by these. A more comprehensive framework for simulating ambiguity and uncertainty is offered by this addition.

Nevertheless, there are other circumstances in which $\rho + \sigma \geq 1$, which IFSs are unable to effectively capture. The Pythagorean fuzzy sets (PFSs) were proposed by Yager [20, 21] to solve this problem. The membership and non-membership degrees of PFSs fulfil $\rho^2 + \sigma^2 \leq 1$, and hence $\rho^2 + \sigma^2 + \pi^2 = 1$, where π is the Pythagorean fuzzy index. More flexibility in conveying ambiguity and uncertainty is possible with this approach.

A new mathematical method for handling uncertainty was introduced: the notion of soft sets. Molodtsov invented the idea of soft sets [17]. Coker et al. [9] examined the fundamental idea of intuitionistic fuzzy point and developed intuitionistic fuzzy topology. Kuratowski was the first to propose the notion of an ideal in topological space [14]. In an ideal topological space, they also have a specified local function. Additionally, in [12], Hamlett and Jankovic investigated the characteristics of ideal topological spaces and presented a new operator known as the ψ-operator.

The concept of Pythagorean fuzzy soft ideal in PyFSS theory is presented in this study. Additionally, we provide the idea of a PyFS local function. The goal of this discussion is to identify a new PyFS topology that differs from the original. This study also examines the fundamental structure, particularly a foundation for such created PyFS topology. Here, the concept of PyFS ideal compatibility with PyFS topology is presented, and some related criteria are created.

II. Preliminaries And Definitions

Definition 2.1 *Let X be a universal set and S a set of parameters. A pair (M, S) is called a soft set over X if and only if M is a mapping from S into the power set of X, that is, $M: S \to P(X)$, where $P(X)$ denotes the collection of all subsets of X.*

Definition 2.2 *An expression of the type*

$$I = \{\langle x, \mu_I(x), \nu_I(x)\rangle : x \in X\},$$

denotes an intuitionistic fuzzy set (IFS) I in X, where $\mu_I: X \to [0,1]$, $\nu_I: X \to [0,1]$ are membership and non-membership grade and, for every $x \in X$, $0 \le \mu_I(x) + \nu_I(x) \le 1$.
The level of uncertainty $\pi_I(x) = 1 Itis - \mu_I(x) - \nu_I(x)$.

Definition 2.3

$$P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\},$$

gives a pythagorean fuzzy set P in a finite universe of discourse X, where $\mu_P: X \to [0,1]$, $\nu_I P: X \to [0,1]$ are membership and non-membership grade provided that $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$.
The level of uncertainty $\sqrt{1 - (\mu_P(x))^2 - (\nu_P(x))^2} = \pi_P(x)$.

Some Operations on Pythagorean Fuzzy Sets
Let P_1 and P_2 be two Pythagorean fuzzy sets, then the following operations and relations can be defined as
(i) $P_1 \subseteq P_2 \;\; iff \;\; (\rho_{P_1}(x) \le \rho_{P_2}(x))$ and $(\sigma_{P_1}(x) \ge \sigma_{P_2}(x))$ (for all $x \in X$)
(ii) $P_1 = P_2 \;\; iff \;\; (\rho_{P_1}(x) = \rho_{P_2}(x))$ and $(\sigma_{P_1}(x) = \sigma_{P_2}(x))$ (for all $x \in X$)
(iii) $P_1 \cap P_2 = \{\langle x. \min(\rho_{P_1}(x), \rho_{P_2}(x)), \max(\sigma_{P_1}(x), \sigma_{PP_2}(x))\rangle : x \in X\}$
(iv) $P_1 \cup P_2 = \{\langle x. \max(\rho_{P_1}(x), \rho_{P_2}(x)), \min(\sigma_{P_1}(x), \sigma_{P_2}(x))\rangle : x \in X\}$

Definition 2.4 *Let S be the set of parameters. Let $P(X)$ represent the set of all subsets of X. Suppose that $A \widetilde{\subseteq} S$. The pythagorean fuzzy soft set over X is a pair (F, A) or (F_A), where F is a mapping provided by $F: A \to P(X)$.*

In general, for every $e \in A$, $F(e)$ is a pythagorean fuzzy set of X and it is termed pythagorean fuzzy valueset of parameter e.

Definition 2.5 *Let F_A and G_B be two PyFSSs over X. Then, F_A is called a pythagorean fuzzy soft subset of G_B and write $F_A \widetilde{\subseteq} G_B$ if*
(i) $A \subseteq B$
(ii) for all $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.6 *Let F_A and G_B be two PyFSSs over X. Then $F_A = G_B$ if $F_A \widetilde{\subseteq} G_B$ and $G_B \widetilde{\subseteq} F_A$.*
Definition 2.7 *Let F_A and G_B be two PyFSSs over X. Then*
(i) $F_A \,\widetilde{\cup}\, G_B = H_C$, where $C = A \cup B$ and for all $e \in C$ and

$$H(e) = \begin{cases} F(e) & if \;\; e \in A - B \\ G(e) & if \;\; e \in B - A \\ F(e) \cup G(e) & if \;\; e \in A \cap B \end{cases}$$

(ii) $F_A \,\widetilde{\cap}\, G_B = H_C$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.8 *Over X, let F_A be a PyFSS. Then F_A^c represents the complement of F_A, which is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \to P(X)$ is a mapping provided by $F^c(e) = (F(e))^c$ for all $e \in A$.*
Definition 2.9 *A PyFSS F_A over the universe set X is said to be null PyFSS and is denoted by $\tilde{0}$ defined as*

$$\tilde{0} = \{\langle x, 0,1\rangle : x \in X\}$$

for all $e \in A$.

Definition 2.10 *A PyFSS F_A over the universe set X is said to be absolute PyFSS and is denoted by $\tilde{1}$ defined as*

$$\tilde{1} = \{\langle x, 1,0\rangle : x \in X\}$$

for all $e \in A$.

Definition 2.11 *Let F_A and G_B be two PyFSSs over X. The difference between F_A and G_B is a PyFSS H_C defined as*
$F_A - G_B = H_C$, where $C = A \cap B$ for all $c \in C$, $x \in X$ and
$\mu_{H(e)}(x) = \min\{\mu_{F(e)}(X), \nu_{G(e)}(X)\}$ and $\nu_{H(e)}(x) = \max\{\nu_{F(e)}(X), \mu_{G(e)}(X)\}$.

Definition 2.12 *A family of PyFSSs is called a Pythagorean fuzzy soft topology τ on (X, E) if following properties are satisfied by Δ over (X, E).*
(i) $\tilde{0}, \tilde{1} \,\widetilde{\in}\, \tau$
(ii) if F_A, $G_B \,\widetilde{\in}\, \tau$, then $F_A \,\widetilde{\cap}\, G_B \,\widetilde{\in}\, \tau$

(iii) if $F_A \tilde{\in} \tau$ for all $\alpha \tilde{\in} \Delta$ an index set, then $\underset{\alpha \tilde{\in} \Delta}{\cup} F_{A_\alpha} \tilde{\in} \tau$.
The triplet (X, τ, E) is called a PyF soft topological space.
If $F_A \tilde{\in} \tau$, then the PyFSS F_A is said to be PyFS open set.
A PyFSS F_A over X is said to be an PyFS closed set in X, if its complement F_A^c belongs to τ.

Definition 2.13 *The PyF soft closure of F_A is defined as the intersection of all PyFS closed sets which contained F_A and is denoted by $cl(F_A)$.*
Therefore, $cl(F_A) = \tilde{\cap}\{G_B : G_B \text{ is PyF soft closed and } F_A \tilde{\subseteq} G_B\}$.

Definition 2.14 *Let (X, τ, E) is a PyF soft topological space and let F_A be a PyFSS over X. The PyFS interior of F_A is defined as the union of all PyFS open sets which contained in F_A and is denoted by $int(F_A)$.*
Therefore, $int(F_A) = \tilde{\cup}\{G_B : G_B \text{ is PyFS open and } G_B \tilde{\subseteq} F_A\}$.

Definition 2.15 *Let (X, τ, E) is a PyFS topological space and let F_A be a PyFSS over X. A PyFS point e_F is said to be in PyFS set G_A, denoted by $e_F \tilde{\in} G_A$ if for the element $e \in A$, $F(e) \leq G_e$.*

Definition 2.16 *A PyFSS F_A is a neighbourhood of a PyFSS G_B if and only if there exists a pythagorean fuzzy open set $Q_c \tilde{\in} \tau$ such that $F_A \tilde{\subseteq} Q_c \tilde{\subseteq} G_B$.*

Definition 2.17 *A PyFSS F_A is a neighbourhood of a PyFS point $e_F \tilde{\in} F_A$ if there exists a pythagorean fuzzy open set $Q_c \tilde{\in} \tau$ such that $e_F \tilde{\subseteq} Q_c \tilde{\subseteq} F_A$.*

III. Main Results

Definition 3.1 *A non-empty collection of PyFSSs $\tilde{P}$ of a soft set X is called PyFS ideal if*
(i) If $F_A \tilde{\in} \tilde{P}$ and $G_B \tilde{\in} \tilde{P}$, then $F_A \tilde{\cup} G_B \tilde{\in} \tilde{P}$
(ii) If $F_A \tilde{\in} \tilde{P}$ and $G_B \tilde{\subseteq} F_A$, then $G_B \tilde{\in} \tilde{P}$.

Example 1 (i) *If $\tilde{P} = \{\tilde{0}\}$, then $\tilde{P}$ is a PyFS ideal on (X, E).*
(ii) If $\tilde{P} = \{\tilde{1}\}$, then $\tilde{P}$ is a PyFS ideal on (X, E).

Definition 3.2 *Let (X, τ, E) is a PyFS topological space and let $\tilde{P}$ be a PyFSS over X. Then the triplet $(X, \tau, \tilde{P})$ is called PyFS ideal topological space.*

Definition 3.3 *Let (X, τ, E) is a PyFS topological space and let $\tilde{P}$ be a PyFS ideal over X. The PyFS local function F with respect to τ and $\tilde{P}$ is denoted by $F_A^*(\tau, \tilde{P})$ or F^* and defined as*

$$F_A^* = \tilde{\cup}\{e_F \tilde{\in} X : F_A \tilde{\cap} Q_c \tilde{\notin} \tilde{P} \ \ for\ every\ \ Q_c \tilde{\in} \tau\}.$$

Theorem 3.1 *Let (X, τ, E) is a PyFS topological space and let $\tilde{P}$ and $\tilde{Q}$ be two PyFS ideal over X. Let F_A and G_B are two PyFSSs. Then*
(i) $\tilde{0}^* = \tilde{0}$
(ii) $F_A \tilde{\subseteq} G_B \Rightarrow F_A^* \tilde{\subseteq} G_B^*$
(iii) $(F_A \tilde{\cup} G_B)^* = F_A^* \tilde{\cup} G_B^*$
(iv) $(F_A \tilde{\cap} G_B)^* = F_A^* \tilde{\cap} G_B^*$
(v) $\tilde{P} \tilde{\subseteq} \tilde{Q} \Rightarrow F_A^* \tilde{\cap} G_B^*$
(vi) $F_A^* \tilde{\subseteq} cl(F_A)$
(vii) F_A^* is a PyF soft closed soft set.

Proof. (i) It is obvious from definition.
(ii) Let $e_F \tilde{\in} X$. Suppose we have $F_A \tilde{\subseteq} G_B$
which implies that $F_A \tilde{\cap} Q_c \tilde{\subseteq} G_B \tilde{\cap} Q_c$, $Q_c \tilde{\in} \tau$.
Now, $F_A^* = \{e_F \tilde{\in} X : F_A \tilde{\cap} Q_c \tilde{\notin} \tilde{P} \ \ for\ every\ \ Q_c \tilde{\in} \tau\}$
$\tilde{\subseteq} \{e_f \tilde{\in} X : G_B \tilde{\cap} Q_c \tilde{\notin} \tilde{P}\ for\ every\ \ Q_c \tilde{\in} \tau\} = G_B^*$.
Therefore $F_A^* \tilde{\subseteq} G_B^*$.
(iii) Let $e_F \tilde{\in} (F_A \tilde{\cup} G_B)^*$
Now, $(F_A \tilde{\cup} G_B)^* = \{e_F \tilde{\in} X : (F_A \tilde{\cup} G_B) \tilde{\cap} Q_c \tilde{\notin} \tilde{P} \ \ for\ every\ \ Q_c \tilde{\in} \tau\}$
$\Rightarrow \{e_F \tilde{\in} X : (F_A \tilde{\cap} Q_c) \tilde{\cup} (G_B \tilde{\cap} Q_c) \tilde{\notin} \tilde{P} \ \ for\ every\ \ Q_c \tilde{\in} \tau\}$
$\Rightarrow \{e_F \tilde{\in} X : (F_A \tilde{\cap} Q_c) \tilde{\cup} (G_B \tilde{\cap} Q_c) \tilde{\notin} \tilde{P} \ \ for\ every\ \ Q_c \tilde{\in} \tau\}$

Therefore, $(F_A \tilde{\cup} G_B)^* \tilde{\subseteq} F_A^* \tilde{\cup} G_B^*$.
Again, $F_A \subseteq F_A \tilde{\cup} G_B$ and $G_B \subseteq F_A \tilde{\cup} G_B$.
From, (ii), we have $F_A^* \subseteq (F_A \tilde{\cup} G_B)^*$ and $G_B^* \subseteq (F_A \tilde{\cup} G_B)^*$.
Therefore, $F_A^* \tilde{\cup} G_B^* = (F_A^* \tilde{\cup} G_B)^*$
Hence, $(F_A \tilde{\cup} G_B)^* = F_A^* \tilde{\cup} G_B^*$.
(iv) We have $F_A \tilde{\cap} G_B \tilde{\subseteq} F_A$ and $F_A \tilde{\cap} G_B \tilde{\subseteq} G_B$.
From (ii), we have $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} F_A^*$ and $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} G_B^*$.
Therefore, $(F_A \tilde{\cap} G_B)^* \tilde{\subseteq} F_A^* \tilde{\cap} G_B^*$.
(v) Let $e_F \tilde{\in} F_A^*(Q)$. Then $Q_C \tilde{\cap} F_A^* \tilde{\notin} \tilde{Q}$ for every $Q_c \tilde{\in} \tau$.
Since $\tilde{P} \tilde{\subseteq} \tilde{Q}$, then $Q_c \tilde{\cap} F_A^* \tilde{\notin} \tilde{P}$ for every $Q_c \tilde{\in} \tau$.
Hence $e_F \tilde{\in} F_A^*(\tilde{P})$.
Thus $F_A^*(\tilde{Q}) \tilde{\subseteq} F_A^*(\tilde{P})$.
(vi) Assume that $e_F \tilde{\notin} cl(F_A)$.
Then there exists $Q_c \tilde{\in} \tau$ such that $Q_c \tilde{\cap} F_A = \phi \tilde{\in} \tilde{P}$.
Hence, $e_F \tilde{\notin} F_A^*$.
Thus, $F_A^* \tilde{\subseteq} cl(F_A)$.
(vii) Clearly, $F_A^* \tilde{\subseteq} cl(F_A^*)$.
So, let $e_F \tilde{\in} cl(F_A^*)$.
Then $Q_c \tilde{\cap} F_A^* \neq \phi$ for every $Q_c \tilde{\in} \tau$.
Hence there exists $e_F \tilde{\in} Q_c \tilde{\cap} F_A^*$.
Thus $e_F \tilde{\in} Q_c$ and $e_F \tilde{\in} F_A^*$.
It follows that $Q_c(e_F) \tilde{\cap} F_A \tilde{\notin} \tilde{P}$ for every $Q_c(e_F) \tilde{\in} \tau$.
This implies that $Q_c \tilde{\cap} F_A \tilde{\notin} \tilde{P}$ for every $Q_c \tilde{\in} \tau$. So $e_F \tilde{\in} F_A^*$.
This means that $cl(F_A^*) \tilde{\subseteq} F_A^*$.
Therefore $F_A^* = cl(F_A^*)$.

Definition 3.4 *Let $\tilde{P}$ be a PyFS ideal over X. The PyFS closure operator of a PyFSS F_A in (X, τ, E) is defined as $cl^*(F_A) = F_A \tilde{\cup} F_A^*$.*

Theorem 3.2 *Let (X, τ, E) is a PyFS topological space and let $\tilde{P}$ be a PyFS ideal over X. If $F_A \tilde{\subseteq} G_B$, then $cl^*(F_A) \tilde{\subseteq} cl * (G_B)$.*

Proof. Since $cl^*(F_A)(x) = (F_A \tilde{\cup} F_A^*)(x) = \max\{F_A(x), F_A^*(x)\} \tilde{\subseteq} \{G_B(x), G_B^*(x)\} = cl^*(G_B)(x)$. Hence, $cl^*(F_A) \tilde{\subseteq} cl^*(G_B)$.

Definition 3.5 *Let $cl^*: P(X) \to P(X)$ be the PyFS closure operator. Then there exists a unique PyF soft topology over X finer than τ, called the * PyFS topology, denoted by$\tau^*(\tilde{P})$ or τ^*, given by $\tau^* \tilde{P} = \{F_A\ PyF\ soft : cl^*(F_A) = F_A\}$.*

Definition 3.6 *The PyF soft topology τ is compatible with the PyF soft ideal $\tilde{P}$, denoted by $\tau \tilde{\tilde{P}}$ if for every PyF soft point e_F there exists $Q_c \tilde{\in} \tau$ such that $Q_c \tilde{\cap} F_A \tilde{\in} \tilde{P}$, then $F_A \tilde{\in} \tilde{P}$.*

Theorem 3.3 *Let (X, τ, E) is a PyFS ideal topological space and let $\tau \tilde{\tilde{P}}$. Then following are equivalent.*
(i) $F_A \tilde{\cap} F_A^* = \phi$
(ii) $(F_A - F_A^*)^* = \phi$
(iii) $(F_A \tilde{\cap} F_A^*)^* = F_A^*$, for every PyFSS F_A.

Proof. (i) ⇒(ii)
Let F_A be a PyFSS. Since, $(F_A - F_A^*) \tilde{\cap} (F_A - F_A^*)^* = \tilde{\phi}$. Then $(F_A - F_A^*)^* = \tilde{\phi}$ by (i).
(ii) ⇒(iii)
Let F_A be a PyFSS. Since,
$F_A = (F_A - (F_A \tilde{\cap} F_A^*)) \tilde{\cup} (F_A \tilde{\cap} F_A^*)$.
Then $F_A^* = ((F_A - (F_A \tilde{\cap} F_A^*)) \tilde{\cup} (F_A \tilde{\cap} F_A^*))^*$
$= ((F_A - (F_A \tilde{\cap} F_A^*))^* \tilde{\cup} (F_A \tilde{\cap} F_A^*)^*) = \tilde{\phi} \tilde{\cup} (F_A \tilde{\cap} F_A^*)^*)$
$= (F_A \tilde{\cap} F_A^*)^*$.
(iii) ⇒ (i)
Let F_A be a PyFSS and let $F_A \tilde{\cap} F_A^* = \phi$. Then $F_A^* = (F_A \tilde{\cap} F_A^*)^* = \tilde{\phi}^* = \tilde{\phi}$.

Theorem 3.4 *Let* (X, τ, E) *is a PyF soft ideal topological space and let* $\tau \sim \tilde{P}$.
Then $(F_A)^* = F_A^*$.
Proof. It is obvious from definition.

Theorem 3.5 *Let* (X, τ, E) *is a PyFS ideal topological space. Then the following are equivalent:*
(i) $\tau \sim \tilde{P}$
(ii) $F_A \tilde{\cap} F_A^* = \phi$
(iii) $F_A - F_A^* \tilde{\in} \tilde{P}$
(iv) $F_A - F_A^* \tilde{\in} \tilde{P}$
(v) if F_A contains no null PyFSS G_B with $G_B \tilde{s}ubseteq G_B^*$, then $F_A \tilde{\in} \tilde{P}$, for every PyFSS F_A.

Proof. (i) ⇒(ii)
Let F_A be PyFSS such that $F_A \tilde{\cap} F_A^* = \phi$. Then for every $e_F \tilde{\in} F_A$ and $e_F \tilde{\notin} F_A^*$, we have $Q_c \tilde{\cap} F_A \tilde{\in} \tilde{P}$ for every $Q_c \tilde{\in} \tau$. Thus, $F_A \tilde{\in} \tilde{P}$ by (i).
(ii) ⇒(iii)
Let F_A be PyFSS. Since $(F_A - F_A^*) \tilde{\cap} (F_A - F_A^*)^* = (F_A \tilde{\cap} F_A^*) \tilde{\cap} (F_A - F_A^*)^* \tilde{\subseteq} (F_A \tilde{\cap} F_A^*) \tilde{\cap} F_A^* = \phi$. Then $F_A - F_A^* \in \tilde{P}$ by (ii).
(iii) ⇒(iv)
Let F_A be τ^*-PyFS closed set. Then $F_A - F_A^* \tilde{\in} \tilde{P}$ by (iii).
(iv) ⇒(i)
Let F_A be a pthagorean fuzzy soft set and assume that for every $e_F \tilde{\in} F_A$ there exists $Q_c \tilde{\in} \tau$ such that $Q_c \tilde{\cap} F_A \tilde{\in} \tilde{P}$. Then $e_F \tilde{\notin} F_A^*$. Hence $F_A \tilde{\cap} F_A^* = \phi$ and since $F_A \tilde{\cup} F_A^*$ is τ^*-PyFS closed set, we have $(F_A \tilde{\cup} F_A^*) - (F_A \tilde{\cup} F_A^*) \tilde{\in} \tilde{P}$ by (iv).
Hence $(F_A \tilde{\cup} F_A^*) - (F_A \tilde{\cup} F_A^*)^* = (F_A \tilde{\cup} F_A^*) - F_A^* = F_A \tilde{\in} \tilde{P}$. Thus, $\tau \sim \tilde{P}$.
(iii) ⇒(v)
Let F_A be a PyFSS such that F_A contains no null PyFSS G_B with $G_B \tilde{\subseteq} G_B^*$. Since, $F_A \tilde{\cap} F_A^* \tilde{\subseteq} F_A^* = (F_A - F_A^*)^*$.
It follows that $F_A \tilde{F}_A^* \tilde{\subseteq} (F_A \tilde{\cap} F_A^*)^*$.
By assumption, $F_A \tilde{\cap} F_A^* = \phi$.
Thus $F_A - F_A^* \tilde{\in} \tilde{P}$ by (iii).
(v) ⇒(iii)
Let F_A be a PyFSS. Since $(F_A - F_A^*) \tilde{\cap} (F_A - F_A^*)^* = \phi$ and $F_A - F_A^*$ contains no null PyFSS G_B with $G_B \tilde{\subseteq} G_B^*$.
Hence, $F_A - F_A^* \tilde{\in} \tilde{P}$ by (iv).

Theorem 3.6 *Let* $(X, \tau, \tilde{P})$ *be a PyF soft ideal topological space and let* $\tau \sim \tilde{P}$. *Then a PyFSS is* τ^**-PyF soft closed set if and only if it is the union of* τ^**-PyF soft closed set and a PyFSS in* $\tilde{P}$.

Proof. Let F_A be a τ^*-PyF soft closed set. Then, $cl^*(\mathrm{F}_A) = \mathrm{F}_A$ and $\mathrm{F}_A \tilde{\mathrm{F}}_A^* = \mathrm{F}_A$.
Hence, $\mathrm{F}_A^* \tilde{\subseteq} \mathrm{F}_A$.
Thus $\mathrm{F}_A = (\mathrm{F}_A - \mathrm{F}_A^*) \tilde{\cup} \mathrm{F}_A^*$, $\mathrm{F}_A - \mathrm{F}_A^* \tilde{\in} \tilde{P}$ and F_A^* is a τ-PyFS closed set and $P_E \tilde{\in} \tilde{P}$.
Then $\mathrm{F}_A^* = (G_B - I_E)^* = G_B^* \tilde{\subseteq} cl(G_B) = G_B \tilde{\subseteq} \mathrm{F}_A$.
Hence, $\mathrm{F}_A \tilde{\cap} \mathrm{F}_A^* = \mathrm{F}_A$. Thus $cl^*(\mathrm{F}_A) = \mathrm{F}_A$.
Therefore, F_A is a τ^*-PyFS closed set.

IV. Conclusion

Finding a new PyFS topology that is distinct from the original is the aim of this debate. The basic structure is also examined in this work, namely a basis for the developed PyFS topology. This article introduces the notion of PyFS ideal compatibility with PyFS topology and develops some associated criteria.

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