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Solution of a Multi-criteria Problem of Choosing the Location of Retail OutletUsing Z-numbers

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Abstract. *This work is devoted to the issues of solving the multi-criteriaretail marketing problem on the rational choice of the location of retail outlets using the Z-number. Along with other methods that were developed by the authors, the early use of this approach allows a more adequate consideration of the uncertainties inherent in the very problem of choosing the location of a retail outlet, is more practical and requires minimal costs to solve such problems. The paper gives a general algorithm and a numerical example of solving the considered problem.*
Keywords:*Commercial establishments, retail marketing, multi-criteria decision-making, fuzzy set theory, Z-number*

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I. Introduction

Retail trade is one of the most dynamically developing sectors of the national economy, which is characterized by high dynamism of development, a wide variety of retail trade networks, a high rate of capital turnover, and service [1,2,3].

The efficiency of retail trade organizations and the rate of capital turnover depends on the rationality of its geographical location. In other words, the correct choice of the location of the outlet determines the potential number of customers and turnover.

Practice shows that despite the available analytical methods [1]: financial analysis method, checklist method, analog approach, regression analysis, Huff gravity model method, revealed preference model methods, basically the choice of the location of the outlet is carried out according to the experience and intuition of others words to the judgments of experts in the field.

In [4, 5], we substantiated the application of the experience and intuition of experts (senior staff and managers) in conditions of uncertainty and presented two approaches to multi-criteria decision-making for this problem based on the assessment of alternative solutions using the theory of fuzzy sets [4] and using the method Saati and the theory of fuzzy sets [5].

In this paper, to solve this problem under conditions of uncertainty, Z-numbers are used, the feasibility of which is confirmed by the fact that Z-numbers are intended to provide a basis for calculations with numbers that are not completely reliable.

The fact is that basically the information that is available to managers, on the basis of which decisions are made, is vague, inaccurate and / or incomplete. Formalizing this information is challenging. It is Z-numbers that allow you to formalize undefined variables.

The work is structured as follows. Section 2 describes an algorithm for evaluating alternatives for the rational choice of the location of a retail outlet. Section 3 shows an example of solving the problem of multi-criteria decision-making on the choice of the location of the outlet using the Z-number. Concluding remarks are included in the conclusion section.

II. General algorithm for evaluating alternatives for the rational choice of the location of a retail outlet

Suppose there are many alternatives (locations of outlets) $V=\{v_1,v_2,...,v_m\}$ (where m is the total number of alternatives) and a set of criteria $K=\{K_1,...,K_n\}$ (where n is the total number of criteria), with the help of which these alternatives are evaluated according to the set goal.
Based on the importance of the criteria, which are described through the corresponding linguistic variables ("Very high") and the degree of confidence in the significance of the criteria, which is also described as a

linguistic variable, each statement for each alternative is linguistically described under the concept of Z-numbers and a decision-making matrix M (Z), which is presented in table 1.
Zadeh [6] defined the Z-number associated with an undefined variable X as an ordered pair of fuzzy numbers ($\widetilde{\boldsymbol{A}}$, $\widetilde{\boldsymbol{B}}$), where $\widetilde{\boldsymbol{A}}$represents the value of the variable and $\widetilde{\boldsymbol{B}}$ represents the idea of definiteness. Zadeh refers to the ordered triplet (X, $\widetilde{\boldsymbol{A}}$, $\widetilde{\boldsymbol{B}}$) as a Z-estimate, which is equal to the statement about.

Table 1. The general structure of the decison matrix using Z-numbers

Alternatives	Criteria			
	K_1 (Z_{k1})	K_2 (Z_{k2})		$K_m(Z_{k3})$
v_1	Z_{11}	Z_{12}		Z_{1m}
$\boldsymbol{v_2}$	Z_{21}	Z_{22}	…	Z_{2m}
…				…
v_n	Z_{n1}	Z_{n2}		Z_{nm}

assignment X is **($\widetilde{\boldsymbol{A}}$, $\widetilde{\boldsymbol{B}}$).** The works [7,8,9,10,11] are devoted to the problems of processing Z-numbers.
In our case, the Z-number can be described as follows:

$$Z_{ij} = \ (\boldsymbol{X = K_i}, \widetilde{\boldsymbol{A_{ij}}}, \widetilde{\boldsymbol{B_{ij}}}\ ) \tag{1}$$

which can be given the following linguistic description: we can say with confidence $\boldsymbol{B_{ij}}$that the linguistic variable $\boldsymbol{X = K_i}$ will receive the value $\widetilde{\boldsymbol{A_{ij}}}$. $\boldsymbol{A_{ij}}$and$\boldsymbol{B_{ij}}$ are triangular fuzzy numbers, which are represented respectively by the following triplet ($\boldsymbol{a_1^{ij}}$, $\boldsymbol{a_2^{ij}}$, $\boldsymbol{a_3^{ij}}$) and ($\boldsymbol{b_1^{ij}}$, $\boldsymbol{b_2^{ij}}$, $\boldsymbol{b_3^{ij}}$), where the membership can be determined as the following equation, which can be shown in Fig. 1. And correspond to the following statements $\boldsymbol{A_{ij}}$is approximately equal to $\boldsymbol{a_2^{ij}}$ and $\boldsymbol{B_{ij}}$is approximately equal to $\boldsymbol{b_2^{ij}}$and are respectively in the intervals **[$\boldsymbol{a_1^{ij}}$,$\boldsymbol{a_3^{ij}}$]** and **[$\boldsymbol{b_1^{ij}}$,$\boldsymbol{b_3^{ij}}$].**
The criterion weights are also represented as Z-number ($\boldsymbol{Z_{ki}}$).
Below is an algorithm for multi-criteria decision-making for the rational choice of the location of outlets based on the method proposed in the work. [10]:
1. The choice of the main criteria-K, on which the correctness of the decision depends. Dividing criteria into benefit criteria — high weight criteria and cost — low weight criteria.
2. Identification of alternative options - To the location of the outlets.
3. Construction of the decision-making matrix M (Z) in the form described in Table 1.

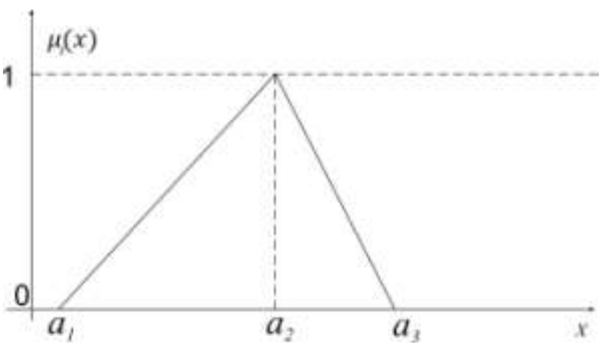

Fig. 1: A triangular fuzzy number.

4. Transformation of linguistic meanings into numerical meanings.
The transformation of the first part of the Z-number is carried out according to the membership function of triangular fuzzy numbers after which the matrix M($\widetilde{\boldsymbol{A}}$)is compiled. To transform the second component of the Z-number, it is necessary to rely on the following correspondence: Very Low - (0,0,0.25), Low - (0,0.25,0.5), Medium - (0.25,0.5,0.75), High - (0.5,0.75 , 1), Very High - (0.75,1,1)..
5. Normalization of the matrix M($\widetilde{\boldsymbol{A}}$). Normalization of local criteria is necessary because of their different dimensions. Normalization is carried out over all elements of the matrix M($\widetilde{\boldsymbol{A}}$)according to the following formulas: - for benefit criteria

$$\widetilde{a_{ij}} = \left(\frac{a_1^{ij}}{a_{max}^{ij}}, \frac{a_2^{ij}}{a_{max}^{ij}}, \frac{a_3^{ij}}{a_{max}^{ij}}\right) \quad (2)$$

- for cost criteria

$$\widetilde{a_{ij}} = \left(\mathbf{1} - \frac{a_3^{ij}}{a_{max}^{ij}}, \mathbf{1} - \frac{a_2^{ij}}{a_{max}^{ij}}, \mathbf{1} - \frac{a_1^{ij}}{a_{max}^{ij}}\right) (3)$$

6. Convert all Z-numbers to crisp numbers. For conversion, the canonical representation of the operation of multiplication by a triangular fuzzy number, which was introduced by Chou [12], is used. For our case, it is necessary to multiply both components $\widetilde{\boldsymbol{A}_{ij}}$and $\widetilde{\boldsymbol{B}_{ij}}$ of each $\boldsymbol{Z}_{ij}$using the following formula:

$$\boldsymbol{c}(\boldsymbol{Z}_{ij}) = ((\boldsymbol{a}_1^{ij} + \mathbf{4}\boldsymbol{a}_2^{ij} + \boldsymbol{a}_3^{ij})/\mathbf{6}) * ((\boldsymbol{b}_1^{ij} + 4\boldsymbol{b}_2^{ij} + \boldsymbol{b}_3^{ij})/\mathbf{6}) \quad (4)$$

7. According to the formula (4), the weight coefficients are also converted to clear numbers $\boldsymbol{c}(\boldsymbol{Z}_{ki})$.

8. In order to ensure that the sum of all weight coefficients should be equal to 1, we normalize the weight coefficients using the following formula:

$$\boldsymbol{w}(\boldsymbol{k}_i) = \frac{c(Z_{ki})}{\sum_{i=1}^{n} c(Z_{ki})} \quad (5)$$

9. Ranking of alternatives. The total of all of each alternative is determined by the following formula:

$$\boldsymbol{r}_i = \sum \boldsymbol{c}(\boldsymbol{Z}_{ij}) * \boldsymbol{w}(\boldsymbol{k}_i) \quad (6)$$

III. Solving the Problem of Choosing The Location of the Outlet

Let us give an example of solving a multi-criteria problem for the rational choice of the location of a retail outlet. As can be seen from Table 2, it is necessary to choose the most rational solution from among the proposed three alternative (v_1, v_2, v_3) options for the location of the outlet, taking into account four factors (criteria): K_1 - The flow of people through the object for 1 hour; K_2 - number of customers ın the effect zone, K_3 - relative purchasing power for a resıdent, K_4 - monthly expenses of the object.

When compiling the decision matrix, the opinions of managers of trade enterprises were used, which were further formulated in the form of Z-numbers. Here are some examples of managers' statements, on the basis ofwhich Z-numbers are formulated:

1. Z_{k1} = The flow of people through the object for 1 hour, high, then with high confidence we can say that the object is quite suitable for selection).
2. Z_{k2} = (Number of customers ın the effect zone, high, then there is a very high probability that a large number of this number of people will enter the outlet)
3. Z_{k3} = (Relative purchasing power for a resident,high, then with high confidence we can say that there will be a very high sale at the outlet).
4. Z_{k4} = (Monthly expenses of the object, low, then there is a high probability that the costs will be very low).

According to the opinions of experts, all alternative options were also analyzed in terms of expected values for all criteria and the membership functions of triangular fuzzy numbers were determined.The resulting decision matrix is presented in Table 2.According to the above algorithm, a multi-criteria problem was solved for the rational choice of the location of the outlet, the solution, which is given below:

1. All linguistic variables that are shown in table 2 are converted to their numerical values (table 3).
2. According to step 5 and formulas 2 and 3, all fuzzy data were normalized and the results of normalization are shown in Table 4.

Table 2. Decision matrix with linguistic values

	Criteria			
	The flow of people through the object for 1 hour (about 500) K_1 (VH,VH)	Number of customers ın the effect zone (about 8000) K_2 (H,VH)	Relative purchasing power for a resident (about 200) K_3 (H,VH)	Monthly expenses of the object K_4 (M,VH)
v_1	((250,270,290),M)	((4000,4600,5000),L)	((150,200,300),VH)	((300,400,500),H)
v_2	((400,430,450),H)	((5500,6000,6300),M)	((90,150,200),M)	((500,550,600),VH)
v_3	((180,200,225),M)	((7000,7600,8000),H)	((110,160,210),H)	((1000,1200,1500),VH)

Table 3. Decision matrix with numerical values

	Criteria			
	Number of people passing by the facility in 1 hour (approximately 500 people) K_1	Number of customers in the impact zone (Norm 8000) K_2	Relative per capita capacity (approximately AZN 200) K_3	Monthly cost of the object (AZN) K_4
	((0.75,1,1),(0.75,1,1))	((0.5,0.75,1),(0.75,1,1))	((0.5,0.75,1),(0.75,1,1))	((0.25,0.5,0.75),(0.75,1,1))
v_1	((250,270,290),(0.25,0.5,0.75))	((4000,4600,5000),(0,0.25,0.5),)	((150,200,300),(0.75,1,1))	((300,,400,500),(0.5,0.75,1))
v_2	((400,430,450),(0.5,0.75,1))	((5500,6000,6300),(0.25,0.5,0.75))	((90,150,200),(0.25,0.5,0.75))	((500,550,600),(0.75,1,1))
v_3	((180,200,225),(0.25,0.5,0.75))	((7000,7600,8000),(0.5,0.75,1))	((110,160,210),(0.5,0.75,1))	((1000,1200,1500),(0.75,1,1))

Table 4. Normalized decision matrix with combines the restraint and reliability

	Criteria			
	Number of people passing by the facility in 1 hour (approximately 500 people) K_1	Number of customers in the impact zone (Norm 8000) K_2	Relative per capita capacity (approximately AZN 200) K_3	Monthly cost of the object (AZN) K_4
	((0.75,1,1),(0.75,1,1))	((0.5,0.75,1),(0.75,1,1))	((0.5,0.75,1),(0.75,1,1))	((0.25,0.5,0.75),(0.75,1,1))
v_1	((0.56,0.6,0.64),(0.25,0.5,0.75))	((0.5,0.58,0.63),(0,0.25,0.5))	((0.5,0.67,1),(0.75,1,1))	((0.67,0.73,0.8),(0.5,0.75,1))
v_2	((0.89,0.96,1),(0.5,0.75,1))	((0.69,0.75,0.79),(0.25,0.5,0.75))	((0.3,0.5,0.67),(0.25,0.5,0.75))	((0.6,0.63,0.67),(0.75,1,1))
v_3	((0.4,0.44,0.5),(0.25,0.5,0.75))	((0.88,0.95,1),(0.5,0.75,1))	((0.37,0.53,0.7),(0.5,0.75,1))	((0,0.2,0.33),(0.75,1,1))

When normalizing the criteria, $\boldsymbol{K_1}$, $\boldsymbol{K_2}$, $\boldsymbol{K_3}$ were considered as a criterion of benefit, and $\boldsymbol{K_4}$ criteria were considered as a criterion for cost.

3. Further, using formula 5, all Z-numbers were converted into crisp numbers, the results of which are shown in Table 5.

Table 5. Decision matrix with the conversion of all Z-numbers into crisp numbers

	Criteria			
	Number of people passing by the facility in 1 hour (approximately 500 people) K_1	Number of customers in the impact zone (Norm 8000) K_2	Relative per capita capacity (approximately AZN 200) K_3	Monthly cost of the object (AZN) K_4
	0.92	0.72	0.72	0.48
v_1	0.3	0.15	0.67	0.55
v_2	0.72	0.38	0.25	0.6
v_3	0.22	0.71	0.4	0.18

4. According to formula 5, the weights were normalized, and using formula 6, the weights were calculated and the alternatives were ranked. ThecalculationresultsareshowninTable 6.

Table 6. Ranked decision matrix by weight coefficients

	Criteria				Priority (R)
	Number of people passing by the facility in 1 hour (approximately 500 people) K_1	Number of customers in the impact zone (Norm 8000) K_2	Relative per capita capacity (approximately AZN 200) K_3	Monthly cost of the object (AZN) K_4	
	0.32	0.25	0.25	0.17	
v_2	0.72	0.38	0.25	0.6	0.49
v_1	0.3	0.15	0.67	0.55	0.39
v_3	0.22	0.71	0.4	0.18	0.38

Analysis of the results of solving a multi-criteria problem shows that: $v_1$afor an alternative–r_1= 0.39; $v_2$for an alternative–r_2=0.49 ;$v_3$for an alternative–r_3=0.38. In ascending order: ;v_3→0.38<v_1→0.39<v_2→0.49. As you can see, the best object among these alternatives is the second object.

IV. CONCLUSION

The results obtained show the reliability and ease of calculation for making a decision using the Z-number. The main thing here is that in everyday life we are all faced with the fact that we reason about all events with a certain certainty. It is the Z-number that allows us to formalize this kind of uncertainty and take them into account when solving multi-criteria decision-making problems.

And from the point of view of the subject area of the considered work, we can say that managers and managers, when evaluating alternatives for choosing the location of a retail outlet, also reason in the form of a Z-number.

On the other hand, the results allow us to justify the effectiveness of multi-criteria decision-making in solving this type of problem with the help of Z-numbers based on the knowledge and experience of managers, rather than a simple method of accumulating points.

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