Constrained Engineering Design Optimization using Average Differential Evolution

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Abstract: The use of metaheuristics has a growing interest in solving constrained optimization problems due to the computational disadvantages of numerical methods. Metaheuristics are a powerful tool in reaching the global optimum. In this work, the Average Differential Evolution (ADE) algorithm, which is one of the newly proposed metaheuristics, has been adapted to the constrained engineering design problems. The ADE algorithm is a population-based approach with a high convergence rate. It uses a mutation operator with collective diversity in the production of candidates. The results show the robustness and effectiveness of the proposed algorithm compared to state-of-the-art algorithms in literature.

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I. Introduction

Constrained engineering design problems (CEDP) are considered as real-world problems with many constraints. Constraints are very important for engineering design problems because they make searching difficult and search method ineffective. Many researchers investigate the solution of these problems and offer different approaches [1-3]. However, because of the complex and nonlinear characterization of the problems, classical optimization techniques may be inadequate to achieve the global optimum. To overcome this, the interest in the use of metaheuristics in this area has been increasing in recent years.

Metaheuristics such as Genetic algorithm (GA) [4], artificial bee colony (ABC) [5], bat algorithm (BA) [6], crow search algorithm (CSA) [7] and particle swarm optimization (PSO) [8] provide remarkable performance in solving engineering design problems. Including natural phenomena, these algorithms essentially maintain a population of solutions that are evolved through random alterations and selection. The differences between these operations lie in the type of alterations used for generating new solutions, and the mechanism employed for selecting new members.

The ADE algorithm is one of the newly proposed metaheuristics and has been introduced as a search method with rapid convergence rate. This metaheuristic, in which new individual production is performed using the average value of the solutions in the population, collectively provides the evolution of the candidate solution [9]. In this study, the ADE algorithm has been applied to well-known CEDP’s. The obtained results have been compared with the results reported in the literature.

II. The ADE Algorithm

The ADE algorithm is a newly proposed metaheuristic algorithm based on population [9]. This metaheuristic, which has a rapid convergence, provides considerable success in solving the system identification problems. It has six computational phases, including initialization, evaluation, improvising of new trial solution, handling of bound, selection and termination.

2.1 Initialization

In initialization phase, the algorithm parameters are initialized and the initial population is randomly generated within the range of boundaries of variables as follows:

\[ x_{i,j}^{i,G} = x_{i,min} + rand \times (x_{i,max} - x_{i,min}) \quad i = 1,2,...,NP \ and \ j = 1,2,...,D \]

where, \( x \) is the set of solution vector, \( NP \) is the population size or the number of solution vectors, \( D \) is the number of variables, \( x_{i,max} \) and \( x_{i,min} \) are the maximum and minimum allowable values for the \( D \) variable, \( rand \) is a random number in the interval [0, 1], and \( G \) is the generation number.
2.2 Evaluation of solution vectors

The fitness values of the solution vectors are determined at this stage. Fitness values actually represent the quality of vectors. Therefore, the value of each solution vector in the objective function of the problem is taken as the fitness value of that vector.

2.3 Improvising of new trial solution

At this phase, the candidate vector for the next generation is created. Firstly, the average vector in the present generation is computed. This vector is calculated by taking the average of the solution vectors in the present population, as in the following:

\[ \tilde{x}_G = \frac{1}{NP} \sum_{i=1}^{NP} x_{i,G} \]  

Here, \( \tilde{x}_G \) shows the average vector of the generation \( G \). \( NP \) shows the solution number in the population, \( x_{i,G} \) is the current solution vector, and \( G \) shows the current generation. Then, a mutant vector is created by the following equation for each solution vector.

\[ \tilde{u}_{i,G+1} = \tilde{x}_{best,G} + \gamma * rand([-1,1]) \times (\tilde{A}_G - \tilde{x}_{i,G}) \]  

where, \( \tilde{u}_{i,G+1} \) is the mutant vector, \( \tilde{x}_{best,G} \) is the best solution vector in generation \( G \), \( \tilde{A}_G \) is the average vector in generation \( G \), \( \tilde{x}_{i,G} \) is the original solution vector in generation \( G \), \( \gamma \) is the scaling factor, and \( rand([-1,1]) \) is the random number in interval between [-1, 1].

Finally, in order to form the trial vector, \( \tilde{x}_{i,G+1} \), the mutant vector \( \tilde{u}_{i,G+1} \) is put on a crossover with \( Cr \) (crossover rate) possibility together with the original solution vector \( \tilde{x}_{i,G} \) as done in the DE algorithm. Each variable belonging to the trial vector is selected with \( Cr \) possibility from the mutant vector and with \( 1-Cr \) possibility from the original solution vector.

2.4 Handling of bound violations

Constraint violations are checked for candidate solutions produced in the previous stage. If any variable of the trial vector is found to be outside the boundaries defined in initialization, then this variable is assigned the nearest limit value.

2.5 Selection

The decision of transferring the candidate solution to the next generation is decided in this process step. As expressed in the following equation, the vector with better fitness function is transferred to the next generation.

\[ \tilde{x}_{i,G+1} = \begin{cases} \tilde{x}_{i,G+1} & \text{if } f(\tilde{x}_{i,G+1}) > f(\tilde{x}_{i,G}) \\ \tilde{x}_{i,G} & \text{otherwise} \end{cases} \]  

where, \( f(\tilde{x}_{i,G+1}) \) and \( f(\tilde{x}_{i,G}) \) represent the fitness function of \( \tilde{x}_{i,G+1} \) and \( \tilde{x}_{i,G} \), respectively.

2.6 Termination

The five stages described above are maintained until the termination criteria are met. When the number of predefined generations is reached, the computation is stopped and the best vector is considered as the global optimum.

III. Results and Discussions

In this section, simulation studies based on some well-known constrained engineering design problems are carried out for investigating the performance of the proposed ADE algorithm. The selected problems are well-known benchmarks studied by various approaches [4, 8, 10-12]. For an accurate comparison, ADE has been run 30 times independently and it has been taken as \( FEs = 5000 \). In all cases, parameters of the ADE are set as follows: \( NP = 25 \), \( Cr = 0.9 \) and \( \gamma = 2 \).

3.1 The design of a tension/compression spring

The tension/compression spring structure, shown in Figure 1, is a design problem. This problem consists of minimizing the weight \( f(x) \) of a tension/compression spring subject to constraints on shear stress, minimum deflection, and surge frequency. It can be stated as following with three design variables such as the wire diameter, \( d(= x_i) \) the mean coil diameter, \( D(= x_j) \) and the number of active coils \( N(= x_k) \) [10].
\begin{equation}
    f_{\text{const}}(x) = (x_3 + 2)x_3x_1^2 
\end{equation}

Subject to

\begin{align}
    g_1(x) &= 1 - \frac{x_3x_1}{71785} \leq 0 \\
    g_2(x) &= \frac{4x_1^3 - x_1x_3}{12566(x_1x_3 - x_4^1)} + \frac{1}{5108x_1^4} \leq 0 \\
    g_3(x) &= 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0 \\
    g_4(x) &= \frac{x_1 + x_2}{1.5} - 1 \leq 0
\end{align}

The variable regions are limited by

\begin{align}
    0.05 \leq x_1 \leq 2, & \quad 0.25 \leq x_2 \leq 1.3, \quad 2 \leq x_3 \leq 15. \\
    \text{Further, Arora [10]} & \text{ has also provided a solution to this problem using nonlinear programming (NP) technique. Also, Coello [4] has solved this problem using a GA-based method. He and Wang [8] have proposed the co-evolutionary PSO for solving this problem. In addition, some researchers have used newer improved DE algorithms and other metaheuristic algorithms to solve this problem [5-7, 11, 13-18].}
\end{align}

Table 1 presents statistical results of ADE. And, results of ADE are compared with solutions reported by other researchers, as shown in Table 2. From Table 2, it can be seen that the best solution obtained by ADE is better than those of the other methods.

\begin{table}
\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
Best & Mean & Worst & S. D. \\
\hline
0.009873 & 0.009906 & 0.010164 & 0.000073 \\
\hline
\end{tabular}
\end{center}
\end{table}

\begin{table}
\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
Methods & Design parameters & \multicolumn{2}{c|}{f_{\text{const}}(x)} \\
\hline
IHS [13] & \begin{align*}
    d &= x_1 \\
    \overline{D} &= x_2 \\
    N &= x_3
\end{align*} & 0.051154 & 0.349871 & 12.076432 & 0.012670 \\
\hline
NP [10] & Continuous & 0.051680 & 0.356532 & 11.313501 & 0.012677 \\
\hline
Discrete & 0.051200 & 0.345400 & 12 & 0.012680 \\
\hline
GA [4] & 0.051480 & 0.351661 & 11.632201 & 0.012704 \\
\hline
CPSO [8] & 0.051728 & 0.357644 & 11.244534 & 0.012674 \\
\hline
IACO [11] & 0.051865 & 0.361500 & 11.000000 & 0.012643 \\
\hline
rank-MDDE [14] & 0.051689 & 0.356717 & 11.288998 & 0.012665 \\
\hline
MAL-DE [15] & 0.051689 & 0.356717 & 11.288955 & 0.012665 \\
\hline
BA [6] & 0.051690 & 0.356730 & 11.2885 & 0.012670 \\
\hline
ABC [5] & 0.051749 & 0.358179 & 11.203763 & 0.012665 \\
\hline
WCA [16] & 0.051680 & 0.356222 & 11.300410 & 0.012665 \\
\hline
CSA [17] & 0.051689 & 0.3567169 & 11.289011 & 0.012665 \\
\hline
IAFSO [17] & 0.051685 & 0.356629 & 11.294175 & 0.012665 \\
\hline
\end{tabular}
\end{center}
\end{table}

It is observed that the number of coils \( N = x_3 \) is not an integer at the optimum design point. This problem was solved by methods in literature as a continuous case study. If the problem is assumed as a discrete case study, the number of active coils should be integer values [10]. In this case, the problem can be re-optimized for discrete values. Thus, best solution of ADE is

\begin{align}
    f_{\text{const}}(x) &= 0.00998 \\
    x_1 &= 0.050634, \quad x_2 = 0.389267, \quad x_3 = 8
\end{align}

with

\begin{align}
    g_1(x) &= -6.93044E-05 \\
    g_2(x) &= 1.53117E-04 \\
    g_3(x) &= 4.8665 \\
    g_4(x) &= 0.706733
\end{align}
3.2 The design of a pressure vessel

The design of a pressure vessel consists of the minimization of the cost of the pressure vessel as shown in Figure 2. The main purpose is to decrease the total cost [19]. There are totally four different design variables namely, $T_s$ is the thickness of the shell ($= x_1$), $T_h$ is the thickness of the head ($= x_2$), $R$ is the inner radius ($= x_3$), and $L$ is the length of the cylindrical section of the vessel except head ($= x_4$). $T_s$ and $T_h$ show the available thickness of rolled steel plates and these parameters are also the definition of integer multiples of 0.0625 inch in scale. Moreover, $R$ and $L$ are the continuous parameters in a regular pressure vessel designs. The problem can be based on the same explanation using by Coello [19] in below;

$$f_{	ext{cost}}(x) = 0.6224x_1x_2x_3 + 1.7781x_1^2x_3 + 3.1661x_1^3x_4 + 19.84x_2^4x_4$$

(7)

Subject to

$$g_1(x) = -x_1 + 0.0193x_3 \leq 0$$

$$g_2(x) = -x_2 + 0.00954x_4 \leq 0$$

$$g_3(x) = -x_2^3x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \leq 0$$

$$g_4(x) = x_4 - 240 \leq 0$$

(8)

The design space is bounded by $1 \times 0.0625 \leq x_1, x_2 \leq 99 \times 0.0625, 10 \leq x_3, x_4 \leq 200$

Various approaches such as GA [12], HSA [13], and discrete programming (DP) [3] were applied to solve this problem. Also, a detailed mathematical analysis of this problem is provided that proves that 6,059.714335 is the global minimum [20].

![Fig. 2: A pressure vessel](image_url)

The statistical results of ADE and the comparison of results are presented in Tables 3 and 4, respectively. The results show that ADE algorithm reached to global optimum. It is worth mentioning that the best objective value obtained by Eskandar et al. [16] is not feasible since design variables $x_1$ and $x_2$ are not integer multiples of 0.0625.

### Table 3: Statistical results of pressure vessel problem by ADE.

<table>
<thead>
<tr>
<th>Method</th>
<th>Best</th>
<th>Mean</th>
<th>Worst</th>
<th>S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6059.714362</td>
<td>6547.574957</td>
<td>10544.421824</td>
<td>1132.581152</td>
<td></td>
</tr>
<tr>
<td>$g_1(x)$</td>
<td>$g_2(x)$</td>
<td>$g_3(x)$</td>
<td>$g_4(x)$</td>
<td></td>
</tr>
<tr>
<td>-0.000000000677</td>
<td>-0.035880829350</td>
<td>-0.005417838693</td>
<td>-63.363402749637</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4: Comparison of results for the pressure vessel problem.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Design parameters</th>
<th>$f_{	ext{cost}}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IHS [13]</td>
<td>$T_s(= x_1)$</td>
<td>$T_h(= x_2)$</td>
</tr>
<tr>
<td>GA [4]</td>
<td>0.8125</td>
<td>0.4375</td>
</tr>
<tr>
<td>CP SO [8]</td>
<td>0.8125</td>
<td>0.4375</td>
</tr>
<tr>
<td>IACO [11]</td>
<td>0.8125</td>
<td>0.4375</td>
</tr>
<tr>
<td>meta-GA [12]</td>
<td>1.1250</td>
<td>0.6250</td>
</tr>
<tr>
<td>rank-MDDE [14]</td>
<td>13.0</td>
<td>7.0</td>
</tr>
<tr>
<td>MAL-DE [15]</td>
<td>0.8125</td>
<td>0.4375</td>
</tr>
<tr>
<td>BA [6]</td>
<td>0.8125</td>
<td>0.4375</td>
</tr>
<tr>
<td>ABC [5]</td>
<td>0.8125</td>
<td>0.4375</td>
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</table>
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<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>0.7781</td>
<td>0.8125</td>
<td>0.8125</td>
<td>0.8125</td>
<td>1.1250</td>
<td>0.8125</td>
</tr>
<tr>
<td>Shear</td>
<td>0.3846</td>
<td>0.4375</td>
<td>0.4375</td>
<td>0.4375</td>
<td>0.6250</td>
<td>0.4375</td>
</tr>
<tr>
<td>Stress</td>
<td>40.3196</td>
<td>42.0984</td>
<td>42.0984</td>
<td>42.0984</td>
<td>48.9700</td>
<td>42.0984</td>
</tr>
<tr>
<td>End Deflection</td>
<td>-200.00</td>
<td>176.6369</td>
<td>176.6366</td>
<td>176.6366</td>
<td>106.72</td>
<td>176.6366</td>
</tr>
<tr>
<td>Buckling Load</td>
<td>5,885.3327</td>
<td>6,059.7143</td>
<td>6,059.7143</td>
<td>6,059.7143</td>
<td>7,980.8940</td>
<td>6,059.7143</td>
</tr>
</tbody>
</table>

3.3 The design of a welded beam

As it can be seen in Figure 3, the welded beam structure is a practical design problem [4]. The objective is to carry out the minimum fabrication cost of the welded beam subject into the constraints on bending stress, $(\sigma)$, shear stress, $(\tau)$, end deflection, $(\delta)$, buckling load, $(P_c)$, and side constraint. There are four design variables: $h(x_1)$, $l(x_2)$, $t(x_3)$, and $b(x_4)$. The cost function is stated in below:

$$f_{cost}(x) = 1.10471x_1^2 + 0.04811(x_1 + x_2)$$

(9)

Subject to

$$g_1(x) = \tau(x) - \tau_{max} \leq 0$$
$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0$$
$$g_3(x) = x_1 - x_2 \leq 0$$
$$g_4(x) = 0.10471x_1^2 + 0.04811x_1x_4(14.0 + x_2) - 5.0 \leq 0$$
$$g_5(x) = 0.125 - x_1 \leq 0$$
$$g_6(x) = \delta(x) - \delta_{max} \leq 0$$
$$g_7(x) = P - P_c(x) \leq 0$$

(10)

The variable regions are limited by $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$, $0.1 \leq x_4 \leq 2$. Coello [4] and Deb [21] have provided a possible solution to find out the GA based method problems. On the other hand, some other researchers have used metaheuristic methods to solve this problem [5-8, 11, 13, 14, 16-18]. The statistical results of ADE and the comparison of results are presented in Tables 5 and 6, respectively.

where,

$$\tau(x) = \sqrt{\left(\frac{\tau^\prime}{2}\right)^2 + 2\tau\tau^\prime \frac{x_1}{2R} + \left(\frac{\tau^\prime}{2}\right)^2}, \quad \sigma(x) = \frac{6PL}{x_1^2 x_4}, \quad \delta(x) = \frac{6PL^3}{E x_1^2 x_4^3}, \quad P_c(x) = \frac{4.013E \sqrt{x_1^2 x_4^2}}{36 \left(1 - \frac{x_1}{2L} \sqrt{\frac{E}{4G}}\right)}$$

$$J = 2 \left(\frac{\sqrt{2x_1 x_2} \left[\frac{x_3}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]}{\sqrt{2x_1 x_2}}\right), \quad M = P \left(\frac{L + x_3}{2}\right), \quad R = \sqrt{\frac{x_1^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$P = 6,000 \text{ lb}, \quad L = 14 \text{ in}, \quad \delta_{max} = 0.25 \text{ in}, \quad E = 30 \times 10^6 \text{ psi}, \quad G = 12 \times 10^6 \text{ psi}, \quad \tau_{max} = 13,600 \text{ psi}, \quad \sigma_{max} = 30,000 \text{ psi}.$$

Fig. 3: Welded beam structure
As it can be seen in Table 6, the best solution found by the ADE algorithm is better than the other solutions utilizing by other techniques.

### IV. Conclusions

Optimization is a significant issue in the design process of engineering optimization problems. An optimizer aims at achieving the optimal solution for design problems that are encountered in several areas. Over the last two decades, metaheuristic algorithms have been successfully applied as an optimum utilized for solving complicated real-world optimization problems. These algorithms have carried out conventional numerical methods and also provide the optimal solution. Thus, researchers have focused on improving these metaheuristic algorithms.

In this study, an effective metaheuristic algorithm (ADE) has been successfully applied to design problems. A comparative study has been carried out to show the effectiveness of the ADE over other methods. The results indicate that the proposed method provides successful results in general. The proposed method is promising for future works especially for the solution of complex real-world problems including optimum design.

### References


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