Application of Fuzzy $m^*_X$ Oscillation in the Field of Face Recognition using Rough Set

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Abstract: The aim of this paper is to introduce the new mathematical concept of Fuzzy $m^*_X$ Oscillation using rough sets. Fuzzy-rough $m^*_X$ Oscillation is a new concept which has been applied on some set of images. With the help of Fuzzy-rough $m^*_X$ Oscillation an unknown face image can be distinguished from a set of known face images. In this paper we introduce a new algorithm based on the theory of Fuzzy-rough $m^*_X$ Oscillation and using MATLAB 7.9 software we implemented this algorithm. Experiments are performed to test the proposed algorithm on Face fix Database and ORL database. Accuracy of the results describes the application of Fuzzy-rough $m^*_X$ Oscillation in the field of face recognition.

Keywords: Face recognition, Fuzzy $m^*_X$ Oscillation, Fuzzy-rough $m^*_X$ Oscillation, Rough set, etc.,

Date of Submission: 07-07-2017
Date of acceptance: 22-07-2017

I. Introduction

The Oscillatory region concept of a fuzzy set which means the oscillation of a fuzzy set occurs between at least two fuzzy open set and two fuzzy closed set, was first introduced A. Mukherjee and S. Halder in 2007[11]. Previously the concept of minimal structure in general topological space was the outcome of research work by Popa and Noiri, presented in 2000[14]. After that in the year 2006 M. Alimohammady and M. Roohi introduced the concept of fuzzy minimal structure[1]. More analysis in Fuzzy minimal structure and fuzzy minimal space is obtained in [1-4]. Lots of research works along with their application are going on in this field.

The concept of $m_X$ open set was introduced by H Maki in 1986[9]. Later on M Alimohammady and M.Roohi presented the concept of fuzzy $m_X$ structure in 2006[1]. The concept of Fuzzy Minimal Structure or Fuzzy Structure Oscillation was first introduced by S. Bhattacharya (Halder) and Susmita Roy in 2010[6]. In that paper authors have only developed the conceptual part with some examples. In the paper introduced by S Bhattacharya (Halder) and Susmita Roy[6], the concept of $m_X^*$ structure has been introduced, since in an image it is not mandatory that a full black and a full white part is present but at least one part may belong, which is the concept of $m_X^*$ structure.

This paper is motivated by concept of Fuzzy $m^*_X$ Oscillation to apply in the domain of Face recognition. In this paper a new algorithm is proposed and for experimental purpose, it is verified using MATLAB 7.9 software. Last portion of this paper contains result of the experiment. It is observed that using this technique face recognition can be done with accuracy depending on the number of feature coordinates.

The aim of this paper is to propose a new algorithm for face recognition and show its functionality while it is used to recognize an unknown image from a set of known images. The theorem proposed in [6] for Fuzzy-rough $m^*_X$ Oscillation was mathematically verified with a single face image with few pixel intensity values. In this paper the proposed algorithm was implemented using Matlab7.9 and verified on various available Face database collected from FacePix database [18] considering a set of important feature values. In section two the preliminaries are described which are required for the proposed algorithm. In section three the concept the Fuzzy-rough $m^*_X$ Oscillation algorithm has been introduced. In section-III one block diagram of the face recognition process is shown. Along with the block diagram the complete algorithm is presented which tested using MATLAB program in the next section. The experimental part is discussed in section IV. Finally the end portion contains the review of literature and conclusion.

II. Preliminaries

The concept of $V^-$ set and $A^-$ set was first introduced by Maki [9]. The similar set can be introduced in fuzzy environment which varies between at least two open and two closed set. The fuzzy set is bounded
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between the infimum of all open set (called fuzzy closed set) containing the set and the supremum of all fuzzy open (called fuzzy closed) contained in the set. This is defined as “fuzzy oscillation” in the paper [11].

In this section we shall illustrate those notions, which are necessary to define concepts used in the paper. The notation and definition of open and closed set, minimal structure, fuzzy oscillation etc. are given below which are back ground of the proposed algorithm.

Definition 2.1[9]: H. Maki called a subset $A$ of an ordinary topological space $(X, T)$ a $\alpha$- set if it is the intersection of open sets containing the set i.e. $\alpha(A) = \cap \{ G : G \supseteq A, G \text{ is an open set} \} = A$. Also $\alpha(A^c) = 1 - V(A)$, CL($A^c$) = $1 - \text{Int}(A)$, $V(A) \subseteq A \subseteq \text{Cl}(A)$, for any subset A of X.

Definition 2.2[14]: A subfamily $m_{aX}$ of $P(X)$ power set of $X$, is called minimal structure on $X$ if $\emptyset \in m_{aX}$ and $X \in m_{aX}$. Each member of $m_{aX}$ is said to be $m_{aX}$-open set and the complement of a $m_{aX}$-open set is said to be a $m_{aX}$-closed set.

Definition 2.3[1]: A family M of fuzzy sets in $P(X)$ is said to be a fuzzy minimal structure on $X$ if $\alpha I_x \in M$. A fuzzy $m_X$ - structure set on $X$ is defined as a set where at least $\mu_x$ and/or $\mu_\alpha$ belongs to $m_X$ with $\alpha \in I^x$. In this case $(X,M)$ is called a fuzzy minimal space.

Definition 2.4[1]: On fuzzy $m_X^*$ oscillation: A subfamily of fuzzy sets is said to be fuzzy structure on $X$ at least $\mu_x$ and/or $\mu_\alpha$ belongs to $m_X^*$ with $\alpha \in I^x$. Members of $m_X^*$ structure are $m_X^*$ open set.

Definition 2.5[1]: A fuzzy-rough $m_{aX}$ set on $X$ is defined as a set where at least $\mu_x$ and/or $\mu_\alpha$ belongs to $m_{aX}$ with $\alpha \in I^x$, where $\alpha \in I^x$. Members of fuzzy-rough $m_{aX}^*$ - structure are fuzzy-rough $m_{aX}^*$ lower approximation and their complements are fuzzy-rough $m_{aX}^*$ upper approximation.

Definition 2.6: The operator $\land : I^1 \rightarrow I^1$ is defined as

\[(i) \land_y (x) = \inf \{ \mu_{a_j} (x) : \mu_{a_j} (x) \geq \mu_{a_j} (x), x \in G, G \text{ is an lower approximation of rough set} \} = \hat{I}, \text{ if no such lower approximation of rough set exists.} \]

\[(ii) \text{Int}_y (x) = \sup \{ \mu_{a_j} (x) : \mu_{a_j} (x) \leq \mu_{a_j} (x), x \in G, G \text{ is an upper approximation of rough set} \} = \phi, \text{ if no such upper approximation of rough set exists.} \]

\[(iii) \text{Cl}_y (x) = \inf \{ \mu_{a_j} (x) : \mu_{a_j} (x) \geq \mu_{a_j} (x), x \in G, G \text{ is an lower approximation of rough set} \} = \hat{I}, \text{ if no such lower approximation of rough set exists.} \]

\[(iv) \text{V}_y (x) = \sup \{ \mu_{a_j} (x) : \mu_{a_j} (x) \leq \mu_{a_j} (x), x \in G, G \text{ is an upper approximation of rough set} \} = \phi, \text{ if no such upper approximation of rough set exists.} \]

Where $\mu_{a_j} (x)$ is the membership value of any particular attribute $a_j$ of any object $x_i$.

Definition 2.7: An operator $O^c : I^1 \rightarrow I^1$ such that $O^a_{a_j}(x) = \land_y (x) - \text{Int}_y (x)$, is said to be fuzzy-rough $m_{a}^*$-lower approximation oscillatory operator and an operator $O^o$: 

$I^1 \rightarrow I^1$ such that $O^o_{a_j}(x) = \text{Cl}_y (x) - \text{V}_y (x)$, is said to be fuzzy-rough $m_{a}^*$-upper approximation oscillatory operator.

Theorem 2.8: For any object y in fuzzy-rough $m_{a}^*$-structure, $O^o_{a_j}(y) = \alpha$ implies $O^o_{a_j}(y') = -\alpha$ and vice versa where $\alpha \in I^x$.

Remark 2.9: Oscillation can never be negative, so $-\alpha$ may indicate oscillation in the opposite direction.

Remark 2.10: (i) If $O^a_{a_j}(y) = O^a_{a_j}(y') = 0$ then $\land_y (y) = \text{Int}_y (y)$ and $\text{Cl}_y (y') = \text{V}_y (y')$.

(ii) If $O^a_{a_j}(y) = O^a_{a_j}(y') = 1$ then $\land_y (y) = \text{Cl}_y (y') = 1$ and $\text{Int}_y (y') = \text{V}_y (y') = 0$.

2.11 Application of fuzzy $m_{a}^*$ oscillation on image comparison: [6]

In Paper [6] author introduced the concept of fuzzy $m_{a}^*$ oscillation. Along with that, some process were described how different values of the oscillation operator used as a measure of similarity or difference.

An image database of different image forms a space where due to brightness or darkness or different pose the pixel with gray level membership value 0 or 1 must lie in the collection. This collection is an $m_{a}^*$ structure and the members are $m_{a}^*$ open set. Suppose a database contains images $I = I_1, I_2$. To compare an unknown image with the known image by the help of fuzzy $m_{a}^*$-oscillation the following cases might appear according to the values of oscillator operator.

For $O^o_{i_1k}(Y_{ij}) = A_{i_1k}(Y_{ij}) \cdot \text{Int}_{i_1k}(Y_{ij})$, the following cases may appear [6]:

1. $O^o_{i_1k}(Y_{ij}) = 0$ or 1
2. $0 < O^o_{i_1k}(Y_{ij}) < 1$
3. $O^o_{i_1k}(Y_{ij}) = A_{i_1k}(Y_{ij}) - \phi$
4. $O^o_{i_1k}(Y_{ij}) = I \cdot \text{Int}_{i_1k}(Y_{ij})$
Case (1): Let possible \( O^0_{ik}(Y_{ij}) = \Lambda_{ik}(Y_{ij}) \cdot \text{Int}_{ik}(Y_{ij}) = 0 \leftrightarrow \text{Image } I_k \) and the unknown image are same at the pixel \((i, j)\).

Now if possible let \( O^0_{ik}(Y_{ij}) = \Lambda_{ik}(Y_{ij}) \cdot \text{Int}_{ik}(Y_{ij}) = 1 \cdot \text{Int}_{ik}(Y_{ij}) = 0 \), i.e. the intensity of the unknown image at pixel \((i, j)\) is not within a known position. So this pixel is not compare with the pixel \((i, j)\) of known image. So this pixel is undefined pixel.

Case (2): Let possible \( 0 < O^0_{ik}(Y_{ij}) < 1 \) i.e \( 0 < \Lambda_{ik}(Y_{ij}) \cdot \text{Int}_{ik}(Y_{ij}) < 1 \).

(i) If \( O^0_{ik}(Y_{ij}) \leq 0.1 \), there may be some similarity between the two images at pixel \((i, j)\).

(ii) If \( O^0_{ik}(Y_{ij}) \geq 0.1 \), we have to check the difference between the intensity of the unknown image at pixel \((i, j)\) and \( \Lambda_{ik}(Y_{ij}) \) or \( \text{Int}_{ik}(Y_{ij}) \). If this difference is \( \leq 0.1 \) then also the images may be similar at the pixel \((i, j)\).

Case (3): \( O^0_{ik}(Y_{ij}) = \Lambda_{ik}(Y_{ij}) - \phi \leftrightarrow In this case we may check the difference between the intensity of the unknown image at pixel \((i, j)\) and \( \Lambda_{ik}(Y_{ij}) \). If this difference is \( \leq 0.1 \) then, the images may be considered as similar at the pixel \((i, j)\). Otherwise different.

Case (4): \( O^0_{ik}(Y_{ij}) = 1 \cdot \text{Int}_{ik}(Y_{ij}) \), In this case we may check the difference between the intensity of the unknown image at pixel \((i, j)\) and \( \text{Int}_{ik}(Y_{ij}) \). If this difference is \( \leq 0.1 \) then, the images may be considered as similar at the pixel \((i, j)\), Otherwise different. The case \( O^0_{ik}(Y_{ij}) = \text{Cl}_{ik}(Y_{ij}) \cdot V_{ik}(Y_{ij}) \) follows similarly as above cases.

### III. Fuzzy rough \( m^x_\lambda \) Oscillation Based Face Recognition

In this section the Fuzzy rough \( m^x_\lambda \) Oscillation based algorithm is introduced and how it can be used to recognize the faces are shown. In initial phase of the algorithm images from any image database are used as training image set. These are considered as known faces. Some location, actually some coordinates in the face image are pointed to be considered as feature coordinates. Coordinates of eyeball, lips corners may be the feature coordinates. Pixel values of these coordinates from every training image are captured (including the face about which decision to be made). These pixel values from known image matrix with which the unknown image matrix is compared at each feature coordinate. First the operator \( \Lambda \) (called \( \text{Cl}_\lambda \) operator for negative images) and then the \( \text{Int} \) (called \( \text{Cl} \) operator for negative images) operators are measured. The difference between the \( \lambda \) and \( \text{Int} \) operator called Open Oscillator Operator (Closed Oscillator Operator for negative images) are estimated. Open and Closed Oscillator are the aimed derived values from both image matrices depending on which different cases(introduced in [6]) would be considered for similarity measurement.

Fig. 1

The Figure 1 indicates one sample image which shows the feature coordinates that is, the pixel values of that particular position which are under consideration. We emphasize on extracting pixel value from eye-ball, lips corners.

### A. Face Recognition Procedure

**Step 1:** Suppose we have an image database consist of \( M \) images, these are considered as input image or Training samples. Some feature values from some fixed coordinate need to be collected from each image of the database. Simultaneously the feature values also collected from the unknown image considering the same coordinate value.
**Step 2:** Now $A$ operator ($\text{LAMDA\_OPTR}$) will be calculated. The operator consist of Imfimum value of training set images feature value matrix and the unknown images feature value matrix. In this way $\text{Int}$ operator ($\text{INT\_OPTR}$) with supremum value value is measured.

**Step 3:** For the complementary image the operator with Imfimum and Supremem value that is $\text{CL\_OPTR}$ and $\text{V\_OPTR}$ respectively need to be calculated. Then the open oscillator $O^o$ value from normal image and the closed oscillator values $C^o$ from the complementary images are calculated.

**Step 4:** After the open oscillator and the closed oscillator is calculated the four cases of similarity measure as stated in [6] are considered. In this phase the decision about similarity and difference is drawn. The decision is made considering the pixel by pixel comparison.

**B. Block-Diagram of Fuzzy rough $m^x_X$ Oscillation Based Face Recognition**

Following block describes each process which ensures the face recognition process.

**C. Algorithm:**

Fuzzy rough $m^x_X$ Oscillation Based Face Recognition

Input: Image data extracted from any Image database, Considered as known images for training.

Output: Decision if any unknown image is SIMILAR or not with the known image.

1. **Input any known image database**
2. $K[m,n] \leftarrow$ Select some feature values (Pixel values of similar coordinates from every image)
3. **Input the known/unknown image database**
4. $UaK[1,n] \Rightarrow$ Retrieve the feature value from that coordinates
5. Then find the new open database ($K[m,n]$) using the Rough membership function theory.
6. **Then Input the target image database**
7. $UaK[1,n] \Rightarrow$ Retrieve the feature value from that coordinates

Apply Fuzzy-rough $m^x_X$ Oscillation on $K[i]$
8. Calculate \( \text{LAMDA\_OPTR}(\text{KI}[m,n], \text{UKI}[1,n]) \)
    \[ a[1,n] \leftarrow \min_{\text{Max}_{\text{Val}}(\text{KI}[m,n], \text{UKI}[1,n])} \]
9. Calculate \( \text{INT\_OPTR}(\text{KI}[1,n], \text{UKI}[1,n]) \)
    \[ \text{Int}[1,n] \leftarrow \max_{\text{Min}_{\text{Val}}(\text{KI}[m,n], \text{UKI}[1,n])} \]
10. Calculate \( \text{Open Oscillator Operator} O^O \)
    \[ O^O[1,n] = a[1,n] - \text{Int}[1,n] \]
11. Calculate the Negative or Complementary image set \( \text{NKI}[m,n] \) and \( \text{NUKI}[1,n] \)
12. Calculate \( \text{CL\_OPTR}(\text{NKI}[1,n], \text{NUKI}[1,n]) \)
    \[ \text{CL}[1,n] \leftarrow \min_{\text{Max}_{\text{Val}}(\text{NKI}[m,n], \text{NUKI}[1,n])} \]
13. Calculate \( \text{V\_OPTR}(\text{KI}[1,n], \text{UKI}[1,n]) \)
    \[ V[1,n] \leftarrow \max_{\text{Min}_{\text{Val}}(\text{NKI}[m,n], \text{NUKI}[1,n])} \]
14. Calculate \( \text{Closed Oscillator Operator} O^C \)
    \[ O^C[1,n] = \text{CL}[1,n] - V[1,n] \]
15. Consider four cases of \( O^O \) and \( O^C \) for pixel by pixel comparison.
16. Decision about Similarity and Difference.

IV. Experiment

The experiment is performed using FacePix database which is a face image database created at the Center for Cognitive Ubiquitous Computing (CubiC) at Arizona State University, and it is available free of charge to the worldwide. The database contains different pose of an individual. Pose of a sample face image vary from angle +90 degrees to -90 degrees, where +90 degrees represents a left profile view, 0 degrees represents a frontal view, and -90 degrees represents a right profile view. (An example of this set is shown below.) Every face image is of 107 x 107 column and row. Each image is unit8 class, and the type of image is true color ‘.png’ image. “Fig. 4” Shows the images set is used as training set.

We use MATLAB 7.9 for our experiment. To test our algorithm we develop a Matlab program. We first manually located 30 feature coordinates from the 0 degrees frontal view of a face image. The pixel values are fetched from coordinates of left eyeball center, right eyeball center, nose tip and two mouth corners as shown in figure 1. After that pixel values from similar coordinate are fetched from every facial pose. After completion of these initial phases one image different individual is compared and the program produce expected result. On the other hand for the same individual, if compared, it declared it as similar face.

Fig 3. Shows Pixel points for the known images (Training Set) of any instance. The image is captured in an in-between stage of the program execution which shows the feature coordinate based pixel values (matrix), which will be individually compared with unknown image pixel values. In this way the supremum value and the infimum value is measured which subsequently produce the oscillator values. According to the value of oscillator similarity or difference is decided. In case of complementary image also, how the oscillator oscillates, give the decision of similarity or something else.
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Table I. For Different face (Comparing with an image of another individual)

<table>
<thead>
<tr>
<th>Number of Coordinate of Feature</th>
<th>Number of Training Set of an Individual</th>
<th>Number of Similar Pixel(s)</th>
<th>Number of Different Pixel(s)</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.79</td>
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<td>20</td>
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<td>0.74</td>
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<tr>
<td>30</td>
<td>15</td>
<td>26</td>
<td>33</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Fig 4: Pixel points for the known images (Training Set) of any instance

Table II. For similar face (Comparing with different image of similar individual)

<table>
<thead>
<tr>
<th>Number of Coordinate of Feature</th>
<th>Number of Training Set of an Individual</th>
<th>Number of Similar Pixel(s)</th>
<th>Number of Different Pixel(s)</th>
<th>Accuracy</th>
</tr>
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</tr>
</tbody>
</table>

V. Review Of Literature

During the last many years different research work is going on in the area of face recognition applying Fuzzy set theory. Many researcher proposed different new mathematical and statistical methods along with fuzzy to introduce better face recognition or face detection techniques. Neural network based techniques combining with Fuzzy is very effective for face recognize, making decision about the known and unknown face. In Neuro-fuzzy techniques Neural network need to be properly trained by different mathematical model. In [7, 15, 13] Combination of these two techniques produce better accuracy but it requires much computational cost and storage. Any appearance based approach collect the whole image information for analyze and it sometime requires the dimension need to reduce. In [16] authors propose appear based approach where the whole image pixel values are stored as one vector. After that fuzzy based nearest neighbor classification is applied on the pixel values. According to value of membership function the belongingness of each pixel to any specific class is defined. Then PCA is used for dimension reduction and Euclidian distance is measured for comparison NN class for known and unknown image. So this procedure produces better accuracy on account of greater computational complexity. In [10] authors apply the visual perception method of newborn to capture the find the feature lines of faces. After applying the dimension reduction algorithm followed by fuzzy classification is incorporated to recognition purpose. [5] In this paper detecting face is done by skin color of face image. It is compared some predefined RGB values. Then entropy based selection is applied in the intermediate data. Finally fuzzy logic is applied to final detection of the face from a whole image. [16] In this paper author explore facial expression
Recognition by fuzzy. Knowledge is formed for facial emotion information storing and according to input data images are classified.

VI. Conclusion

In our paper we select important information carrying pixels. So the whole image is needed not to be considered. Therefore the stage of dimension reduction is not under consideration here. The concept of fuzzy $m^x$ and fuzzy $m^y$ Oscillation is a new mathematical model which is applied in the field of face recognition. It involves fuzzy rough $m^x$ Oscillation for image comparison which is done by applying oscillation operator on the important feature carrying pixel (pixel from eyes, nose, mouth ….). It produces better accuracy with less computational cost.

We propose an algorithm based on the concept of Fuzzy rough $m^x$ Oscillation. We verify our algorithm developing Matlab program and test it on different available Face database. We receive expected result in most of the cases. Increasing the number of training set accuracy of expected result also increases. Therefore this paper shows Fuzzy rough $m^x$ Oscillation successfully recognizes faces. In our future work, we plan to carry out further experiments with Fuzzy rough $m^x$ Oscillation for the whole faces of a person and get better result with more accuracy.

References