Outage Probability Performance Analysis of a New Hybrid Relay Selection Protocol

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Abstract: In this paper, the diversity gains of wireless relay networks were exploited. The proposed cooperative scheme using the first best relay for providing the highest SNR at the destination, whereas the second best relay is used for suboptimal SNR at the destination. The source communicates with the single antenna destination, with the help of multiple AF relays, equipped with a single antenna. Specifically, the BEP and outage probability performance of AF cooperation with the first and second best-relay selection scheme were investigated. Analysis proves that multiple antennas at the source node with best-relay selection can achieve full diversity for transmission, which significantly improves the received quality at destination. In particular, closed-form expressions for the probability density function (PDF) and cumulative density function (CDF) of SNR at the destination are derived. Subsequently, closed-form expressions for the outage probability are obtained.

Keywords: BEP; hybrid relay selection; outage probability.

I. Introduction

Cooperative diversity has recently been widely discussed in wireless networks as a promising technique to overcome the limitations imposed by multiple-input-multiple-output (MIMO) systems. However, the advantage of cooperative diversity comes at the cost of a reduction in spectral efficiency. To alleviate these drawbacks, the technique of best relay selection can be used to improve transmission performance [1, 2, 3, 4, 5]. Performance analysis of best-relay selection schemes has been widely undertaken in the research literature [1, 2, 6]. The first best-relay scheme is proposed by Bletsas et al. in [1], where the best-relay with the highest SNR is chosen at the destination to assist in the transmission of source signals. It is shown that the proposed system achieves the same diversity order as the regular cooperative diversity in terms of the outage probability. In [2], closed-form expressions of the symbol error rate (SER), error probability, and outage capacity of AF cooperative diversity systems with best-relay selection, are derived. However, in some practical cases, the best relay may be unavailable due to some scheduling or load balancing conditions. As a result, the selection may be made to choose the second or more generally the Nth best relay. In [6], the performance of AF with Nth best-relay selection over independent and identical Rayleigh fading channels is analyzed, where approximate expressions of the error probability and outage probability are derived.

Recently, an alternative relay selection scheme based on a hybrid relay protocol has received a lot of attention. The scheme divides the relays into two groups, an amplify-and-forward (AF) relay group and a decode-and-forward (DF) relay group, in order to exploit the merits of both relay groups. By incorporating relay selection with hybrid relaying protocol, overall system performance can be further improved. This process is known as hybrid relay selection [7, 8, 9, 10, 11]. In [7, 8], the hybrid relay selection scheme has been proposed. In this scheme, all relays are classified into two relay groups (AF and DF relay groups) based on whether it decodes correctly or not. Among all relays in both AF and DF groups, the best relay will be selected in [7], while in [8], all relays of two groups can participate in communication. In [10], the authors analyzed the performance of hybrid relaying protocol with distributed turbo coding. In order to avoid channel estimation, differential modulation with hybrid relay selection was proposed in [11]. The single relay scenario was extended to a multiple relay scenario in [9], where all relays participate in the transmission process without relay selection.

The contributions of this paper are four-fold:
1- The proposed system is based on an improved hybrid relay selection protocols proposed in [12] and [13], where all relays are classified into two groups, referred to as AF group and DF group. The key difference of the proposed protocol in this paper, compared with those in [7, 14, 9], is that the best relay is chosen from each group;
2- Closed-form expressions for the probability density function (PDF) and cumulative density function (CDF) of the SNR at the destination, are derived;
3- Using the PDF and CDF, closed-form expressions for the outage probability is obtained;
4- Subsequently, results of the outage probability and PSNR are presented to demonstrate the proposed scheme in terms of the received signal quality.
II. Proposed Hybrid relay selection protocol

The proposed design of the transmission system is aimed to improve the overall performance by considering the hybrid relay selection (HRSP). The proposed HRSP has the advantages of both AF and DF relay schemes and it is assumed that the proposed system uses the best DF and AF relays for transmission. Each node in the proposed two-hop multi relay network is assumed to be equipped with a single antenna and the half-duplex transmission mode is considered. All links will experience quasi-static Rayleigh fading, where the channel coefficients will remain unchanged throughout one block, but change independently from block to block. The block diagram of the proposed system is shown in "Fig. 1"

![Fig. 1: The proposed Hybrid Relay Selection.](image)

The corresponding received signal at the relay $i$, at time $t$, denoted by $y_{s,r_i}(t)$, can be expressed as:

$$y_{s,r_i}(t) = \sqrt{P_{s,r_i}}h_{s,r_i}x(t) + w_{s,r_i}(t),$$

where $P_{s,r_i}$ is the received signal power at the relay $i$ and $x(t)$ is the signal transmitted by the source at time $t$. $h_{s,r_i}x(t)$ denote the channel gains between the source and relays, and the relay nodes and destination, respectively.

$w_{s,r_i}(t)$ denote the AWGN with zero-mean and with variance of $N_0$ between the source and destination and the relay and the destination, respectively.

The received signal represented by $y_{r_i,d}$ at the destination, can be written as:

$$y_{r_i,d}(t) = h_{r_i,d}x_r(t) + w_{r_i,d}(t),$$

where $h_{r_i,d}$ is the channel fading between the relay $i$ and the destination and $w_{r_i,d}(t)$ is the AWGN at the destination.

The received signal at destination from AF group can be written as:

$$y_{r_i,d}(t) = \beta_i h_{r_i,d}x_r(t) + w_{r_i,d}(t),$$

$$y_{r_j,d}(t) = \beta_i h_{r_j,d}y_{r_i,d}(t) + w_{r_j,d}(t),$$

where $h_{r_j,d}$ is the channel fading between the relay $i$ and the destination and $w_{r_j,d}(t)$ is the AWGN at the destination.

The received signal at destination from DF group can be expressed as:

$$y_{r_j,d}(t) = \sqrt{P_{r_j,d}}h_{r_j,d}s(t) + w_{r_j,d}(t),$$

where $y_{r_j,d}(t)$ is the instantaneous SNR of the $r_{j-1-D}$ link.

The overall received signal at destination after the selection of the two relays can be written as:

$$\gamma_{AF} = \max_i \frac{P_{s,r_i}|\beta_i h_{r_i,d}|^2}{N_0}$$

From equation (5), the instantaneous SNR at the destination from $i^{th}$ relay in AF group can be evaluated as:

$$\gamma_{DF} = \max_i \frac{P_{r_i,d}|\beta_i h_{r_i,d}|^2}{N_0}$$

From equation (5), the corresponding SNR at the destination from $j^{th}$ relay in DF group is equal to:

$$\gamma_{DF} = \max_i \frac{P_{r_j,d}|\beta_i h_{r_j,d}|^2}{N_0}$$

where $y_{r_j,d}$ is the instantaneous SNR of the $r_{j-1-D}$ link.
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The maximum instantaneous SNR of the selected relay from AF group can be approximated at high SNR as follows:

\[ F_{\gamma_{\text{AF},i}}(\gamma) = \prod_{j=1}^{q} \left[ 1 - \left( 1 - F_{\gamma_{\text{AF},j}}(\gamma) \right) \left( 1 - F_{\gamma_{\text{DF},i}}(\gamma) \right) \right]. \]  

(8)

In the same manner, the CDF of DF group can be approximated as:

\[ F_{\gamma_{\text{DF},(n-q)}}(\gamma) = \prod_{j=q+1}^{n} F_{\gamma_{\text{DF},j}}(\gamma). \]  

(9)

By using the convolution method, the PDF at the destination is obtained from:

\[ f_{\gamma_{\text{HRSP}}}(\gamma) = \int_{0}^{\gamma} f_{\gamma_{\text{DF},(n-q)}}(\gamma - \gamma') f_{\gamma_{\text{AF},i}}(\gamma') d\gamma'. \]  

(10)

By substituting (8) and (9) into (10), then the PDF at the destination is obtained as follows:

\[ f_{\gamma_{\text{HRSP}}}(\gamma) = \sum_{i=1}^{q} \left( \frac{\gamma}{H \cdot R_{S}} \right) \left( \frac{n-q}{i} \right) \left( n-q \right) \left( 1 + i \right) \int_{0}^{\frac{\gamma_{\text{eq}}}{\gamma}} \int_{0}^{\frac{\gamma_{\text{rd}}}{\gamma}} e^{-i \frac{\gamma_{\text{eq}}}{\gamma}} e^{-i \frac{\gamma_{\text{rd}}}{\gamma}} d\gamma_{\text{eq}} d\gamma_{\text{rd}}. \]  

(11)

The outage probability is defined as the probability that the end-to-end instantaneous SNR falls below a certain predefined threshold value. Thus, the outage probability \( P_{\text{out}} \) is given by:

\[ P_{\text{out}} = P_{\gamma_{\text{HRSP}}(\gamma < \gamma_{\text{th}})} = \int_{0}^{\gamma_{\text{th}}} f_{\gamma_{\text{HRSP}}}(\gamma) d\gamma. \]  

(12)

Given the PDFs of \( \gamma \) in (8), (9), and (11), the \( P_{\text{out}} \) can be obtained as:

\[ P_{\text{out}} = \sum_{i=1}^{q} \sum_{j=q+1}^{n} \left( \frac{\gamma}{j} \right) \left( \frac{n-q}{i} \right) \left( n-q \right) \left( 1 + i \right) \int_{0}^{\frac{\gamma_{\text{eq}}}{\gamma}} \int_{0}^{\frac{\gamma_{\text{rd}}}{\gamma}} e^{-i \frac{\gamma_{\text{eq}}}{\gamma}} e^{-i \frac{\gamma_{\text{rd}}}{\gamma}} d\gamma_{\text{eq}} d\gamma_{\text{rd}}. \]  

(13)

**IV. Results**

Fig. 2" represents the average PSNR performance against SNR for HRSP with 2 and 4 relays in each group. It is observed that the proposed scheme has the best performance compared with EEP for all SNR values. The performance gap widens with the increase of SNR. For example, the proposed HRSP scheme outperforms EEP in the SNR range of 6 - 12 dB by about 0.6 - 1.7 dB, when number of relays is 2 \((M = 2)\). When SNR is high (i.e. SNR = 12 - 18 dB), the HRSP has a PSNR gain of about 1.7 - 4.3 dB. This indicates that the HRSP results in a better performance, especially at moderate-to-high SNRs, compared to EEP.

"Fig. 3" shows the outage probability performance of the proposed HRSP discussed in Section III with different numbers of relays versus SNR. "Fig. 3" plots \( P_{\text{out}} \) and \( P_{\text{out}}^\text{direct} \) with respect to \( M \) and \( N \). The curves compare the outage probabilities obtained via the theoretical expressions and simulations for non-cooperative (direct transmission) and cooperative schemes. As expected, the outage probabilities decrease with \( M \). It can also be observed that the cooperative scheme outperforms the non-cooperative scheme in outage performance. Furthermore, "Fig. 3" demonstrates that the simulation curves nearly overlap with their analytical counterparts, which validates the outage probability analysis in Section III.

![Fig. 2: PSNR performance of HRSP with different numbers of relays.](https://www.iosrjournals.org)

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In this paper, the diversity gains of wireless relay selection were exploited. Specifically, a new hybrid relay selection was proposed. Closed-form expressions for the PDF, CDF, and MGF of the end-to-end SNR were deduced. Then, closed-form expression of the outage probability is derived for the proposed system in Rayleigh fading channels. Results demonstrate that the outage probability performance of the proposed system decreases significantly with the increase in the number of relays $M$, and increases linearly with $N$. Also, it can be seen that the new hybrid relay selection scheme achieves significant PSNR performance gains comparable to those of EEP, particularly at medium to high SNRs.

References