# Simulation Based Analysis of the Hierarchical Timed Coloured Petri Net Model of the Nigerian Voting System

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Abstract: During Nigerian 2015 general elections, in an attempt to exercise their civic rights in voting for the candidates of their choices, many electorates exercised the limitations of their patience by tiredly waiting on queue for their turn at the poll centers. In order to study the aforementioned scenario, this paper carried out a simulation based analysis of a Hierarchical Timed Coloured Petri Net (HTCPN) model, which abstracted the Re-Modified Open-Secret Ballot System (REMOBS) adopted in the 2015 Nigerian general elections. Using CPN Tools, ten simulation runs were carried out on the HTCPN model to experiment the effect 300 and 750 voters on service time, queue length and average waiting time while dedicating Assistant Polling Officers I, II and III (APOI, APOII and APOIII) or four polling officers (two APOI and two APOII) to each polling unit. The validation of the HTCPN model was determined by carrying out a statistical analysis (T-Test) between the simulated and observed voters' arrival times. By employing three polling officers (APOI, APOII and APOIII) at the polling unit, the simulation results revealed that service times of 5 hours and 13 hours 57 minutes would be required to accredit 300 and 750 intending voters, respectively. The simulation results also established that a maximum queue length of 105 and 480 voters was reached during accreditation of 300 and 750 voters, respectively. Similarly, a maximum waiting time of 1 hour 20 minutes and 10 hours was recorded in the process of accrediting of 300 and 750 voters, respectively. With four polling officers (two APOI and two APOII) per polling unit, the simulation results revealed that service times reduced from 13 hours 57 minutes to 6 hours 47 minutes to completely accredit 750 voters. The simulation results also established that average queue length reduced from 240 to 155 voters during accreditation of 750 voters. Similarly, the maximum waiting time was reduced from 13 hours 53 minutes to 1 hour 14 minutes in the process of accrediting of 750 voters. Statistically, there were no significant differences between the simulated and observed number of the voters arriving at the polling unit at 5% level. Thus, the results obtained from the simulation based analysis of the HTCPN model could serve as a basis for decision makers to improve and manage the issues of prolonged queue which often disfranchise intending voters during elections.

Keywords: Voters, Election, Petri Nets, modularized-model, voter arrival rate.

### I. Introduction

Political conditions in Africa are the greatest impediment to development [1]. It has been established that the best approach for the emergence of political leadership in modern world is through an electoral process. Many countries all over the world are therefore constantly refining their electoral processes to ensure the emergence of the right kind of political leadership. From the birth of democracy in Athens in sixth century BC and the first form of electoral laws, electoral systems have been designed and developed according to country particularities in democratic governments worldwide [2]. In Nigeria, the process of refining the electoral process has been a recurrent one. Since the return to civil rule in 1999 after many years of military rule, the electoral law guiding the conduct of elections has been amended after every election leading to the enactment of the 2001, 2006 and 2010 electoral acts [1].

Elections in Nigeria have always been with one challenge or the other, most importantly is the issue of voters' disenfranchisement which has nearly cripple the stability of the country. The visibility of long lines is a visible sign of the administrative friction of managing elections and therefore a convenient symbol for those who seek to improve election administration [3]. It discourages some people from voting, undermines confidence in the electoral system, and imposes economic costs on voters. There is very little data to explain the operational inefficiencies that lead to long queues at polling stations. [4] concluded that waiting in line to vote is the most visible sign of the administrative friction of managing elections.

However, the last few years have brought a renewed focus onto the technology used in the voting process and a hunt for voting systems that engender confidence [5]. There are a number of voting systems adopted all over the world with each of them having its peculiar problems [6]. Voting systems involve the behavior of all the voting components (be they electronic, mechanic or human), and so their assurance requires one to investigate all the aspects in an integrated way. Voting systems are complex systems that involve

activities that take place before, during and after election. They are also referred to as safety critical system in that if they fail or are compromised, could significantly affect the social, economic wellbeing and security of the nation [7].

The integrity and assurance of a complex and safety-critical system's correct behavior with respect to a specification can be achieved if good engineering practices are appropriately devised and used [8]. Formal methods are part of the standard recommendations for developing and certifying the integrity of security-critical systems [9]. Much of the field research on elections focuses on activities that take place before elections, such as voter registration and poll worker training, and uses information that is available after the election, such as turnout and election results, but treat the polling station as a black box [10]. [11] developed a non-modular simulation-optimization model for determining voting material requirements at the polls. Also [12] simulated the voting process using a simulation model that allows the use of non-stationary arrivals and non-steady-state queues. Thus, most existing voting system models cannot be easily modified to accommodate any future modification in voting systems owing to the fact that they conceptualized on non-modular formalisms. The lack of modularity limits the flexibility and reusability of such system models. As a result [13] developed a Hierarchical Timed Coloured Petri Net (HTCPN) model of a Nigerian voting system using the Re-Modified Open-Secret Ballot System (REMOBS) adopted in the Nigerian 2015 general elections as a case study. Colored Petri net is a powerful modeling language for the analysis of complex distributed systems [14]. REMOBS is a voting system developed by the Nigerian Independent National Electoral Commission and used for the first time in 2015.

This research effort is aimed to find suitable operating scenarios to improve performance and reduce voter disenfranchisement using the Nigerian Voting System. The developed model was used to experiment the effect different voters population (300 and 750 voters) on service time, queue length and average waiting time while dedicating three polling officers: Assistant Polling Officers I, II and III (APOI, APOII and APOIII), and four polling officers (two APOI and two APOII) to each polling unit. Compared to the existing literature, this paper puts more emphasis on using features of CPN (color, time, and hierarchy) to capture the complex nature of the system. Hierarchical Timed Coloured Petri Net models are simple, and particularly suitable for the modeling, simulation and analyzing of timed concurrent Systems. Some verification and analysis methods have been developed around them and many mature analysis tools are available [15]. CPN Tools is an appropriate and powerful tool to generate interactive simulations of the modeled systems and apply a wide range of formal analysis alternatives. CPN Tools is the result of a research project, the CPN2000 project, at the University of Aarhus, sponsored by the Danish National Centre for IT Research (CIT), George Mason University, Hewlett-Packard, Nokia, and Microsoft [16]. This tool provides convenience for qualitative and quantitative analysis for the system. This paper validates and analyzes the performance of the developed HTCPN model [13] via simulation based analysis technique using CPN Tools.

# II. Research Methodology

In this paper, a new approach to explore different scenarios during elections that is based on Colored Petri Nets (CPNs) is proposed. CPNs provide a graphical language to describe and analyze models of concurrent systems [17]. CPNs have been used in a wide variety of domains, however this paper is the first to recommend their use for election management and modelling.

### 2.1 Simulation Analysis of the Developed HTCPN Model

Fig. 1 shows the main page of the developed HTCPN [13] for the Re-Modified Open-Secret Ballot System (REMOBS) adopted for the Nigerian 2015 general elections. In this paper, the developed HTCPN model in [13] was implemented and simulated within CPN Tools environment. For the simulation, the CPN Tools generated as an output a set of simulated reports. The simulations were captured graphically and its snapshots are as shown in Figs. 2, 3, 4 and 5. The input data used were those obtained from Nigerian Independent National Electoral Commission's (INEC) election manual for 2015 elections [18], INEC election manual for 2011 in [19], NEPAD 2012 report in [20] and [13]. The outputs of the CPN Tools were processed with Microsoft Office Excel to obtain a set of graphs plotted for easy interpretation. In the model, the effects of different input parameters such as polling officers' response times, voters' response times and population of voters' arrival rates were simulated to show how the system performed under different load condition. Owing to the fact that the simulation model is stochastic, it is necessary to execute several simulation runs in order to compute mean values [21]. Based on one simulation run, it might not be possible to make any logical conclusions. That is, in another simulation run, the previous results could be different. Therefore, there is the need to compute confidence intervals. This was done by repeating the simulation experiment several times to get an idea of the reliability of the results. This experiment was replicated 10 times and standard statistical methods were used to compute confidence intervals based on sub-runs. Confidence intervals allowed us to compare different alternatives. CPN Tools automatically calculated these intervals by using the command:

CPN'Replications.nreplications10 [22].

For each population of voters, the percentage turnout rate observed from the 2011 election and presented in [11], were used as input in the model. For different simulations, the INEC expected standard population sizes of 300 and 750 voters per polling station as indicated in INEC election manual for 2015 elections [20] and INEC election manual for 2011 in [21] respectively, and 561 voter populations recommended in [11], were used. The observed percentage turnout rate for past elections was also used to generate a table of expected population for other regions.

The polling officers (APOI and APOII) were assumed to maintain an average of 75 and 60 seconds for accreditation respectively; these were choosing according to data released during field trials carried out by INEC for the card readers [23]. Five hours were dedicated to the arrival of intending voters during accreditation exercise (8.00am - 1.00pm) while seven hours were dedicated to the arrival of voters during the voting process (1.30pm - 7.30pm). The voters' arrival pattern for accreditation and voting processes being assumed followed the observed pattern monitored and reported in [11]. Each hourly period is assumed to follow a Poisson pattern of voter arrival for the voter population within the hour. The modelled voters' arrival pattern is consistent with the observation made in [11]. Monitors facilities of Coloured Peti Net Tools (CPN Tools) were used to obtain the required data from the developed HTCPN model during the simulation runs. Ten replications of the simulation run were executed each for the accreditation and voting processes. The simulations were coded to allow easy remembrance of the data as follows:

- a) ACC3007560: Accreditation Process with Voter Population = 300 voters, Service Time for APOI= 75sec, Service Time for APOII =60sec
- b) VOTING3007560: Voting Process with Voter Population = 300 voters, Service Time For APOI= 75sec, Service Time for APOII =60sec
- c) ACC5617560: Accreditation Process with Voter Population = 560 voters, Service Time For APOI=75 sec, Service Time for APOII =60sec
- d) VOTING750109060: Voting Process with Voter Population = 750 voters, Service Time For APOI = 10sec, Service Time for APOII =90sec, Balloting Time= 60sec
- e) VOTING551: Voting Process with Voter Population = 551 voters, Service Time for APOI=75sec, Service Time for APOII =60sec
- ACC561: Accreditation Process with Voter Population = 561 voters, Service Time for APOI=75 sec, Service Time for APOII =60sec



Fig. 1. Main Page of the Developed HTCPN Model [13]



Fig. 2. Simulation of the Voting booth (Operation) of the developed HTCPN Model for Accreditation process



Fig. 3. Simulation of the Voting Station (Arrival) of the developed HTCPN Model for Accreditation process



Fig. 4. Simulation of the Voting Station (Arrival) of the developed HTCPN Model for Voting process



Fig. 5. Simulation of the Voting booth (Operation) of the developed HTCPN Model for Voting Process

### 2.2 Validation of the developed HTCPNs model

For the correctness and credibility of the model, validation of the developed model was carried out. Validation is to determine that the developed model is a representation of the real system [24]. Thus, the proposed Hierarchical Timed Coloured Petri Net model was validated by comparing the output of the simulation of the developed model (i.e. number of voters' arrival within different time frame) for both the accreditation and the voting processes with the real values of the actual system.

### III. Results and Discussion

Outputs of the simulations were captured using advanced features of Coloured Petri Nets. These flexibilities make CPNs the perfect tool for the implementation of this kind of model while being executable. CPN tools, an appropriate and powerful tool to generate interactive simulations [16], relies on standard ML as the implementation and interpretation language in addition to the simulation engine [22]. With these modern implementations, several monitoring tools are also available. Advanced monitors are present in CPN tools and custom observers implemented by the users [25].

# 3.1 Simulation of the Voters' Arrival Pattern

Voters arrival pattern were generated according to the arrival rate modelled using the arc inscriptions 1 + i@+ round (exponential  $(1.0/\lambda)$ ). Figs. 6 and 7 are graphs of the simulated voters' arrival rate within each 60 minutes for the accreditation and voting using an expected total voter population of 750. The average arrival rate for each hour is also presented as obtained during the simulation. This is of importance to predict average voter arrival rate within each hourly division during the process. The arrival of voters per model time for the accreditation and the voting processes using an expected total population of 750 voters were plotted in Figs. 6 and 7. From these the total population of voters that has arrived into the polling unit at any time instance within the process period can be determined. A Poisson arrival process was assumed to program the transition that abstracts the arrival process.



Fig. 6. Simulated Arrived Voter population versus Model Time for Voting Process with a voter population of 750



Fig. 7. Simulated Arrived Voter population versus Model Time for Accreditation process with a voter population of 750

# 3.2 Simulation Result of the Developed HTCPNs Model

Performance evaluation in service oriented systems focuses on service time and queue length as major quality of service parameter. Disenfranchising eligible and willing voters is one of the ways rigging elections. Voting is a fundamental rite of individual to prevent unwanted politicians from gaining access to the position of governance, choose new representatives, and take a stand on important issues [26]. The model developed used the monitoring capability of the CPNs Tools system kit to monitor and store some of these important features of the modelled system for validation and analysis. Simulations were carried out using the Re-modified open-secret ballot system given by INEC in the manuals for 2015 elections as case study.

## 3.2.1 Effect of Voters' Population on Queue Length

Figs. 8 and 9 show simulation results of voters' queue length at different model time for the accreditation process with a voter's population turnout of 300 and 750 voters respectively. The processing time for APOI and APOII were taken to be 75 second and 60 seconds respectively. A peak queue length of approximately 105 voters was observed at model time 3000 seconds. Not less than 40 voters were still on queue till model time 14,000 seconds, while the last person left the queue place around model time 17,500. This shows that the accreditation will still end before 1.00 pm given as the time to complete the process. Though this is obtainable only if the last person to join the queue was admitted by model time 14400 i.e. 12.00 noon. The simulation result also established that a maximum queue length of 105 and 480 voters was reached during accreditation of 300 and 750 voters, respectively. The same pattern was observed for the voting process. This is shown in Figs 10 using a voter population of 750



Fig. 8. Simulated Queue Length Versus Model Time for Accreditation process with a voter population of 300



Fig. 9. Simulated Queue Length versus Model Time for Accreditation process with a voter population of 750



Fig. 10. Simulated Queue Length for Voting Process for a Population of 750 Voters (Voting 750109060)

# 3.2.2 Effect of Voters' Population on Queue Delay (Waiting time)

Figs. 11, 12 and 13 show the effect of population on queue delay. Fig. 11 indicated that a peak waiting time of 4800 seconds (1 hour 20 minutes) would be reached between model times 5000 and 8000 second. The last person to leave the queue would leave after spending 3000 seconds queuing at model time 17,500 second. The set of voters that waited for the longest period on queue spent 4800 seconds on queue. This means that with a voter turnout population of 300, nobody will spend more than 1 hour 20 seconds on the queue and all on queue would have been attended to by the end of 18,000 second. The time for the last voter to leave the queue is seen to be reduced from 50000seconds (13 hour) for voters' population of 750 to 41000 seconds (11 hour) for voter population of 551 and finally 17500seconds (5 hour) for a population of 300 and 750 voters, respectively. This shows plainly that the 300 voters' population given by INEC in 2011 will be very good to finish accreditation within the stipulated four hours, while the 750 voter will still have people on queue even after 16 hours (1.00am midnight of second day).

### 3.2.3 Effect of Voters' Population on Service Time Distribution

With the basic assumptions of Poisson arrival for each hourly period respectively during accreditation and employing three polling officers (APOI, APOII and APOIII) at each polling unit with process time of 75 and 60 seconds for APOI and APOII, the simulation result revealed that service times of 5 hours and 13 hours 57minutes would be required to accredit 300 and 750 intending voters, respectively. The results showed that the higher the population of voters, the longer the waiting time, the queue length and the time that would be required to complete the accreditation exercise. Figs. 14 and 15 further established this, 750 voters population was too much to be handled within the service time given. Simulation result showed that accreditation cannot be completed until around 50,000 second (13 hours 53minutes) which means voters will still be on queue till midnight for accreditation. The question "Will the maximum wait times occur for only a few voters?" would have a No for an answer. It is evident that the buildup and decay of waiting times – the development and contraction of long lines – responds to voter turnout with unchanged number of polling staff.



Fig. 11. Simulated Waiting Time Versus Model Time for Accreditation process with a voter population of 300



Fig. 12. Simulated Queue Delay versus Model Time for Accreditation process with a voter population of 750



Fig. 13. Simulated Queue Delay versus Model Time for voting process with a voter population of 750



Fig. 14. Simulated Population of Voters serviced versus Model Time for Accreditation process with a voter population of 750



Fig. 15. Simulated Population of Voters serviced versus Model Time for Voting process with a voter population of 750

# 3.2.4 Effect of Varying Polling Staff Number and Voter Population

Figs. 16 and 17 show the extreme sensitivity of the formation of long lines and waiting times to polling place conditions. The number of polling officers (APOI, APOII) was increased to two from one each. With four polling officers (two APOI and two APOII) per polling unit, the simulation result revealed that service times reduced from 13hrs 57minutes to 6hrs 47minutes to completely accredit 750 voters. The simulation result also established that average queue length reduced from 13hrs 53 minutes to 1hr 14 minutes in the process of accrediting of 750 voters. It therefore shows that APOI and APOII are the major queue formation units and hence the potential choke points (bottleneck) for voters. The sensitivity of waiting times to small changes in

voter number and processing time with the officers indicated that for an average processing time of 75 second with APOI and 60 second with APOII, a voter population of 300 will maximally utilize the studied system. This established that instead of creating another voting unit for accreditation process when voter population exceeds 300 voters, increasing the number of APOI and APOII to two will reduce the average queue length from 235 voters to 150 voters while APOIII and PO are retained as one each.

The Nigerian case with almost 2000 polling units and almost 80 million registered voters, and some of these with well over 300 voters even if 50% of voters turn out to vote, if all the voting points with above 450 voter had long line and serious delays, there would be significant effect on local, regional and national election and consequences on political dispute.



Fig. 16. Result of the simulated model comparing Queue Lengths of a set of one officers each (APOI and APOII) each and two officers each



Fig. 17. Comparative Result of a set of one officers each (APOI, APOII) each and two officers each.

### 3.2.5 Effect of Rate of Arrival on Percentage of Disenfranchised Voters

After developing the HTCPN model presented in [13] as shown in Figs. 1 to 5 and adding the monitors, it is easy to explore various alternative input variables and compare them. In the developed HTCPNs model, the maximum number of voters (capacity) in queue place before balking was set to 100 voters, indicating that at most 100 voters can queue before any voter will be able to think of going away from the system (balking). If there were 100 voters in the queue and another one arrives, the transition Defranchised voter becomes enabled which indicated that the 101th voter could choose not to join the queue and quits. If such leaves the queue, the voter is withdrawn from the queue and sent to the Balked (go-out) place. Table 1 showed the Simulated Voters' Arrival Pattern for voting process of a population of 750 Voters and Table 2 shows the Simulated Voters' arrival pattern for accreditation process for a population of 750 Voters. The population of voters that arrived within each time frame (period) is shown. This is higher than the observed population for 300 voters populations because the higher the overall population, the higher the turnover rate at the periodic time (3600 sec). Despite the fact that more voters are being attended to, waiting times and the average queue length get longer, thus increasing number of disenfranchised voters. Increasing polling staff to two, indicates two card readers per voting booth. Increasing the token (card reader) in place *free-officer* to two gave a reduction in flow time from approximately 18557 seconds to 6470 seconds. The immediate effect was that the percentage of disenfranchised voters was decreased and waiting time was reduced.

<b>Table 1.</b> Simulated Voters'	Arrival Pattern for	r Accreditation	Process for	or a Population o	of 750 Vo	ters (ACC
		7500060)				

7509000)					
Transition	Time Interval	Average Voters' population			
PreloadArrival_1_count_iid	Before 8.00am	135.00			
Arriva11_1_count_iid	8.00-9.00am	275.30			
Arrriva2l_1_count_iid	9.00-10.00am	52.00			
Arriva3l_1_count_iid	10.00-11.00am	185.20			
Arrriva41_1_count_iid	11.00-12noon	96.60			

**Table 2.** Simulated Voters' Arrival Pattern for Voting Process for a Population of 750 Voters (Voting 7509060)

Transition Monitor	Time Interval	Average Voter's Population
PreloadArrival_1_count_iid	Before12 noon	112.00
Arrival1_1_count_iid	12.00-1.00pm	152.70
Arrival3_1_count_iid	1.00-2.00pm	209.50
Arrrival2_1_count_iid	2.00-3.00pm	83.80
Arrrival4_1_count_iid	3.00-4.00pm	57.40
Arrrival5_1_count_iid	4.00-5.00pm	72.80
Arrrival6_1_count_iid	5.00-6.00pm	44.40

### 3.3 Validation of the Developed HTCPN Model

The statistical analysis of the results and the test data used were carried out using SPSS (Statistical Package for the Social Sciences) software. The result of the T-Test for the validation is presented in Table 3. The T-Test shows that there were no significant differences between the outputs of the simulation and the actual data at 5% significance level. Hence, the developed HTCPN model is valid and accurately represents the actual system. Figs. 18 and 19 graphically show the relationship between the observed and the simulated hourly arrival rates.

	Table 1. Statistical analysis (T-Test) of the validation result: Paired Samples Test           Paired differences								
		devi Ma		Std. error mean	95% confidence interval of the difference		L O	ui	Sig. (2- tailed)
		an	d. ation	d. Ation	Lower	Upper			
Pair 1	observe - actual	.75714	2.15396	.81412	-1.23493	2.74922	.93	6	.388



Fig. 18. Validation Result: Simulated and observed (real) voter arrival population at different periodic time during Accreditation Process with 561 population





# **IV.** Conclusion

The developed HTCPNs simulation model revealed valid representation of the voters' arrival pattern process of the considered voting center. The analysis of HTCPNs simulation reports generated revealed that;

- (i) A population of 750 voters is far more than can be handled by the current system within the stipulated hours.
- (ii) More APO I and APOII (validation and statistic) officer will be needed to attain reasonable voters waiting time.
- (iii) Voters' population and voters' arrival pattern affect queue buildup while the response speed of APOI and APOII affect queue's rate of disappearance.
- (iv) The optimal voter population is 300 voters.

This research work also used a running model of the Re-Modified Open-Secret Ballot System (REMOBS) adopted in Nigeria to explain several design patterns for discrete event modelling in terms of HTCPNs. These patterns will be a guide to researchers in modelling complex processes that require interplay of control flow and data flow. It is hoped that this work will help students, researchers, system designers and process analysts to make better models in less time. It is recommended that future research may be geared towards developing a modularized model for electronic voting system.

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