Lossless and Lossy Polynomial Image Compression

Ghadah Al-Khafaji and Maha A. Rajab
Department of Computers, College of Science, Baghdad University, Baghdad, Iraq
Department of Computers, College of Ibn Al-Haytham, Baghdad University, Baghdad, Iraq

Abstract: This paper introduced a new hybrid image compression system of the lossless non-linear polynomial coding base and lossy linear polynomial base. The results are promising, since it would achieve high compression ratio with excellent medical image quality.

Keywords: lossless and lossy, linear and non-linear polynomial coding.

I. Introduction

Nowadays, compression is become an essential requirement of transmission applications and storage. Image compression is the application of data compression on digital images [1], that aims to eliminate the data redundancies, which simply categorized into three essential redundancies: inter-pixel redundancy, coding redundancy and psycho-visual redundancy[2-3], for more details see [4-5].

In general, the image compression techniques classified into two types: lossless and lossy depending on the redundancies utilized, the first one also called information preserving or error free techniques, as their name indicates that no loss of information and the reconstructed image is identical to the original one and based on using the inter pixel redundancy and/or coding redundancy, that characterized by low compression ratio, with techniques such as Huffman coding, arithmetic coding and run length coding [6-7]. The second one, where some information may be lost through the processing but the distortion level of the reconstructed image must be acceptable and cannot be known by the human visual system. In other words, the original image can not be reconstructed exactly from the compressed data, where there is some degradation on image quality based on using of psycho-visual redundancy, either alone or combined with statistical redundancy with high compression ratio [8-9]. Review on various image compression techniques can be found in [6-10-11].

The linear polynomial coding is a modern effective image compression technique used by a number [12-13-14] based on using the image spatial domain that work either of linear base model or of nonlinear base model [15-16-17].

This paper is completely dedicated to the investigation of the hybrid compression system to compress the images effectively, using the lossy linear polynomial coding technique (first order Taylor series) and lossless non-linear polynomial coding technique (second order Taylor series).

This paper is organized as follows; section 2 discussed the proposed compression system. Section 3 explained experimental results and discussion. Conclusions are shown in Section 4.

II. The Proposed Compression System

The hybrid proposed compression system mixed the lossy and lossless types along with utilizing of polynomial coding of linear (need three coefficients $a_0, a_1, a_2$) and nonlinear base (i.e., require six coefficients $a_0, a_1, a_2, a_3, a_4, a_5$). The source coding and decoding of the proposed system are illustrated in figure 1and figure 2 respectively. Figure 3, shows an example of the proposed compression techniques. The following steps are illustrated the proposed hybrid image compression system:

1. Load the input uncompressed grayscale image $I$ of size $(N \times N)$.
2. Partition the image ($I$) into nonoverlapped blocks of fixed size $n \times n$, such as $(8 \times 8)$, then create an image called sampled image ($S$) of size quarter than the partitioned image $I$. In other words, the technique used two images one corresponding to the partitioned image $I$ and the sampled image of small redundancy[7].
3. Apply non-linear polynomial coding technique on the partitioned sampled image $S$ of fixed size $n \times n$ to compute the estimated coefficients according to equations (1-17) below [18].

\[
\begin{align*}
\begin{bmatrix}
    V_1 & W_2 & W_2 \\
    V_2 & W_3 & W_4 \\
    V_3 & W_4 & W_3
\end{bmatrix}
\end{align*}
\]

\[
a_0 = \begin{bmatrix}
    W_1 & W_2 \\
    W_2 & W_3
\end{bmatrix} \begin{bmatrix}
    W_1 \\
    W_2
\end{bmatrix}
\]
Lossless and Lossy Polynomial Image Compression

\[ a_1 = \frac{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} S(x, y)(x - xc)}{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2} \quad \ldots \quad (2) \]

\[ a_2 = \frac{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} S(x, y)(y - yc)}{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (y - yc)^2} \quad \ldots \quad (3) \]

\[ a_3 = \begin{bmatrix} W_1 & V_1 & W_2 \\ W_2 & V_2 & W_3 \\ W_3 & V_3 & W_4 \end{bmatrix} \quad \ldots \quad (4) \]

\[ a_5 = \frac{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} S(x, y)(x - xc)(y - yc)}{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2(y - yc)^2} \quad \ldots \quad (5) \]

\[ a_4 = \begin{bmatrix} W_1 & W_2 & V_1 \\ W_2 & W_3 & V_2 \\ W_3 & W_4 & V_3 \end{bmatrix} \quad \ldots \quad (6) \]

\[ xc = yx = \frac{n - 1}{2} \quad \ldots \quad \ldots \quad (7) \]

\[ V_1 = a_0 W_1 + a_3 W_2 + a_4 W_2 \quad \ldots \quad \ldots \quad (8) \]

\[ V_2 = a_0 W_2 + a_3 W_3 + a_4 W_4 \quad \ldots \quad \ldots \quad (9) \]

\[ V_3 = a_0 W_3 + a_3 W_4 + a_4 W_4 \quad \ldots \quad \ldots \quad (10) \]

\[ W_1 = n \times n \quad \ldots \quad \ldots \quad \ldots \quad (11) \]

\[ W_2 = \sum_{x=0}^{n-1} (x - xc)^2 = \sum_{y=0}^{n-1} (y - yc)^2 \quad \ldots \quad \ldots \quad (12) \]

\[ W_3 = \sum_{x=0}^{n-1} (x - xc)^4 = \sum_{y=0}^{n-1} (y - yc)^4 \quad \ldots \quad \ldots \quad (13) \]

\[ W_4 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2 (y - yc)^2 \quad \ldots \quad \ldots \quad (14) \]

Where

\[ V_1 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} S(x, y) \quad \ldots \quad \ldots \quad \ldots \quad (15) \]

\[ V_2 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2 S(x, y) \quad \ldots \quad \ldots \quad \ldots \quad (16) \]

\[ V_3 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (y - yc)^2 S(x, y) \quad \ldots \quad \ldots \quad \ldots \quad (17) \]

Where \( S \) represent the sampled image. Where \( xc \) equal to \( yc \) that represents a block center of size \( (n \times n) \).

4. Create the predicted image sampled as a nonlinear combination polynomial model of coefficients and pixel distance as equation (18), such as [18]:

\[ \tilde{S} = a_0 W_1 + a_1 (x - xc) + a_2 (y - yc) + a_3 (x - xc)^2 + a_4 (y - yc)^2 + a_5 (x - xc) (y - yc) \quad \ldots \quad (18) \]

5. Find the residual image \( Res \) or prediction error (as difference between original and prediction image), as equation (19).

\[ Res(x, y) = S(x, y) - \tilde{S}(x, y) \quad \ldots \quad \ldots \quad \ldots \quad (19) \]

6. Encode the non-linear estimated coefficients and resultant residual image losslessly, using the run length coding (RLC) which is passed through Huffman coding to remove the coding redundancy.
7. Apply linear polynomial coding technique on the original partitioned image $I$ to compute the estimated coefficients $(a_0, a_1, a_2)$ according to equations (20-22)\[19].

$$a_0 = \frac{1}{n} \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} I(x,y)$$

$$\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} I(x,y) \times (y \times xc)$$

$$a_1 = \frac{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} I(x,y) \times (y \times xc) + \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} I(x,y) \times (y \times yc) \times (x \times yc)}{\sum_{x=0}^{n-1} \sum_{y=0}^{n-1} I(x,y) \times (x \times yc)^2}$$

where $I(x,y)$ is the original image of size $N \times N$ that partitioned into fixed block size $n \times n$. Where $xc$ equal to $yc$ that represents a block center of size $(n \times n)$ as equation (7).

8. Using the uniform scalar quantization to quantize the estimated coefficients of the linear base, where each coefficient is quantized using different quantization level, The quantizer/dequantizer as illustrated in equations (23-25).

$$Q_{a_0} = round\left(\frac{a_0}{QL_{a_0}}\right) \rightarrow DQ_{a_0} = Q_{a_0} \times QL_{a_0}$$

$$Q_{a_1} = round\left(\frac{a_1}{QL_{a_1}}\right) \rightarrow DQ_{a_1} = Q_{a_1} \times QL_{a_1}$$

$$Q_{a_2} = round\left(\frac{a_2}{QL_{a_2}}\right) \rightarrow DQ_{a_2} = Q_{a_2} \times QL_{a_2}$$

Where $Q_{a_0}$, $Q_{a_1}$, $Q_{a_2}$ are represented the quantized value of the coefficients $(a_0, a_1, a_2)$ respectively, while $QL_{a_0}$, $QL_{a_1}$, $QL_{a_2}$ are represented the quantization levels of the parameters. $DQ_{a_0}$, $DQ_{a_1}$, $DQ_{a_2}$ represents the dequantization value of the estimated parameters.

9. Create the predicted image value $\hat{I}$ using the dequantized polynomial coefficients for each encoded block as equation (26).

$$\hat{I} = DQ_{a_0} + DQ_{a_1}(y - xc) + DQ_{a_2}(x - yc)$$

10. Find the residual or prediction error as difference between the original image and the predicted one $\hat{I}$ as equation (27).

$$Residual(x,y) = I(x,y) - \hat{I}(x,y)$$

11. Apply then uniform scalar quantization to quantize the residual as equation (28) shown quantizer/dequantizer of residual.

$$Q_{Residual} = round\left(\frac{Residual}{QL_{Residual}}\right) \rightarrow DQ_{Residual} = Q_{Residual} \times QL_{Residual}$$

12. Encode the estimated coefficients of linear polynomial model and residual image lossily, also by utilizing both the run length coding (RLC) and Huffman coding.

13. The decoder reconstruct the inverse images, correspond to the identical $\hat{S}$ image and the approximated $\hat{I}$ using the nonlinear/linear coefficients and residuals images as equations (29-30).

$$\hat{S}(x,y) = Res(x,y) + \hat{S}(x,y)$$

$$I(x,y) = DQ_{Residual}(x,y) + \hat{I}(x,y)$$

14. The reconstructed hybrid image of lossy and lossless base as equation (31).

$$\hat{M}(x,y) = \hat{S}(x,y) + \hat{I}(x,y)$$
Lossless and Lossy Polynomial Image Compression

Figure 1- The proposed source coding of compression system.

Figure 2- The proposed source decoding of compression system.

Figure 3- Example of the proposed system structure.
III. Experimental and Results

Three standard medical images are selected for testing the proposed hybrid compression system, the images of 256 gray levels (8 bits/pixel) of size 256×256 (see figure 4 for an overview). To evaluate the performance of the proposed compression system, the compression ratio used (CR) which is the ratio between the original image size and the compressed size (see equation 32), also the peak signal to noise ratio (PSNR) along with normalized root mean square error (NRMSE) adopted, see equations (33-34), where a large PSNR value implicitly means high image quality and close to the original image and vice versa, while NRMSE where the range of the values between 0 and 1, if the value is close to zero refers to high image quality and vice versa [7].

\[ \text{Compression Ratio} = \frac{\text{Size of Original Image}}{\text{Size of Compressed Information}} \]  \hspace{1cm} (32)

\[ \text{PSNR} = 10 \cdot \log_{10} \left( \frac{255^2}{\frac{1}{N \times N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (M(i,j) - I(i,j))^2} \right) \] \hspace{1cm} (33)

\[ \text{NRMSE}(I, \tilde{M}) = \sqrt{\frac{\sum_{x=0}^{N-1} \sum_{y=0}^{N-1} (M(x,y) - I(x,y))^2}{\sum_{x=0}^{N-1} \sum_{y=0}^{N-1} I(x,y)^2}} \] \hspace{1cm} (34)

The result of the proposed hybrid compression system indicates that the high image quality is achieved because of utilization of effective non-linear lossless polynomial coding technique along with the efficient linear lossy polynomial coding technique.

The results showed in table (1) of block sizes 8×8. It is obvious that the blocks size and the quantization step affected the technique performance, where the quantization process utilized for the linear polynomial model only, so the quantization levels of the coefficients and the residual affects the image quality and compression ratio.

Figures 5 illustrated the results of the compressed three tested images of block sizes 8×8, and quantization level of three coefficients as, Qa0, Qa1, Qa2={1,2,2} and quantization level of residual equal to {30}.

![Overview of the tested images](image-url)
Table 1- Illustrated the results of three images for block size (8×8) and value of quantization levels of coefficients equals to 1,1,1 and 1,2,2, with different quantization levels of residual image.

<table>
<thead>
<tr>
<th>Test Images</th>
<th>Quantization Residual</th>
<th>Block Size (8×8)</th>
<th></th>
<th></th>
<th>CR</th>
<th>PSNR</th>
<th>NRMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee</td>
<td>5</td>
<td>0.0946</td>
<td>27.0276</td>
<td>8.1600</td>
<td>0.0950</td>
<td>26.9927</td>
<td>8.3560</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.0528</td>
<td>32.0880</td>
<td>8.2701</td>
<td>0.0531</td>
<td>32.0379</td>
<td>8.4010</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0268</td>
<td>37.3597</td>
<td>8.6350</td>
<td>0.0290</td>
<td>37.3014</td>
<td>8.9300</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0198</td>
<td>40.5962</td>
<td>9.1633</td>
<td>0.0200</td>
<td>40.5307</td>
<td>9.3650</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.0151</td>
<td>42.9651</td>
<td>9.4210</td>
<td>0.0152</td>
<td>42.8877</td>
<td>9.6630</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.0122</td>
<td>44.7930</td>
<td>9.5901</td>
<td>0.0124</td>
<td>44.7101</td>
<td>9.8913</td>
</tr>
<tr>
<td>Brain</td>
<td>5</td>
<td>0.2186</td>
<td>21.0490</td>
<td>10.4050</td>
<td>0.2189</td>
<td>21.0374</td>
<td>10.2561</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.1222</td>
<td>26.0966</td>
<td>10.1352</td>
<td>0.1224</td>
<td>26.0822</td>
<td>10.3035</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0617</td>
<td>32.0426</td>
<td>10.4600</td>
<td>0.0618</td>
<td>32.0247</td>
<td>10.6333</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0408</td>
<td>35.6381</td>
<td>10.5459</td>
<td>0.0408</td>
<td>35.6235</td>
<td>10.7989</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.0304</td>
<td>38.1716</td>
<td>10.6761</td>
<td>0.0306</td>
<td>38.1395</td>
<td>10.8055</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.0243</td>
<td>40.1437</td>
<td>10.7002</td>
<td>0.0243</td>
<td>40.1277</td>
<td>10.8952</td>
</tr>
<tr>
<td>Knees</td>
<td>5</td>
<td>0.1248</td>
<td>27.5953</td>
<td>9.1097</td>
<td>0.1250</td>
<td>27.5805</td>
<td>10.0045</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.0621</td>
<td>33.6622</td>
<td>9.3822</td>
<td>0.0622</td>
<td>33.6355</td>
<td>10.2593</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0328</td>
<td>39.2129</td>
<td>9.4982</td>
<td>0.0329</td>
<td>39.1631</td>
<td>10.5476</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0223</td>
<td>42.5402</td>
<td>9.7101</td>
<td>0.0226</td>
<td>42.4497</td>
<td>10.6680</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.0171</td>
<td>44.8702</td>
<td>9.8050</td>
<td>0.0173</td>
<td>44.7642</td>
<td>10.7999</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.0138</td>
<td>46.7091</td>
<td>9.8832</td>
<td>0.0140</td>
<td>46.6091</td>
<td>10.9079</td>
</tr>
</tbody>
</table>

Figure 5- Overview the results of the tested three images for block size (8×8), and Coefficients Quantization level, Qa0, Qa1, Qa2~(1,2,2) and Quantization Residual level – (20).

IV. Conclusions

This paper attempts to exploit the hybrid image compression system based on lossless non-linear polynomial coding to compress the sampled image and lossy linear polynomial coding to compress the original image. The experimental results clearly showed that when increased the block size the compression ratio increased and also reduced image quality and vice versa. Also shown when increase the quantization levels of residual the image quality increased and CR decreased and vice versa.

References

Lossless and Lossy Polynomial Image Compression


