IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 18, Issue 2, Ver. II (Mar-Apr. 2016), PP 60-68
www.iosrjournals.org

Numerical Analogy of q-Function

Prashant Singh[1], Prof.P.K.Mishra[2]
[1]*(Department of Computer Science,Institute of Science,Banaras Hindu University,India)*
[2]*(Department of Computer Science,Institute of Science,Banaras Hindu University,India)*

Abstract : *This paper is a collection of q analogue of various classical methods for finding solutions of algebraic and transcendental equations. It also deals with comparing classical methods with q methods proposed by us. We have discussed few problems where these methods are equivalent and also cases where q method is better. We also provide a short explanation about the need of iterative methods in scientific and engineering problems. We have introduced q analogues of some classical methods and discussed their merits and demerits. We have presented the basic definitions and classification of iterative methods and discussed some elementary concepts and definitions regarding roots and systems of nonlinear equations. The fundamental concepts and classification of iterative methods and their significant features are also stated in this paper. We have also used various techniques, which are being used by researchers to produce higher order iterative methods such as functional approximation, sampling, composition, geometric approaches.*
Keywords: *q analogue , basic analogue , q method ,classical method*

I. Introduction and Literature Survey

C.F.Gauss[1,11] started the theory of q hypergeometric series in 1812 and worked on it for more than five decades.E.Heine[1,11] extended this theory and worked on it for more than three decades. Later on F.H.Jackson[1,11,17,18,19] in the beginning of twentieth century started working on q function and proposed q-differentiation and q-integration and worked on transformation of q-series and generalized function of Legendre and Bessel. G.E.Andrews[11,51,52]contributed a lot on q theory and worked on q-mock theta function,problems and prospects on basic hypergeometric series,q-analogue of Kummer's Theorem and on Lost Notebook of Ramanujan. G.E.Andrew[11,51,52] with R.Askey[1] worked on q extension of Beta Function. J.Dougall[1] worked on Vondermonde's Theorem. H.Exton[1] worked a lot on basic hypergeometric function and its applications.T.M.MocRobert worked on integrals involving E Functions, Confluent Hypergeometric Function,Gamma E Function,Fourier Series for E Function and basic multiplication formula. M.Rahman with Nassarallah worked on q-Appells Function,q-Wilson polynomial,q-Projection Formulas.He also worked on reproducing Kample and bilinear sums for q-Racatanad and q-Wilson polynomial. I Gessel with D.Stanton [14,15]worked on family of q-Lagrange inversion formulas.T.M.MacRobert worked on integrals involving E Functions and confluent hypergeometric series.D.Stanton[14,15]worked on partition of q series.Studies in the nineteenth century included those of Ernst Kummer, and the basic characterization by Bernhard Riemann of the F-function by means of the differential equation it satisfies. Riemann showed that the second-order differential equation for F, examined in the complex plane, could be characterised by its three regular singularities: that effectively the entire algorithmic side of the theory was a upshot of basic facts and the use of Möbius transformations as a symmetry group.

A generalization, the q-series analogues, called the basic hypergeometric series, were given by Eduard Heine[1,11] in the late nineteenth century. During the twentieth century this was a prolific area of combinatorial mathematics, with many connections to other fields. There are plethora of new definitions of hypergeometric series, by Aomoto, Israel Gelfand and others; and applications for example to the combinatorics of arranging a number of hyperplanes in complex N-space (see arrangement of hyperplanes).

q series can be developed on[11] Riemannian symmetric spaces and semi-simple Lie groups. Their impact and role can be understood through a special case: the hypergeometric series 2F1 is directly related to the Legendre Polynomial and when used in the form of spherical harmonics, it expresses, in a certain sense, the symmetry properties of the two-sphere or equivalently the rotations given by the Lie group SO(3) Concrete representations are analogous to the Clebsch-Gordan. A number of hyper-geometric function[1,11] identities were exposed in the nineteenth and twentieth centuries, One conventional[11] list of such identities is Bailey's list. It is at present understood that there is a vast number of such identities, and several algorithms are now known to generate and prove these identities. In a certain sense, the situation can be likened to using a computer to do addition and multiplication; the actual value of the resulting number is in a sense less significant than the various patterns that come out; and so it is with hypergeometric identities as well.

Among Indian researchers R.P.Agrawal[53,54,55,56,57] gave a lot to q function . He worked on fractional q-derivative, q-integral, mock theta function, combitorial analysis, extension of Meijer's G

Function,Pade approximants,continued fractions and generalized basic hypergeometric function with unconnected bases.W.A.Al-Salam [2,3] and A.Verma[2,3] worked on quadratic transformations of basic series.N.A.Bhagirathi [38,39]worked on generalized q hypergeometric function and continued fractions.V.K.Jain and M.Verma[58] worked on transformations of non termating basic hypergeometric series,their contour integrals and applications to Rogers ramanujan's identities.S.N. Singh worked on transformation of abnormal basic hypergeometric functions, partial theorems, continued fraction and certain summation formulae. K.N.srivastava and B.R.Bhonsle worked on orthogonal polynomials. H.M.Srivastava with Karlsson worked on multiple Gaussians Hypergeometric series, polynomial expansion for functions of several variables. S Ramanujan in his last working days worked on basic hypergeometric series.G.E.Andrews[11,51,52] published an article on "The Lost Note Book of Ramanujan". H.S.Shukla worked on certain transformation in the field of basic hypergeometric function.A.Verma and V.K.Jain worked on summation formulas of q-hypergeometric series, summation formulae for non terminating basic hypergeometric series,q analogue of a transformation of Whipple and transformations between basic hypergeometric series on different bases and identities of Rogers-Ramanujan Type.B.D.Sears worked on transformation theory of basic hypergeometric functions.P.Rastogi worked on identities of Rogers Ramanujan type. A.Verma and M.Upadhyay worked on transformations of product of basic bilateral series and its transformations. Generally speaking[1,11] in particular in the areas of combitorics and special functions, a q-analog of a theorem, identity or expression is a simplification involving a new parameter q that returns the novel theorem, identity or expression in the limit as q → 1 (this limit is often formal, as q is often discrete-valued). Typically, mathematicians are interested in q-analogues that occur naturally, rather than in randomly contriving q-analogues of recognized results. The primary q-analogue studied in detail is the q hypergeometric series, which was introduced in the nineteenth century. q-analogs find applications in a number of areas, including the study of fractals and multi-fractal measures, and expressions for the entropy of chaotic dynamical systems. The relationship to fractals and dynamical systems[1,11] results from the fact that many fractal patterns have the symmetries of Fuchsian groups in general and the modular group in particular. The connection passes through hyperbolic geometry and ergodic theory, where the elliptic integrals and modular forms play a prominent role; the q-series themselves are closely related to elliptic integrals. q-analogs also come into sight in the study of quantum groups and in q-deformed super algebras. The connection here is alike, in that much of string theory is set in the language of Riemann surfaces, ensuing in connections to elliptic curves, which in turn relate to q-series.

II. Failure of Classical Methods and our proposed q methods

Let us capture an example of Newton's Method [60] for solving algebraic and transcendental problem and the state where it fails. Newton's method is assured to converge under certain circumstances. One well-liked set of such conditions is this: if a function has a root and has a non-zero derivative at that root, and it is continuously differentiable in some interval around that root, then there exist some neighborhood of the root so that if we choose our preliminary point in that region, the iterations will converge to the given root. Conditions where a classical method fails are when the derivative is zero at the root i.e.the function fails to be continuously differentiable; and also when we have selected a starting point which is not apt, i.e. one that lies outside the range of guaranteed convergence. Degenerate roots [61](those where the derivative is 0) are "uncommon" in general. On the other hand, most functions are not continuous or differentiable at all. The choice of starting point may be understandable if you have an idea about the rough location of the root, or it could be totally hit-and-miss. There are other condition sets which may be more or less helpful; there is no all-encompassing way that captures correctly when the method fails. Generally speaking, if our function is reasonably smooth (differentiable) and we begin at a arbitrary location, function will most likely converge to some root. Some times we may unfortunately select initial point that is stationary or lies in some short cycle.

III. q analogues of some iterative methods using single and double parameters

3.1 q-analogue of Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)x_n(1-q)}{f(x_n)-f(qx_n)} \quad \textbf{(1)}$$

3.2 q-analogue of Newton Raphson Method having multiplicity

$$x_{n+1} = x_n - m\frac{f(x_n)x_n(1-q)}{f(x_n)-f(qx_n)}$$ **, where m is multiplicity.** **(2)**

3.3 q-analogue of Newton Raphson Method for multiple root

$$\mathbf{x_{k+1}=x_k} - \frac{m(3-m)^1}{[2;q]}\frac{f(k)}{D_q f(k)} - \frac{m^2}{[2;q]}\left(\frac{f(k)}{D_q f(k)}\right)^2\frac{D_q^2\ f(k)}{D_q f(k)} \quad \textbf{(3)}$$

3.4 q analogue of Euler[1,11,44] Method

It is cubically convergent method with efficiency index $3^{1/3.}$It requires three evaluations $f, \overline{B_q^-} f, \overline{B_q^2} f$. It is obtained by considering the parabola $x^2+ax+by+c=0$ and imposing tangency conditions.

$$x_{k+1} = x_k - \frac{2(q-1)x_k f(x_k)}{1+\left(f(x_k q)-f(x_k)\right)\sqrt{1-2\emptyset(x_k)}} where \emptyset(x_k) = \frac{f(x_k)(q-1)^2 x_k^2}{(f(qx_k)-f(x_k))^2} \frac{\left(f(q^2 x_k-[2;q]f(qx_k)+qf(x_k)\right)}{q(q-1)^2 x_k^2} \quad (4)$$

3.5 q analogue of Euler [1,11,44]Method using two parameters

$$x_{k+1} = x_k - \frac{2(q1-q2)x_k f(x_k)}{1+\left(f(x_k q1)-f(q2x_k)\right)\sqrt{1-2\emptyset(x_k)}} where \emptyset(x_k) = \frac{f(x_k)(q1-q2)^2 x_k^2}{\left(f(q1x_k)-f(q2x_k)\right)^2} \frac{[q_2 f(q_1^2 x_k)+q_1 f(q_2^2 x_k)-q_1 f(q_1 q_2 x_k)-q_2 f(q_1 q_2 x_k)]}{(q_1-q_2)^2 {x_k}^2} \quad (5)$$

3.6 q analogue of Halley[44] Method

$$x_{k+1} = x_k - \frac{2(q-1)x_k f(x_k)}{\left(f(x_k q)-f(x_k)\right)(2-\emptyset(x_k))} where \emptyset(x_k) = \frac{f(x_k)(q-1)^2 x_k^2}{(f(qx_k)-f(x_k))^2} \frac{[q_2 f(q_1^2 x_k)+q_1 f(q_2^2 x_k)-q_1 f(q_1 q_2 x_k)-q_2 f(q_1 q_2 x_k)]}{(q_1-q_2)^2 {x_k}^2} \quad (6)$$

Its efficiency index is also $3^{1/3.}$It requires three evaluations $f, \overline{B_q^-} f, \overline{B_q^2} f$. It is obtained by considering the hyperbola axy+bx+c=0 and imposing tangency conditions and calculating next iterate.

3.7 q analogue of Halley[44] Method using two parameters

$$x_{k+1} = x_k - \frac{2(q1-q2)x_k f(x_k)}{\left(f(x_k q1)-f(q2x_k)\right)(2-\emptyset(x_k))} where \emptyset(x_k) = \frac{f(x_k)(q1-q2)^2 x_k^2}{(f(q1x_k)-f(q2x_k))^2} \frac{\left(f(q^2 x_k-[2;q]f(qx_k)+qf(x_k)\right)}{q(q-1)^2 x_k^2} \quad (7)$$

3.8 q analogue of Traub Steffensen Method

If the derivative of $f(x_k)$ is approximated by $\frac{f(x+\lambda f(x_k)-f(x_k)}{\lambda f(x_k)} for nonzero \lambda$.Then we get

$$x_{k+1} = x_k - \frac{f(x_k)}{f[x_k, w_k]}, k \epsilon N. where p_k = x_k + \lambda f(x_k) \quad (8)$$

.This method has order of convergence 2 and efficiency index is $2^{1/2}$.

3.9 q analogue of method proposed by Homeir[40]

$$p_k = x_k - \frac{1}{[2;q]} \frac{f(x_k)}{f(qx_k)-f(x_k)}(q-1)x_k \quad (9)$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f(qp_k)-f(p_k)}(q-1)p_k$$

This method is of order 3.

3.10 q analogue of method proposed by Homeir[40] using two q parameters

$$p_k = x_k - \frac{1}{[2;q]} \frac{f(x_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \quad (10)$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f(q1p_k)-f(q2p_k)}(q1-q2)p_k \quad (11)$$

This method is of order 3.

3.11 q analogue of method proposed by Kou[41,62,63]

$$p_k = x_k + \frac{f(x_k)}{f(qx_k)-f(x_k)}(q-1)x_k \quad (12)$$

$$x_{k+1} = p_k - \frac{f(p_k)}{f(qx_k)-f(x_k)}(q-1)x_k \quad (13)$$

This method is of order 3.

3.12 q analogue of method proposed by Kou[41,62,63] using two q parameters.

$$p_k = x_k + \frac{f(x_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \quad (14)$$

$$x_{k+1} = p_k - \frac{f(p_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \qquad (15)$$

This method is of order 3

3.13 q analogue of method proposed by Ozban[42]

$$p_k = x_k - \frac{f(x_k)}{f(qx_k)-f(x_k)}(q-1)x_k \qquad (16)$$

$$x_{k+1} = x_k - \frac{1}{[2;q]}\frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)}\frac{(q-1)p_k}{f(qp_k)-f(p_k)}\left[(f(qx_k)-f(x_k))\frac{1}{(q-1)x_k} + (f(qp_k) - f(p_k))\frac{1}{(q-1)p_k}\right] \qquad (17)$$

This method is of order 3.

3.14 q analogue of method proposed by Ozban[42] using two parameters

$$p_k = x_k - \frac{f(x_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \qquad (18)$$

$$x_{k+1} = x_k - \frac{1}{[2;q]}\frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}\frac{(q1-q2)p_k}{f(q1p_k)-f(q2p_k)}\left[(f(q1x_k)-f(q2x_k))\frac{1}{(q1-q2)x_k} + (f(q1p_k)-f(q2p_k))\frac{1}{(q1-q2)p_k}\right] \qquad (19)$$

This method is of order 3.

3.15 q analogue of method by Weerakoon[43] and Fernando[43]

$$p_k = x_k - \frac{f(x_k)}{f(qx_k)-f(x_k)}(q-1)x_k \qquad (20)$$

$$x_{k+1} = x_k - \frac{[2;q]f(x_k)}{\left[(f(qx_k)-f(x_k))\frac{1}{(q-1)x_k}+(f(qp_k)-f(p_k))\frac{1}{(q-1)p_k}\right]} \qquad (21)$$

This method is of order 3.

3.16 q analogue of method by Weerakoon[43] and Fernando[43]

$$p_k = x_k - \frac{f(x_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \qquad (22)$$

$$x_{k+1} = x_k - \frac{[2;q]f(x_k)}{\left[(f(q1x_k)-f(q2x_k))\frac{1}{(q1-q2)x_k}+(f(q1p_k)-f(q2p_k))\frac{1}{(q1-q2)p_k}\right]} \qquad (23)$$

This method is of order 3.

3.17 q analogue of method proposed by Bi Wu Ren [66] (Order 8)

$$p_k = x_k - \frac{f(x_k)}{f(qx_k)-f(x_k)}(q-1)x_k \qquad (24)$$

$$z_k = p_k - \frac{2(f(x_k)-f(p_k))f(p_k)}{(2fx_k-5f(p_k))}\frac{1}{f(qx_k)-f(x_k)}(q-1)x_k \qquad (25)$$

$$x_{k+1} = z_k - \frac{f(x_k)+(2+\lambda)f(z_k)}{f(x_k)+\lambda f(z_k)}\frac{f(z_k)}{f[z_k,p_k]+f[z_k,x_k,x_k](z_k-p_k)}, \lambda \epsilon R \qquad (26)$$

3.18 q analogue of method proposed by Bi Wu Ren [66] using double parameter

$$p_k = x_k - \frac{f(x_k)}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \qquad (27)$$

$$z_k = p_k - \frac{2(f(x_k)-f(p_k))f(p_k)}{(2fx_k-5f(p_k))}\frac{1}{f(q1x_k)-f(q2x_k)}(q1-q2)x_k \qquad (28)$$

$$x_{k+1} = z_k - \frac{f(x_k)+(2+\lambda)f(z_k)}{f(x_k)+\lambda f(z_k)}\frac{f(z_k)}{f[z_k,p_k]+f[z_k,x_k,x_k](z_k-p_k)}, \lambda \epsilon R \qquad (29)$$

3.19 q analogue of method proposed by Cordero-Torregrosa-Vassileva [67]

$$p_k = x_k - \frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)} \qquad (30)$$

$$z_k = x_k - \frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)}\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))} \qquad (31)$$

$$x_{k+1} = u_k - \frac{f(z_k)}{\beta_1}\frac{\frac{(q-1)x_k}{f(qx_k)-f(x_k)}3(\beta_3+\beta_2)(u_k-z_k)}{\beta_1(u_k-z_k)+\beta_2(p_k-x_k)+\beta_3(z_k-x_k)} \text{ where } \beta_i \in R \text{ and } \beta_2+\beta_3 \neq 0 \qquad (32)$$

and $u_k = z_k - \frac{f(z_k)}{\frac{(q-1)x_k}{f(qx_k)-f(x_k)}}\left(\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))} + \frac{1}{2}\frac{f(z_k)}{(f(p_k)-2f(z_k))}\right)^2$ **(33)**

3.20 q analogue of method proposed by Cordero-Torregrosa-Vassileva[67] using two parameters

$p_k = x_k - \frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}$ **(34)**

$z_k = x_k - \frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))}$ **(35)**

$x_{k+1} = u_k - \frac{f(z_k)}{\beta_1}\frac{\frac{(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}3(\beta_3+\beta_2)(u_k-z_k)}{\beta_1(u_k-z_k)+\beta_2(p_k-x_k)+\beta_3(z_k-x_k)}$ $where\ \beta_i \in R\ and\ \beta_2+\beta_3 \neq 0$ **(36)**

and $u_k = z_k - \frac{f(z_k)}{\frac{(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}}\left(\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))} + \frac{1}{2}\frac{f(z_k)}{(f(p_k)-2f(z_k))}\right)^2$ **(37)**

3.21 q analogue of Thukral's[65] Method

$p_k = x_k - \frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)}$ **(38)**

$z_k = p_k - \frac{f(x_k)^2+f(p_k)^2}{(f(x_k)-f(p_k))D_q(f(x_k))}$ **(39)**

$$x_{k+1} = z_k - \left[\frac{f(z_k)\frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)}}{1}\right]\left[\left(\frac{1+{\mu_k}^2}{1-\mu_k}\right)^2 - 2\mu_k^2 - [3;q]!\,\mu_k^3 + \frac{f(z_k)}{f(p_k)} + \frac{4f(z_k)}{f(x_k)}\right]$$

where $\mu_k = \frac{f(w_k)}{f(x_k)}$ **(40)**

3.22 q analogue of Thukral's[65] Method using two parameters

$p_k = x_k - \frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}$ **(41)**

$z_k = p_k - \frac{f(x_k)^2+f(p_k)^2}{(f(x_k)-f(p_k))D_q(f(x_k))}$ **(42)**

$$x_{k+1} = z_k - \left[\frac{f(z_k)\frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}}{1}\right]\left[\left(\frac{1+{\mu_k}^2}{1-\mu_k}\right)^2 - 2\mu_k^2 - [3;q]!\,\mu_k^3 + \frac{f(z_k)}{f(p_k)} + \frac{4f(z_k)}{f(x_k)}\right]$$

where $\mu_k = \frac{f(w_k)}{f(x_k)}$ **(43)**

3.23 q analogue of method proposed by Liu-Wang[41,62,63,64]

$p_k = x_k - \frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)}$ **(44)**

$z_k = p_k - \frac{f(p_k)(q-1)x_k}{f(qx_k)-f(x_k)}\frac{(f(x_k))}{(f(x_k)-2f(p_k))}$ **(45)**

$$x_{k+1} = z_k - \frac{f(z_k)(q-1)x_k}{f(qx_k)-f(x_k)}\left[\left(\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))}\right)^2 + \left(\frac{f(z_k)}{(f(w_k)-\alpha f(z_k))}\right) + \left(\frac{4f(z_k)}{(f(x_k)+\beta f(z_k))}\right)\right]$$

where $\propto, \beta \in R$ **(46)**

3.24 q analogue of method proposed by Liu-Wang[41,62,63,64] using two parameters

$$p_k = x_k - \frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)} \quad \textbf{(47)}$$

$$z_k = p_k - \frac{f(p_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)} \frac{(f(x_k))}{(f(x_k)-2f(p_k))} \quad \textbf{(48)}$$

$$x_{k+1} = z_k - \frac{f(z_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)}\left[\left(\frac{(f(x_k)-f(p_k))}{(f(x_k)-2f(p_k))}\right)^2 + \left(\frac{f(z_k)}{(f(w_k)-\alpha f(z_k))}\right) + \left(\frac{4f(z_k)}{(f(x_k)+\beta f(z_k))}\right)\right]$$

where $\propto, \beta \in R$ **(49)**

3.25 q analogue of method by Kou-Wang-Li[62,63]

$$p_k = x_k - \frac{f(x_k)(q-1)x_k}{f(qx_k)-f(x_k)} \quad \textbf{(50)}$$

$$z_k = p_k - \frac{f(p_k)(q-1)x_k}{f(qx_k)-f(x_k)} \frac{(f(x_k))}{(f(x_k)-2f(p_k))} \quad \textbf{(51)}$$

$$x_{k+1} = z_k - \frac{f(z_k)}{\frac{f(p_k)-f(z_k)}{p_k-z_k} - c(p_k-z_k) - d(p_k-z_k)^2}$$

where, $d = \frac{d_q f(x_k)-h}{(p_k-x_k)(z_k-x_k)} - \frac{h-k}{(z_k-x_k)^2}$

$$c = \frac{h-k}{x_k-z_k} - d(x_k+w_k-2z_k)$$

$$h = \frac{(f(x_k)-f(p_k))}{x_k-p_k}$$

$$k = \frac{f(p_k)-f(z_k)}{p_k-z_k} \quad \textbf{(52)}$$

3.26 q analogue of method by Kou-Wang-Li[62,63] with two parameters

$$p_k = x_k - \frac{f(x_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)} \quad \textbf{(53)}$$

$$z_k = p_k - \frac{f(p_k)(q1-q2)x_k}{f(q1x_k)-f(q2x_k)} \frac{(f(x_k))}{(f(x_k)-2f(p_k))} \quad \textbf{(54)}$$

$$x_{k+1} = z_k - \frac{f(z_k)}{\frac{f(p_k)-f(z_k)}{p_k-z_k} - c(p_k-z_k) - d(p_k-z_k)^2}$$

where ,$d = \frac{d_q f(x_k)-h}{(p_k-x_k)(z_k-x_k)} - \frac{h-k}{(z_k-x_k)^2}$

$$c = \frac{h-k}{x_k-z_k} - d(x_k+w_k-2z_k)$$

$$h = \frac{(f(x_k)-f(p_k))}{x_k-p_k}$$

$$k = \frac{f(p_k)-f(z_k)}{p_k-z_k} \quad \textbf{(55)}$$

IV. Problems

Problem 1

$f(x) = (x^3 - 9x^2 + 24x - 20)^{1/3} + e^{x/2}$ =0

This function is not differentiable at x=2 and if we choose x_0=2 Classical Newton Raphson Method can not be applied on this problem but q analogue of Newton Raphson Iterative Method is applicable.

x_0	q	$f(x_0)$	$f(qx_0)$	x_1	q	$f(x_1)$	$f(qx_1)$	x_2
2	0.999	2.7182818	2.69266553	1.787769	0.999	1.91963	1.914404	1.131105
2	0.999999	2.7182818	2.71805016	1.976533	0.999999	2.568049	2.56804	1.432849
2	0.989	2.7182818	2.57503132	1.582534	0.989	1.36483	1.321015	1.040274
2	0.99	2.7182818	2.58473299	1.592916	0.99	1.391157	1.350822	1.043515
2	0.98	2.7182818	2.4950233	1.51298	0.98	1.192108	1.118805	1.02088
2	0.97	2.7182818	2.41543867	1.461448	0.97	1.067887	0.96446	1.008759
2	1.001	2.7182818	2.69811227	2.269543	1.001	2.527245	2.52767	-11.2261
2	1.01	2.7182818	2.63957183	2.690708	1.01	2.80673	2.836214	0.129279

$f(x_2)$	$f(qx_2)$	x_3	$f(qx_3)$	$f(x_3)$	x_4	$f(x_4)$	$f(qx_4)$	x_5
0.330953	0.328578	0.284836	-1.25014	-1.24966	1.028571	0.11914	0.11705	0.969947
1.000197	1.000194	0.703346	-0.51168	-0.51168	0.979244	0.01963	0.019628	0.969441
0.14297	0.119668	0.13332	-1.50261	-1.50023	1.05498	0.173038	0.149299	0.970391
0.149585	0.128311	0.139067	-1.49313	-1.49086	1.053903	0.170832	0.149277	0.970378
0.103525	0.062251	0.098332	-1.56017	-1.55699	1.061658	0.186739	0.143278	0.970425
0.078992	0.018132	0.075917	-1.59681	-1.59314	1.066041	0.195745	0.130305	0.970378
-14.1551	-14.1664	3.580464	4.47567	4.465953	1.934963	2.396389	2.403667	1.297878
-1.5068	-1.5047	-12.302	-15.3608	-15.2371	2.860043	3.01337	3.053192	0.695774

Table 1: Calculation of x_1,x_2,x_3,x_4,x_5 (q analogue of Newton Raphson Method) for different values of q

Problem 2

Proceeding in this way we will get the solution after five iterations i.e x=0.969426 is the solution.
Basic Analogue of Newton Raphson Method:
Let us solve one more problem $f(x)= xe^x-1$
Let $x_0=1$
$f(1)= (e-1)$
$f(q)=qe^q-1$

If we calculate x_1 by Classical Newton Raphson Method we will get $x_1=0.6839397$ and $f(x_1)=0.3553424$

q	x_1
0.96	0.6743415
0.97	0.676762956
0.98	0.679169779
0.99	0.681562017
0.999999999999	0.683945343
1.01	0.686302938
1.09	0.704693177
0.95	0.671905363
0.9	0.659502787
0.8	0.633569653
0.7	0.606095896
0.6	0.577041065
0.5	0.546369238
0.4	0.514049563
0.3	0.480056757
0.2	0.444371563
0.1	0.40698115

Table 2: Calculation of x_1 by iterative method(q-analogue of Numerical Methods for different values of q)

Value of x_1 by Newton Raphson Method is 0.6839397.
$f(x_1)$ by Newton Raphson Method is 0.3553424
$f(x_1)$ by q analogue ot Newton Raphson Method at q=0.99 is 0.347423143
$f(x_1)$ by q method at q=0.97 is 0.33153.
We can observe that value of $f(x_1)$ using q method is closer to zero which means it is more accurate and converges rapidly towards solution.

V. Conclusion

q-analogue of iterative methods for solving algebraic and transcendental equations gives the same result as classical methods do but it converges more rapidly towards solution and errors associated with these methods are comparatively lesser if value of q is chosen accordingly and this method is very appropriate for solving transcendental equations .By using single parameter we have to choose value of q very close to one but for double parameter we can get accurate result for most of the values of q_1.Problems have been solved using C++ Programming Language .Open methods (Newton, Halley etc.) differ from the bracketing methods

(Bisection, Regula-Falsi etc.) in the sense that they use information at single point or multiple points. Although it leads to quicker convergence, but it also includes a possibility that the solution may diverge. In general, the convergence of open techniques is partially dependent on the quality of the initial guess and the nature of the function. The closer an initial guess is to a true root, the more likely it is that the methods will be convergent. However, for a given nonlinear equation, it is rather hard to choose an initial approximation near a root. In general, any iterative scheme may be divergent if initial approximation is far from the root.

References

[1]. Exton, H. (1983)., q-Hypergeometric functions and applications Ellis Harwood Ltd., Halsted, John Wiley & Sons, New York
[2]. Al-Salam W.A. (1966) Proc. Amer. Math. Soc., 17, 616-621.
[3]. Al-Salam W.A. and Verma A. (1975) Pacific J. Math., 60, 1-9.
[4]. Annaby M.H. and Mansour Z.S. (2008) J. Math.Anal. Appl., 344, 472-483.
[5]. Atkinson K.E. (1989) An Introduction to Numerical Analysis, John Wiley & Sons, New York, USA.
[6]. Bouzeffour F. (2006) J. Nonlinear Math. Phys., 13(2), 293-301.
[7]. Burden R.L. and Faires J.D. (2005) Numerical Analysis, Thom-as Brooks/Cole, Belmont, CA.
[8]. Ernst T. (2003) J. Nonlinear Math. Phys., 10, 487-525.
[9]. Ernst T. (2002) A New Method for q-Calculus., A Doctorial Thesis., Uppsala University.
[10]. Fitouhi A., Brahim, Kamel B., Bettaibi N. (2005) J. Nonlinear Math. Phys., 12, 586-606.
[11]. Gasper G. and Rahman M. (2004) Basic Hypergeometric Se-ries., Cambridge University Press, Cambridge.
[12]. Hämmerlin G. and Hoffmann K.H. (2005) Numerical Methods., Cambridge University Press, Cambridge.
[13]. Ismail M.E.H. (2005) Classical and Quantum Orthogonal Poly-nomials in One Variable, Cambridge University Press, Cam-bridge.
[14]. Ismail M.E.H. and Stanton D. (2003) J. Comp. Appl. Math., 153, 259-272.
[15]. Ismail M.E.H. and Stanton D. (2003) J. Approx. Theory, 123, 125-146.
[16]. Jackson F.H. (1910) Quart. J. Pure and Appl. Math., 41, 193- 203.
[17]. Jackson F.H. (1909) Messenger Math., 39, 62-64.
[18]. Jackson F.H. (1908) Trans. Roy. Soc. Edinburgh, 46, 64-72.
[19]. Jackson F.H. (1904) Proc. Roy. Soc. London, 74, 64-72.
[20]. Kac V., Cheung P. (2002) Quantum Calculus. Springer-Verlag, New York.
[21]. Kajiwara K., Kimura K. (2003) J. Nonlinear Math. Phys., 10(1), 86-102.
[22]. Kim T. (2006) J. Nonlinear Math. Phys., 13, 315-322.
[23]. Lo'pez B., Marco J.M. and Parcet J. (2006) Proc. Amer. Math. Soc., 134(8), 2259-2270.
[24]. Marco J.M. and Parcet J. (2006) Trans. Amer Math. Soc., 358 (1), 183-214.
[25]. S. Ramanujan(1957), Notebooks, **2** Volumes, Tata Institute of Fundamental Research, Bombay.
[26]. Bailey W.N. (1950), On the basic bilateral hypergeometric series 2ψ2, Quart. J.Math. Oxford Ser. **2(1)** 194 − 198.
[27]. Bailey W.N. (1952), A note on two of Ramanujan's formulae, Quart. J. Math. **2(3)**, 29 − 31.
[28]. Bailey W.N. (1952), A further note on two of Ramanujan's formulae, Quart. J.Math.**(2)**,3, 158 − 160.
[29]. Berndt B. C. (1985),Ramanujans notebooks. Part I, Springer-Verlag, New York.
[30]. Berndt B. C. (1989),Ramanujans notebooks. Part II, Springer- Verlag, New York.
[31]. Berndt B. C. (1991),Ramanujans notebooks. Part III, Springer- Verlag, New York**.**
[32]. Berndt B. C. (1994),Ramanujans notebooks. Part IV, Springer- Verlag,New York.
[33]. Berndt B. C. (1998),Ramanujans notebooks. Part V, Springer- Verlag,New York**.**
[34]. Berndt B. C. and S.H. Chan(2007), Sixth order mock theta functions, Adv. Math.**216**, 771 − 786.
[35]. Berndt B. C., Chan S.H. B.P. Yeap , and A.J. Yee(2007),, A reciprocity theorem for certain q-series found in Ramanujans lost notebook, Ramanujan J. **13**, 27 − 37.
[36]. Berndt B. C. and Rankin R.A. (1995), Ramanujan: Letters and Commentary,American Mathematical Society, Providence, RI, 1995; London Mathematical Society, London**.**
[37]. Berndt B.C. and Rankin R.A. (2001), Ramanujan: Essays and Surveys, American Mathematical Society, Providence, 2001; London Mathematical Society,London**.**
[38]. Bhagirathi N. A. (1988),, On basic bilateral hypergeometric series and continued fractions, Math. Student, **56** 135 − 141.
[39]. Bhagirathi N. A. (1988),, On certain investigations in q-series and continued fractions, Math. Student, **56**, 158 − 170.
[40]. Homeier, H.H.H. (2003).A modified Newton method for root finding with cubic convergence.J.Comput. Appl. Math. 157: 227–230
[41]. Kou, J., Li, Y. and Wang, X. 2006. A modification of Newton method with third order convergence. Appl. Math. Comput.181: 1106–1111
[42]. Ozban, A.Y. (2004).Some new variants of Newton's method.Appl. Math. Lett.17:677–682
[43]. Weerakoon, S. and Fernando, T.G.I. 2000. A variant of Newton's method with accelerated third-order convergence. Appl. Math. Lett.13: 87–93
[44]. Melman, A. (1997).Geometry and convergence of Euler's and Halley's methods. SIAM Rev. 39: 728–735
[45]. Davies, M. and Dawson, B. (1975). On the global convergence of Halley's iteration formula. Numer.Math.24: 133–135
[46]. Traub, J.F. 1964. Iterative Methods for the Solution of Equations. Prentice-Hall, Englewood Cliffs
[47]. Johnson, L.W. and Riess, R.D. (1977). Numerical Analysis. Addison-Wesley, Reading.
[48]. W. HAHN, Uber Orthogonalpolynome, die q-Differenzengleichungen gen¨ ugen.¨ Mathematische Nachrichten **2**, 1949, 4–34.
[49]. Askey, R. and Wilson, J(1997), "Some basic hypergeometric Polynomials" that generalize Jacobi polynomials" Men. Amer. Math. Soc. 54, 319.
[50]. Watson, G.N. (1929)A new proof of Rogers-Ramanujan identities, J. London Math. Soc. 13, 4-9.
[51]. Andrews G.E. and Askey R.(1985), Classical orthogonal polynomials. In: Polynomes Orthogonaux et Applications (eds. C. Brezinski et al.). Lecture Notes in Mathematics **1171**, SpringerVerlag, New York, 1985, 36–62.
[52]. Andrews G.E. and Askey R.and Roy R.(1999), Special Functions. Encyclopedia of Mathematicsand Its Applications **71**, Cambridge University Press, Cambridge.
[53]. Agarwal, R.P.(1965): "An extension of Meijer's G-function" proc. Nat. Inst. Sci (India), Vol (31) A 536-46.
[54]. Agarwal, R.P (1968): "Certain basic hypergeometric identities associated with mock theta functions", Quart.J.Math. (Oxford) 20 , 121-128.
[55]. Agarwal, R.P(1976): "Fractional q-derivative and q-integrals and certain hypergeometric transformation", Ganita, Vol. 27, 25-32.

[56]. Agarwal, R.P.(1991):"Ramanujan's last gift", Math, Student, 58 , 121-150.

[57]. Agarwal, R.P :Generalized hypergeometric series and its applications to the theory of Combinatorial analysis and partition theory.

[58]. Verma, A. and Jain, V.K.(1981) , "Some summation formulae of Basic hyper geometric series", Indian J. of Pure and applied Math. 11 (8), 1021-1038.

[59]. Dougall, J. (1907),,"On Vondermonde's theorem and some more general expansions", Proc. Edin. Math. Soc. 25, 114-132.

[60]. Singh Prashant, Mishra P.K. and Pathak R.S. (2013),q-Iterative Methods, IOSR Journal of Mathematics (IOSR-JM), Volume 8, Issue 6 (Nov. – Dec. 2013), PP 01-06

[61]. Kendall E. Atkinson, *An Introduction to Numerical Analysis*, (1989) John Wiley & Sons, Inc, ISBN 0-471-62489-6.

[62]. Kou, J., (2007). The improvements of modified Newton's method. Appl. Math. Comp. 189: 602–609 [117]

[63]. Kou, J. 2007. Some new sixth-order methods for solving non-linear equations. Appl. Math. Comput. 189: 647–651

[64]. Kou, J., Li, Y. and Wang, X. (2006). Efficient continuation Newton-like method for solving systems of non-linear equations. *Appl. Math. Comput.* 174: 846–853

[65]. Thukral, R. (2008). Introduction to a Newton-type method for solving nonlinearequations. *Appl. Math. Comp.* 195: 663–668

[66]. Bi, W., Ren, H. and Wu, Q.(2008). New family of seventh-order methods for nonlinear equations. *Appl. Math. Comput.* 203: 408–412

[67]. Cordero, A., Torregrosa, J.R., Vassileva, M.P. (2011). A famliy of modified Ostrowski's methods with optimal eighth order of convergence. *Appl. Math. Lett.* 24:2082–2086