3d Object Recognition Using Centroidal Representation

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Abstract: Three dimensional object recognition from two-dimensional image is implemented in this work with the use of 512 different poses of the object (which is represented by an airplane or a cube or a satellite). The object is rotated in the three directions (x,y,z) by an angle of $2\pi/8$ rad. ($45^\circ$). Centroidal sample representation was used to represent the data extracted from the object. Also four or five variables for each object are calculated. These variables represent fast-calculated features of the object that can speed up the recognition process. These variables are scaled and position in variant representation of the object. The mathematical model that represents the object was calculated in two methods. These were the Wavelet Transform Model and the Auto regression Model.

Keywords: Three Dimensional Objects, Two-Dimensional Image, the Mathematical Model, Wavelet Transform, Autoregression Model

I. Introduction

Recognition of an object image requires associating that image with views of the object in computer memory (Database which contains just enough different poses of the object under consideration). These views are called the Object Model and it is needed because as in real world man cannot recognize objects that he had never seen before [1]. The object is attached to the model image with the minimum distance if the distance is less than a predefined value. In real world, rendering an image is done with speed of light shining on the objects and reflecting into human eyes, but rendering of virtual worlds take much more time. This is the reason that the variable rendering quality has always been a biased balance between the computation time and the displayed effects. This setting is differed according to the application [2]. When large object is used searching will become extremely time consuming and hence not realistic. Consequently, the main concern here is to solve the combinatorial time complexity problem in an optimal manner, so that, the correct matching (recognition) is found as fast as possible [3].

II. Proposed Method

The proposed method presents new approach for recognition three-dimensional object from two-dimensional image. Many stages are implemented in this work as shown in figure 1.

Filtering

Spatial filters have been used to remove various types of noise in digital images. These spatial filters typically operate on a small neighborhood area in the range from 3×3 pixels to 11×11 pixels. If the noise added to each pixel is assumed to be additive white Gaussian, then the intensity of each pixel can be represented as follows [4]:

\[ d(x,y) = I(x,y) + n(x,y) \]  

where \( x \) and \( y \) are the coordinate of the pixel under consideration in the image. 

\( d(x,y) \) = Degraded pixel intensity in the image.  

\( I(x,y) \) = Original pixel intensity in the image.
n(x,y) = Additive White Gaussian function value with zero mean which represents the noise intensity added to the pixel (x,y).

Here through this work adaptive Alpha-trimmed filtering is suggested with the condition that if the filtering action is made over an edge pixel, then the filtering action will be canceled and the pixel point will have the same former value in the few filtered image. The decision for an edge point or not is made when the sum of the maximum three pixels values exceeds the sum of the three minimum pixels by a predefined threshold value which is determined experimentally. Figure 2 illustrates the filtered object.

![Filtered Object](image)

**Figure 2: Filtered Object**

**Binary Conversion**

In order to distinguish the object under consideration by its boundary, the image is transferred to a binary image after filtering process. The black level represents anything other than the object in the image and the maximum shiny white level represents the object. This procedure helps in eliminating the surface details of the object and keeps only the boundary information of the object. In some application this conversion may not be used. Figure 3 show binary image after process of filtered object.

![Binary Conversion](image)

**Figure 3: Binary Conversion**

**Edge Detection**

In this work, the main interest is in the presence of an edge not in its direction, so that Roberts operator (which is a (2×2) operator) would work well for the purpose of edge detection. This operator indicates the presence of an edge not its direction. If the magnitude of this operator exceeds some predefined value then an edge point will be detected. The magnitude of this operator is defined as:

\[ \text{Magnitude} = |I(x,y)-I(x-1,y-1)| + |I(x,y-1)-I(x-1,y)| \]  

...(2)

Robert operator is used here because of its computational efficiency. When the magnitude of this operator exceeds some predefined threshold, then an edge will be detected [5].

**Center Point Calculation**

The fastest method to calculate the center of the object in an image is to imagine a rectangle that surrounds the object. The cross point of the diagonals of the rectangle will be the center point of the object, which can be calculated quickly by calculating its coordinates as follows [6]:

\[ X_c = \frac{x_1 + x_2}{2} \]  

...(3)

\[ Y_c = \frac{y_1 + y_2}{2} \]  

...(4)

Where \( x_1 \) and \( x_2 \) are the X coordinates of the first and second vertical lines of the rectangle respectively, which represent the X coordinates of the first left and the first right points of the object in the image respectively. \( y_1 \) and \( y_2 \) are the Y coordinates of the first and second horizontal lines of the rectangle respectively which represent the Y coordinates of the highest top and the lowest bottom points of the object respectively.

**Data Acquisition and Data Normalization**

The data that represent some predefined features of the object is obtained from the object image. All the representation used is one-dimensional representation in which a single set of variables represents the object under consideration.
The Centroidal Representation [7]: In this type of representation, the center of the object is determined first. Then, the distances from that center to the object boundary is calculated every \((2\pi/N)\) angle from a predefined starting point on the boundary, where \(N\) is the number of the samples and it must equal to \((2n)\) (where \(n=1,2,...\)). Through this work \(n=4\) and \(n=5\) were used \((N=16\) and \(N=32\)). The variables (samples) obtained in this type of representation are scale variant, and so, some forms of normalization processes must be introduced [8].

Two types of normalization processes are suggested. The first type is object dependent normalization in which each samples. Dividing it by the first one normalizes the last sample in the set such shown in figure 4. The second type of normalization process is to calculate the average value of all samples and then divide all samples by the average value such shown in figure 5.

Mathematical Models

After the data extraction from the image of the object, data manipulation must be applied in order to have a suitable Mathematical Model for the object.

a) Autoregression Model

The measured data can be represented or approximately represented by selected Autoregression Model. The parametric spectral can be made in a three-step procedure.

1- The model set is selected (AR (all pole Autoregression)).
2- The model parameters are estimated from the data.
3- The spectral estimation is made by the estimated model parameter.

For linear systems with random input, the measurement of the output spectrum is related to the input spectrum by the factorization model [9].

\[
S_yy(Z) = H(Z)H^*(Z)S_{xx}(Z) = |H^2(Z)|S_{xx}(Z)
\]  \(\ldots(5)\)

Figure 6 shows the schematic diagram of the AR model.

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Figure 6 : Autoregression Model
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The model parametric estimation is [10]:

1- Select the representative model (AR Model).
2- Estimation the model parameter from data, that is given \(y(t)\), estimate \((A(q^{-1}))\), where AR: \(H(\Omega)=1/A(\Omega)\) (all pole model).

The all pole model is defined as:

\[
y(t) A(q^{-1}) = e(t)
\]

Where

- \(y(t)\) : is the measured data
- \(e(t)\) : is the zero mean, white noise sequence with variance \(R_{ee}\).
A(q^{-1}) is an Nth order polynomial in backward shift operator. The basic parameter estimation problem for the Autoregression in the general case (infinite covariance) is given as minimum (error) variance solution to

$$\min_{a} J(t) = E(e^2(t))$$  \(\text{(6)}\)

$$e(t) = y(t) - y(t)$$  \(\text{(7)}\)

y(t) is the minimum error between estimated and real value. y(t) is actually the one step predictor. The one step predictor estimates y(t/t-1) based on the past data samples. This operation is called linear predictor. The coefficients of the predictor were calculated using Durban's recursive algorithm. The order of each predictor depends on the number of samples (N) used in calculating the predictor coefficients and equals to the first integer \(> (\ln(N))\).

b) Wavelet Transform Model

Two forms of the discrete wavelet transform are used throughout this work. These are Haar wavelet transform and Daubechies wavelet transform:

**Haar Wavelet Transform** is a cold topic, their graphs are made from flat pieces \{1's and 0's\}. When dealing with a four elements one-dimensional signal. The Haar Wavelet Transform of this signal is a scaling function and with three Wavelet coefficients. If these scaling and wavelet coefficients are multiplied with Haar basis vectors then, the original signal vector will be retrieved. The Haar four element basis vectors form a matrix of the following form:

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$  \(\text{(10)}\)

As it can be seen the basis vectors of the Haar transform are orthogonal. This means that if any vector is multiplied by the inner product with any vector other than itself the result will be zero. (D = the signal vector \& S = Wavelet transform vector)

$$HS = D$$  \(\text{(11)}\)

$$H^{-1}HS = H^{-1}D$$  \(\text{(12)}\)

$$S = H^{-1}D$$  \(\text{(13)}\)

where

$$H^{-1} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix}$$  \(\text{(14)}\)

with

$$S = \begin{bmatrix} S \\ W_1 \\ W_2 \\ W_3 \end{bmatrix} \quad \text{&} \quad D = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$  \(\text{(15)}\)

Here throughout this work, Fast Haar algorithm is used which will transfer the four elements one dimensional signal vector to a scaling function with three-wavelet coefficient. This type of transform is done by the use of the Pyramid algorithm. The component of the signal vector are a1,a2,a3 and a4. The pyramid algorithm is applied as in figure 7 [11].
It was found through the mathematical calculations that the ratio of the scaling function over the first wavelet coefficient with the sum of wavelet coefficient from the second wavelet coefficient to the last one divided by the first wavelet coefficient gives a good normalized (scale invariant) mathematical representation of the data vector. This system is tested on different data sets having the same elements but with different sequences and it gives a good discrimination between these vectors. The first ratio equals:

$$\text{Ratio}_1 = \frac{S}{W_1}$$

...(16)

The second ratio equals:

$$\text{Ratio}_2 = \frac{W_1 + W_2 + \ldots + W_n}{W_1}$$

...(17)

Where n is the number of elements in the data vector. This representation was extended to 32 and 64 element vector.

Some malfunction may appear in the theoretical calculation of the fast Haar transform (such as when dealing with the two vectors of data (5,4,2,1) and (6,3,2,1). These two vectors have different elements but they have the same first ratio ($S/W_1$). A second ratio ($W_2 + W_3 + \ldots + W_n/W_1$) must be used in order to distinguish between the two vectors. A modification over the Harr basis vectors is assigned in order to distinguish between the two vectors from the first ratio. An example of this modification is given below for four elements.

$$H = \begin{bmatrix}
1 & -2 & 1 & 0 \\
2 & 1 & 0 & 1 \\
2 & 1 & 0 & -1 \\
1 & -2 & -1 & 0 
\end{bmatrix}$$

...(18)

$$H^{-1} = \begin{bmatrix}
0.1 & 0.2 & 0.2 & 0.1 \\
-0.2 & 0.1 & 0.1 & -0.2 \\
0.5 & 0 & 0 & -0.5 \\
0 & 0.5 & -0.5 & 0 
\end{bmatrix}$$

...(19)

From this for elements modified Haar transform the scaling function and the three wavelet coefficients are calculated as follows:

$$S = \frac{a_1 + 2(a_2 + a_3 + a_4)}{10}$$

...(20)

$$W_1 = \frac{2(a_1 + a_2) + a_3 + a_4}{10}$$

...(21)

$$W_2 = \frac{a_1 - a_4}{2}$$

...(22)

$$W_3 = \frac{a_2 - a_3}{2}$$

...(23)

The second proposed modified Harr vectors give the following form for four-element signal vector.

$$H = \begin{bmatrix}
1 & 2 & 2 & 0 \\
2 & -1 & 0 & 1 \\
1 & 2 & -2 & 0 \\
2 & -1 & 0 & -1 
\end{bmatrix}$$

...(24)
These proposed wavelet transforms were extended to 16 and 64 element wavelets. **Daubechies wavelet transform:** Three conditions must be satisfied when building the Daubechies Matrix of basis vectors these condition are:

1- The basis vectors must be orthogonal, which mean that the inner product of any two different vector is zero.

2- The difference between the odd and even coefficients is zero.

3- The summation of the coefficients must be bounded (through references, the select number for bounding is (2)).

Three approaches for modified Daubechies wavelet basis vectors are proposed for the 16-element data vector. Each one uses different conditions from the others.

\[
H^{-1} = \begin{bmatrix}
0.1 & 0.2 & 0.2 & 0.1 \\
0.2 & -0.1 & 0.2 & -0.1 \\
0.25 & 0 & -0.25 & 0 \\
0 & 0.5 & 0 & -0.5
\end{bmatrix}
\] ...

The sets of equations that must be fulfilled in order to solve for the \( \alpha \) are:

\[
d_1d_{15}+d_2d_{16} = 0 \\
d_9d_{15}+d_{10}d_{16} = 0 \\
d_1d_{15}+d_2d_{16} = 0 \\
d_3d_{15}+d_{16} = 0 \\
d_1d_9+d_{10}d_{15}+d_{12}d_{16} = 0 \\
d_1d_9+d_{10}d_2+d_5d_{15} = 0 \\
d_1d_9+d_{10}d_6+d_6d_{15}+d_{12}d_4+d_1d_k+d_1d_2+d_1d_1 = 0
\]

The preceding equations satisfy the following Daubechies condition:

\[
\sum_{k=1}^{N} d_k d_{k+2m} = 0 \quad (m \neq 0)
\]

Where \( N \) is the number of elements in each one of the Daubechies basis vectors

\[
d_{16}+d_{15}+d_{14}+d_{13}+d_{12}+d_{11}+d_{10}+d_9+d_8+d_7+d_6+d_5+d_4+d_3+d_2+d_1 = 0
\]

which satisfies the Daubechies condition for wavelet basis vectors elements:

\[
\sum_{k=1}^{N} d_k = 2
\]

\[
d_{16}d_5+d_{15}d_3+d_{14}d_1+d_{10}d_9+d_8d_7+d_6d_5+d_4d_3+d_2d_1 = 0
\]

which satisfies the Daubechies condition for wavelet basis vectors elements:

\[
\sum_{k=1}^{N} (-1)^k k^m d_k \quad \text{for } m = 0,1,2,3,\ldots,(N/2)-1
\]

Equation (37) (n) is taken when the value (0).

\[
d_1d_9-d_3d_7 = 0 \\
d_1d_9-d_3d_3 = 0 \\
d_1d_9-d_3d_{16} = 0
\]
Distance Measurement

When the Mathematical Model for the object is built, then the recognition process can be fulfilled. The decision of the object being recognized or not will depend on three types of distance measurement, these are:

1- When dealing with a single element set such as the variable ratio or angle1 or angle2,...,etc., the direct absolute difference is used. Which is given as:

$$\text{Distance} = |R_1 - R_2|$$  \hspace{1cm} (47)

2- When dealing with one-dimensional vector with more than one element, such as Autoregression Model Euclidean distance is a good measure for the distance between two vectors. If the two vectors are \((a_1, a_2, \ldots, a_n)\) and \((b_1, b_2, \ldots, b_n)\), then the Euclidean distance between them is defined as [7]:

$$\text{Distance} = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_n - b_n)^2}$$  \hspace{1cm} (48)

3- For the same preceding case, Non-metric measure similarity function is good distance measure also. It is defined as \(S(A, B)\).

$$S(A, B) = \frac{\text{Average Crossing}}{\| \text{autocorrelation} \|} = \cos \theta$$  \hspace{1cm} (49)

This measurement gives high value for \(\cos \theta\) when the similarity between the two vectors \(A\&B\) is high.

Object Model

The first step after the filtering and edge extraction processes is the calculation of four variables. These variables represent some fast-calculation features of the designed object. These are the first four variables in the Mathematical Model of each pose in the Object Model. In the recognition process, each one of these variables for the object under consideration is compared with corresponding variable for the pose in the Object Model. If the distance between the two corresponding variables is higher than a predefined value then the recognition process with that pose is terminated. These four variables are defined as Ratio, Angle1, Angle2 and ratio3. In some applications a fifth variable is calculated which is the Average Crossing. The variables are:

- **Ratio**: It is the ratio of the line connecting the highest pixel to the lowest pixel in the object image to the line connecting the first left pixel to first right pixel in the object image. (when more than one pixel appear at any one of the four sides, then for the Top side it is the first left pixel. For the Bottom side it is the first right pixel. For the Right side, it is the first bottom pixel and for the Left side, it is the first top pixel).

- **Angle1**: It is the angle between the line connecting the top pixel to the bottom pixel in the object image to the horizontal axis.

- **Angle2**: It is the angle between the line connecting the first left pixel to the first right pixel in the object image to the horizontal axis.

- **Ratio3**: It is the ratio of the line between the object boundaries and passing through the center of the object horizontally to that passing through the center vertically.

**Average Crossing**: One type of scale invariant representation that may be used in the Object Model building is the Average Crossing, which is used with centroidal representation of the object. This variable is calculated as follows:

The average of centroidal samples is calculated. After that the number of the times that the centroidal samples switches between values higher or less than the average value of the samples is calculated which will give a scale invariant representation of the object.

III. Results

Every proposed algorithm in this work has its own sequential database, which represent the Object Model containing the mathematical representation of all possible 512 poses of the object. Different types of desired and undesired object images were tested. When the following distances were used, the algorithms failed to recognize the correct objects and failed to reject the undesired objects.

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Maximum allowable distance for variable Ratio = 0.3.
Maximum allowable distance for variable Angle1 = \( \pi \).
Maximum allowable distance for variable Angle2 = 2\( \pi \).
Maximum allowable distance for variable Ratio3 = 0.35.
Maximum allowable distance for variable Average Crossing = 0.

For the Wavelet Transform Model and Centroidal Representation for 16 samples and average normalization, the maximum allowable distances as shown in table 1.

**Table 1** maximum allowable distances when using 16 samples and average normalization

<table>
<thead>
<tr>
<th>S/W ( W_i )</th>
<th>( (W_1+...+W_n)/W_i )</th>
<th>1M.H.T</th>
<th>2M.H.T</th>
<th>1M.D.T</th>
<th>2M.D.T</th>
<th>3M.D.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.15</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

For the Wavelet Transform Model and Centroidal Representation for 16 samples and proceeding sample normalization, the maximum allowable distances as shown in table 2.

**Table 2** maximum allowable distances when using for 16 samples and proceeding sample normalization

<table>
<thead>
<tr>
<th>S/W ( W_i )</th>
<th>( (W_1+...+W_n)/W_i )</th>
<th>1M.H.T</th>
<th>2M.H.T</th>
<th>1M.D.T</th>
<th>2M.D.T</th>
<th>3M.D.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.1</td>
<td>0.09</td>
</tr>
</tbody>
</table>

For the Wavelet Transform Model and Centroidal Representation for 32 samples and average normalization, the maximum allowable distances are:

**Table 3** maximum allowable distances when using 32 samples and average normalization

<table>
<thead>
<tr>
<th>S/W ( W_i )</th>
<th>( (W_1+...+W_n)/W_i )</th>
<th>1M.H.T</th>
<th>2M.H.T</th>
<th>1M.D.T</th>
<th>2M.D.T</th>
<th>3M.D.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.55</td>
<td>0.25</td>
<td>0.3</td>
<td>0.1</td>
<td>0.09</td>
<td>0.75</td>
</tr>
</tbody>
</table>

For the Wavelet Transform Model and Centroidal Representation for 32 samples and proceeding sample normalization, the maximum allowable distances are:

**Table 4** maximum allowable distances when using for 32 samples and proceeding sample normalization

<table>
<thead>
<tr>
<th>S/W ( W_i )</th>
<th>( (W_1+...+W_n)/W_i )</th>
<th>1M.H.T</th>
<th>2M.H.T</th>
<th>1M.D.T</th>
<th>2M.D.T</th>
<th>3M.D.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.55</td>
<td>0.25</td>
<td>0.35</td>
<td>0.13</td>
<td>0.05</td>
<td>0.085</td>
</tr>
</tbody>
</table>

**For Autoregression Model:**
Maximum Euclidean distance for Centroidal representation of 16 samples with average normalization = 0.8.
Maximum Euclidean distance for Centroidal representation of 16 samples with proceeding sample normalization = 0.85.
Maximum Euclidean distance for Centroidal representation of 32 samples with average normalization = 0.8.
Maximum Euclidean distance for Centroidal representation of 16 samples with proceeding sample normalization = 1.
Maximum non-metric measure of similarity distance for Centroidal representation with 16 sample and average normalization = 0.1.
Maximum non-metric measure of similarity distance for Centroidal representation with 16 sample and proceeding sample normalization = 0.15.
Maximum non-metric measure of similarity distance for Centroidal representation with 32 sample and average normalization = 0.1.
Maximum non-metric measure of similarity distance for Centroidal representation with 32 sample and proceeding sample normalization = 0.1.

**IV. Conclusion**

In this work, simple, easy implemented and robust techniques for solving the 3D-object recognition are proposed. These techniques are Object Model based. The use of adaptive filter through this work saves the object edges and high frequency component from being blurred. The fastest method for calculating the data concerned with object was the Centroidal representation with 16 samples. When dealing with objects of high
boundary details this method fail to give a good representation to 32, 64 or even more samples which increases the time consumption of the algorithms. The four or five variables used in each pose mathematical model in Object Model are scale and position invariant sine they represent ratio and angles.

Simple Haar Wavelet transformation is the fastest method for calculating the Mathematical Model for the object. This transformation does not contain any multiplication or division processes. The malfunction of this type of transformation is that it needs to calculate two ratios for its mathematical model. The modified Haar and Daubechies transforms proposed through this work give a good mathematical representation for the object by using only Ratio variable, which is S/W1. The modified Daubechies Wavelet coefficients were calculated analytically not numerically which can simplify the calculations.

References

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