PageRank algorithm and its variations: A Survey report

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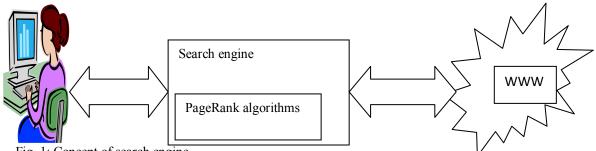
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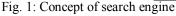
Abstract: This survey report basically gives a comparison report on different page ranking algorithm on the basis of numerical analysis. Four popular algorithms used for this purpose PageRank, Weighted PageRank, PageRank using VOL and finally weighted PageRank using VOL. This report will give brief introduction of web mining concept because web mining is core content in page rank calculation .Many aspect related to the page ranking will be discussed by taking PageRank algorithm and its variation as a basis for this purpose. A detailed comparison table of PageRank, Weighted PageRank, PageRank using VOL and finally weighted PageRank using VOL will be used for better analysis.

Keywords: PageRank, Weighted pageRank, PageRank based on visits of links, Weighted page rank based on visits of links, Web mining

I. Introduction

WWW (World Wide Web), a technique which change the world of information, change the life style of internet world. People could easily got information they did wanted. This huge web world has billions of web structures and within one structure there would be thousands or millions of web resources (links, contents) may exist. But there is a problem with WWW, its size, diversity, heterogeneous nature, non relevancy. So if user wants to access result with more accuracy then problem comes into picture because at this time search engine work starts and result depends on the capability of search engine.





This picture only show about the basic idea behind search engine, there are many other things also participate in this searching technique, crawler, indexer, query processor etc. But our paper is based upon page ranking algorithm. So I am not going to discuss these things but in this paper it will cover page ranking algorithm and its variations.

This paper has six section excluding section 1 introduction, section 2 explain history of pagerank algorithm, section 3 represent basic overview of web mining and kinds of web mining, section 4 give explanation about different pagerank algorithms, section 5 explain numerical part of these algorithms, section 6 represent a comparison table for these algorithms and finally section 7 has conclusion.

II. History of PageRank algorithm

PageRank algorithm is heart of Google. Google began in March 1996 as a research project by Larry Page and Sergey Brin PhD students at Stanford working on the Stanford Digital Library Project (SDLP). The SDLP's goal was "to develop the enabling technologies for a single, integrated and universal digital library" and was funded through the National Science Foundation among other federal agencies. In search for a dissertation theme, Page considered among other things exploring the mathematical properties of the World Wide Web, understanding its link structure as a huge graph. His supervisor Terry Winograd encouraged him to pick this idea (which Page later recalled as "the best advice I ever got") and Page focused on the problem of finding out which web pages link to a given page, considering the number and nature of such backlinks to be valuable information about that page [9]. PageRank would produce better results than existing techniques (existing search engines at the time essentially ranked results according to how many times the search term appeared on a

page). Google search engine arranges the retrieved documents based on the page rank and relevance with higher priority of page rank which means that the page with the highest page rank will appear first even if its relevance is not sufficient [3].

Wenpu Xing and Ali Ghorbani proposed an extension to standard PageRank called Weighted PageRank (WPR) in 2004. It assumes that if a page is more popular, more linkages other web pages tend to have to it or are linked to by it. This algorithm does not divide the rank value of a page evenly among its outgoing linked pages, rather it assigns larger rank values to more important pages [1].

Gyanendra Kumar, Neelam Duhan, A. K. Sharma proposed PageRank based on VOL in 2011. Search engines generally return a large number of pages in response to user queries. To assist the users to navigate in the result list, ranking methods are applied on the search results. Most of the ranking algorithms proposed in the literature are either link or content oriented, which do not consider user usage trends. In this paper, a page ranking mechanism called Page Ranking based on Visits of Links(VOL) is being devised for search engines, which works on the basic ranking algorithm of Google i.e. PageRank and takes number of visits of inbound links of Web pages into account. This concept is very useful to display most valuable pages on the top of the result list on the basis of user browsing behaviour, which reduces the search space to a large scale [2].

Neelam Tyagi, Simple Sharma proposed weighted pagerank based on VOL in 2012.

A page ranking mechanism called Weighted PageRank Algorithm based on Visits of Links (VOL) is being devised for search engines, which works on the basis of weighted pagerank algorithm and takes number of visits of inbound links of web pages into account. The original Weighted PageRank algorithm (WPR) is an extension to the standard PageRank algorithm. WPR takes into account the importance of both the inlinks and outlinks of the pages and distributes rank scores based on the popularity of the pages. This algorithm is used to find more relevant information according to users query. So, this concept is very useful to display most valuable pages on the top of the result list on the basis of user browsing behaviour, which reduce the search space to a large scale [6].

In spite of these there is one more important algorithm exists whose name is A-Pagerank algorithm. In this algorithm, the PageRank value of the source page is distributed to its Link-out pages according to the topic similarity [4].

III. Web mining

Due to rapid growth of web data, information, files on the internet throughout the world, web mining came into picture. Web data's on the internet are heterogeneous, diverse and large, so arrangement of different data's must be compulsory to provide these data's to different group of users efficiently, so there is need of data mining on these data by the help of it, user gets relevant information from the web. Hence in web mining different techniques of data mining are applied on the web data's. But it is not the only body that actually fit for this purpose, besides data mining, artificial intelligence, information retrieval, natural language processing technique can be used efficiently[web mining today and tomorrow]. Web mining gives sophisticated result while accessing the web by the users.

In research paper the exact definition of web mining given as follows:

"Web mining is the application of data mining techniques to find interesting and potentially useful information from the web. It is normally expected that either the hyperlink structure of the web or web log data or both have been used in mining process [8]."

"Web mining is based on knowledge discovery from web, extract the knowledge framework represents in proper way. Web mining is like a graph and all pages are node and each connects with hyperlinks. Web mining is useful to extract the information, image, text, audio, video, documents and multimedia [7]."

Web mining can be categorized in to three area of interest based on which part of the web to mine [7]:

- a) Web Content Mining
- b) Web Structure Mining
- c) Web Usage Mining

3.1 Web Content Mining: Web Mining is basically extract the information on the web. Which process is happen to access the information on the web. It is web content mining. Many pages are open to access the information on the web. These pages are content of web. Searching the information and open search pages is also content of web [7]. Since there are various contents present in web in the form of text, image, video, sound etc. But the primary resources that are mined in web content mining are individual pages [8].

3.2 Web Structure Mining: We can define web structure mining in terms of graph. The web pages are representing as nodes and Hyperlinks represent as edges. Basically it's shown the relationship between user & web. The motive of web structure mining is generating structured summaries about information on web

pages/webs. It is shown the link one web page to another web page [7]. Web structure mining actually focuses on link information.

1.3 Web Usage Mining: Web mining usage aims at utilize data mining techniques to discover the usage patterns from web based application. It is technique to predict user behaviour when it is interact with the web [7].

IV. PageRank

Page ranking algorithms are the heart of search engine and give result that suites best in user expectation. Need of best quality results are the main reason in innovation of different page ranking algorithms, HITS, PageRank, Weighted PageRank, DistanceRank, DirichletRank Algorithm, Page content ranking are different examples of page ranking used in different scenario. Since GOOGLE search engine has great importance now days and this affect many web users now days, so page rank algorithm used by GOOGLE become very important to researches. This paper demonstrate actual page rank algorithm use by GOOGLE that was developed by Larry page, this is the reason why page rank algorithm got this name "PageRank" and its different improved version that was introduced by different researcher.

4.1 PageRank algorithm

PageRank was proposed by S. Brin and L. Page at Stanford University. PageRank is used by very popular search engine of today, GOOGLE. The basic concept behind GOOGLE is, it combined the actual page rank value of page with the matching text value of query and find the overall score of a page [6] [5].

PageRank deals with the link structure of web page. It gives more importance to back links of a web page. If a page has important back links that means this page automatically gets more importance and provide more importance to other pages which this page is pointing. Basic fo

$$pr(u) = \sum_{v \in B(u)} pr(v)/l(v)$$
 (1)

But in some research paper it was given differently

$$pr(u) = c \sum_{v \in B(u)} pr(v) / l(v) \qquad (2)$$

Where:

Pr(u) = page rank of page u that we want to find out

B(u) = Back link set of page u

Pr(v) = Page rank value of page v that is pointing to page u

L(v) = Number of link present in page v

C = Normalization factor

Since these formula basically holds for those users which follow the link structure of page. But for WWW user do not follow the link structure; they randomly click web pages, so for that the modified pagerank formula is (3)

$$pr(u) = (1-d) + d\sum_{v \in B(u)} pr(v)/l(v)$$

Where;

D=damping factor (Whose value is 0.85(approx))

In Equation (3) we can see that (1-d+d) = 1 so it simply multiply 1 with the equation (1) and (2). Here d means user actually follows the link structure and (1-d) means user is randomly select any page on WWW. This formula actually calculated in iterative manner. And after conversation, the actual value of page rank is calculated. PageRank Value converges roughly in logn [1].

4.2 Weighted PageRank algorithm

Weighted pagerank was proposed by Wenpu Xing and Ali Ghorbani [1] [5]. It is the modified version of traditional PageRank algorithm. In this technique, it does not divide page rank value equally in between number of page links available in the page. Unlike the traditional Pagerank algorithm it gives more value to the important pages.

Each out link page gets a value proportional to its popularity (its numbers of in links and out links). Popularity of web pages can be determine by the help of outgoing links and in coming links to the web pages, so this algorithm determine this value by the help of two functions, namely

 $W_{in}(v,u) =$ weight of the link (v, u) calculated based upon number of in links of page u and the number of in links of all references pages of page v.

 $W_{in}(v, u) = I_u / \sum_{p \in R(v)} I_p$

Where;

 $I_u =$ Number of inlinks of page u.

 $I_p =$ Number of inlinks of page p.

(4)

 $W_{out}(v,u)$ = weight of the link (v,u) calculated based upon number of out links of page u and the number of out links of all references pages of page v.

 $W_{out}(v, u) = O_u / \sum_{p \in R(v)} O_p$

Where;

 $O_u =$ Number of inlinks of page u.

 $O_p =$ Number of inlinks of page p.

So the modified PageRank algorithm is as given below $wpr(u) = (1 - d) + d \sum_{v \in R(u)} wpr(v) W_{in}(v, u) W_{out}(v, u)$

Where:

wpr (u) = page rank of page u that we want to find out

wpr (v) = Page rank value of page v that is pointing to page u

4.3 PageRank based on visits of links (vol)

This algorithm was proposed by Gyanendra Kumar, Neelam Duhan, A. K. Sharma in 2011 at International Conference on Computer & Communication Technology (ICCCT)-2011 [2].

Unlike traditional PageRank algorithm, it does not divide page rank value equally between outgoing links. Instead of this it assign more rank value to the outgoing links which is most visited by users. So in this manner page rank is calculated based on visits of inbound links.

The formula behind this algorithm is given below.

$$PR(u) = (1 - d) + d\sum_{v \in B(u)} (PR(v)/TL(v))L_{u}$$

Where;

4.4 Weighted PageRank based on visits of links (vol)

It was proposed by Neelam Tyagi, Simple Sharma in 2012 at International Journal of Soft Computing and Engineering (IJSCE). In traditional weighted pagerank algorithm, page rank value was assigned to its outgoing links according to its popularity and this popularity can be recorded by two functions namely W_{in} and W_{out} [6].

But this algorithm did not consider the popularity of outlinks.

This new algorithm do consider the weight age of page as well as visits of links (i.e. user behaviour). In this algorithm it assigns more rank value to the outgoing links which is most visited by users and received higher popularity from number of inlinks.

The main work behind this algorithm is to provide relevant information to users when they do searching on search engine and get result based on user behaviour, So that the searching time gets reduced.

The formula behind this algorithm is

WPR_{vol} (u) = (1-d) + $d\sum_{v \in B(u)} (L_u \text{ WPR}_{vol} (v) W_{in} (v, u)) / \text{TL} (v)$

Where;

V. Numerical analysis of different PageRank algorithm

5.1. Example

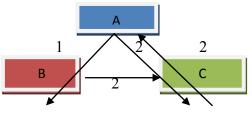


Fig. 2: An Example of Simple Web Pages interaction

5.1.1. PageRank Algorithm:

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Given formula
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pr(u) = (1 - d) + d \sum_{v \in B(u)} pr(v)/l(v)
Initially we assume that
Pr (A) = 1
Pr (B) = 1
Pr (C) = 1
Pr (A) = (1-d) + d (pr (C)/L(C))
Pr (A) = (1-.85) + .85 (1/1)
Pr (A) = 1
Pr (B) = (1-d) + d (pr (A)/2) = (1-.85) + .85 (.5)
Pr (B) = 0.575
Pr (C) = (1-d) + d (pr (A)/2 + pr (B)/1) = 0.15 + 0.85 (0.5 + 0.575)
Pr (C) = 1.063
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5.1.2. Weighted PageRank Algorithm:

Given formula is $wpr(u) = (1 - d) + d\sum_{v \in B(u)} wpr(v)W_{n}^{i}(v, u) W_{out}(v, u)$ Initially we assume that Wpr (A) = 1 $O_A = 2$ $I_A = 1$ WPr (B) = 1 $O_B = 1$ $I_B = 1$ WPr (C) = 1 $I_C = 2$ $O_{C} = 1$ Wpr (A) = $(1 - d) + d (wpr(C)W_{in}(C, A) W_{out}(C, A))$ $W_{in}(C, A) = I_A / I_A = 1/1 = 1$ $W_{out}(C, A) = O_A / O_A = 2/2 = 1$ Wpr (A) = 0.15 + 0.85 (1*1*1)Wpr (A) = 1Wpr (B) = $(1 - d) + d (wpr(A)W_{in}(A, B) W_{out}(A, B))$ $W_{in}(A, B) = I_B / I_{B+} I_C = 1/1+2=1/3$ $W_{out}(A, B) = O_B / O_{B^+} O_C = 1/1 + 1 = 1/2$ Wpr (B) = 0.15 + .85 (1*1/3*1/2)Wpr (B) = 0.2916 $Wpr(C) = (1 - d) + d ((wpr(A)W_{in}(A, C) W_{out}(A, C) + (wpr(B)W_{in}(B, C) W_{out}(B, C)))$ $W_{in}(A, C) = I_C / I_{B+} I_C = 2/2 + 1 = 2/3$ $W_{out}(A, C) = O_C / O_{B+} O_C = 1/2$ In the same manner $W_{in}(B, C) = 1$ $W_{out}(B, C) = 1$ Wpr (C) = 0.15 + 0.85 ((1*2/3*1/2) (0.2916*1*1))Wpr (C) = 0.6811

5.1.3. PageRank based on visits of links (VOL):

Given formula is

 $PR(u) = (1 - d) + d\sum_{v \in B(u)} (PR(v)/TL(v)) L_u$

PR (A) = $(1 - d) + d ((PR(C)/TL(C))* L_A$ $L_A/TL(C) = 2/2 = 1$ PR (A) = 0.15 + 0.85 * (1*1) = 1PR (B) = $(1 - d) + d ((PR(A)/TL(A))* L_B$ $L_B/TL (A) = 1/3$ PR (B) = 0.15 + 0.85 * (1/3 * 1)PR (B) = 0.4333PR (C) = $(1 - d) + d ((PR(A)/TL(A))* L_C) + (PR(B)/TL(B))* L_C)$ $L_C/TL (A) = 2/3$ $L_C/TL (B) = 2/2$ PR (C) = 0.15 + 0.85 ((2/3*1) + (2/2*0.4333)) = 1.0849

5.1.4. Weighted PageRank based on visits of links (VOL):

Given formula is

 $WPR_{vol(u)} = (1-d) + d\sum_{v \in B(u)} (L_u WPR_{vol}(v) W_{in}(v, u)) / TL(v)$ Initially all value are 1 $WPR_{vol} (A) = (1-d) + d (L_A WPR_{vol}(C) W_{in} (C, A))/TL(C))$ $W^{in}(C, A) = I_A / I_A = 1/1=1$ $L_A/TL(C) = 2/2 = 1$ Substituting all these value we'll get $WPR_{vol}(A) = 0.15 + 0.85(1 * 1 * 1) = 1$ $WPR_{vol}(B) = (1-d) + d (L_B WPR_{vol}(A) W_{in}(A, B)) / TL (A))$ $W_{in}(A, B) = I_B / I_{B+} I_C = 1/1 + 2 = 1/3$ $L_{\rm B}/TL(A) = 1/3$ Hence WPR_{vol} (B) = 0.15 + 0.85 (1 * 1/3 * 1/3) = 0.24445 $WPR_{vol}(C) = (1-d) + d((L_C WPR_{vol}(A)W_{in}(A,C))/TL(A)) + ((L_C WPR_{vol}(A)W_{in}(B,C))/TL(B)))$ $W_{in}(A, C) = I_C / I_{B+} I_C = 2/2 + 1 = 2/3$ $L_{C}/TL(A) = 2/3$ $W_{in}(B, C) = 1$ $L_{\rm C}/{\rm TL}$ (B) = 2/2 = 1 $WPR_{vol}(C) = 0.15 + 0.85 ((2/3*2/3*1) + (1*1*0.24445)) = 0.73556$

5.2 Analysis of PageRank value of different PageRanking algorithm on different iteration

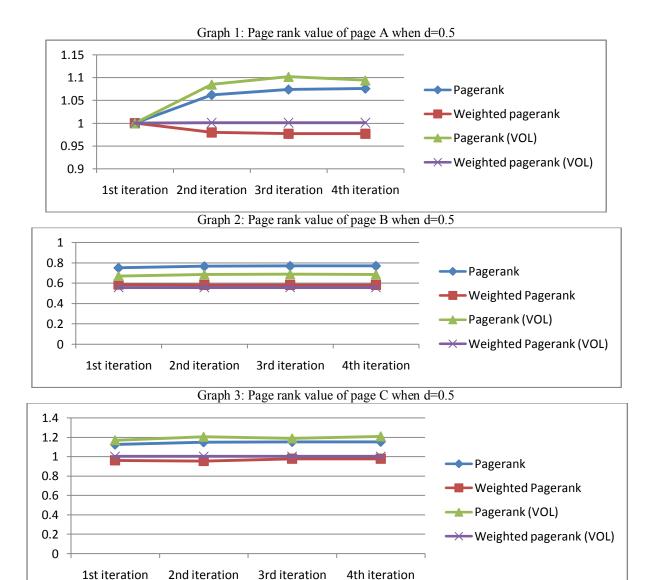
Table 1: Values of page A on different pagerank algorithms and different d values

	Pagerank			Weighted pagerank		Pagerank (VOL)		Weighted pagerank (VOL)				
d	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25
1 st	1	1	1	1	1	1	1	1	1	1	1	1
2 nd	1.053	1.062	1.023	0.728	0.980	1.007	1.072	1.085	1.031	0.775	1.001	1.014
3 rd	1.089	1.074	1.022	0.636	0.977	1.008	1.128	1.102	1.033	0.692	1.001	1.014
4 th	1.114	1.076	1.024	0.606	0.977	1.008	1.167	1.094	1.033	0.659	1.001	1.014

Table 2: Values of page B on different pagerank algorithms and different d value

	Pagerank			Weighted pagerank			Pagerank (VOL)			Weighted pagerank (VOL)		
d	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25
1 st	0.575	0.75	0.875	0.291	0.585	0.792	0.433	0.670	0.835	0.244	0.557	0.778
2^{nd}	0.597	0.765	0.877	0.253	0.583	0.792	0.459	0.684	0.837	0.226	0.557	0.779
3 rd	0.613	0.768	0.878	0.242	0.583	0.792	0.476	0.687	0.837	0.218	0.557	0.779
4 th	0.623	0.769	0.878	0.237	0.583	0.792	0.487	0.685	0.837	0.214	0.557	0.779

		Pagerank		Weighted pagerank		Pagerank (VOL)		Weighted pagerank (VOL)		(VOL)			
	d	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25	0.85	0.5	0.25
Γ	1 st	1.063	1.125	1.093	0.681	0.960	1.318	1.084	1.170	1.126	0.735	1.003	1.056
ſ	2 nd	1.105	1.148	1.097	0.572	0.955	1.032	1.151	1.205	1.132	0.638	1.003	1.058
Γ	3 rd	1.134	1.152	1.097	0.537	0.977	1.032	1.197	1.188	1.132	0.599	1.003	1.058
	4 th	1.153	1.153	1.097	0.524	0.977	1.032	1.229	1.209	1.132	0.584	1.003	1.058



	VI. C	Comparison of Pagel	Rank algorithms	
Algorithm	PageRank	Weighted PageRank	PageRank based on visits of links (VOL)	Weighted PageRank based on visits of links(VOL)
Year of introduce	1996	2004	2011	2012
Author name	Larry page and Sergey brin	Wenpu Xing and Ali Ghorbani	Gyanendra Kumar, Neelam Duhan, A. K. Sharma	Neelam Tyagi, Simple Sharma
Formula used	$pr(u) = (1-d) + d \sum_{v \in B(u)} pr(v)/l(v)$	$wpr(u) = (1 - d) + d\sum_{v \in B(u)} wpr(v)W_{in}(v, u)W_{out}(v, u)$	$PR(u) = (1 - d) + d\sum_{v \in B(u)} (PR(v)/TL(v))L_u$	$WPR_{vol}(u) = (1-d)$ + $d\sum_{v \in B(u)} (L_u$ WPR _{vol} (v) W _{in} (v, u))/ TL (v)
Scope	Web pages	Web pages	Web pages	Web pages
Description/ Working style	Calculate page rank based upon number of backlinks	Assign more page rank value to popular page	Assign more rank value to the outgoing links which is most visited by users	Assign more rank value to the outgoing links which is most visited by users and received higher popularity from number of inlinks

PageRank algori	thm and its variations.	A Survey report
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Technique used	Web structure mining	Web structure mining	Web structure mining, web usage mining	Web structure mining, web usage mining
I/P parameters	Backlinks	Backlinks, Forward links	Backlinks, Forward links	Backlinks, Forward links
Importance	More	More than PR	More than PR	More than WPR
Advantage	High quality results, backlink predictor, advertising business, frequently indexing	More relevant page than traditional pagerank algorithm	This concept is very useful to display most valuable pages on the top of the result list on the basis of user browsing behaviour, which reduces the search space to a large scale.	The proposed algorithm is used to find more relevant information according to user's query.
Disadvantage	False page rank or spoof page rank, equal distribution of page rank	Does not consider user access pattern	Not none	Not none
Efficiency	Moderate	High	Moderate	High
Similarities	Damping factor	Damping factor	Damping factor	Damping factor
Dissimilarities	Basic formula	Introduce two entities W ⁱⁿ and W ^{out}	Introduce L_u and $TL(v)$	It combined L _u , W ⁱⁿ and TL (v) all together
Convergence	Slow as compare to weighted pagearnk and weighted pagerank (VOL)	Very fast	Slow as compare to weighted pagearnk and weighted pagerank (VOL)	Very fast

VII. Conclusion

From the graph we can analyze that pagerank and pagerank (VOL) gives high result as compare to both weighted pagerank and weighted pagerank (VOL) for d=0.5. But when we compare Pagerank and Pagerank (VOL), these two algorithms may pass one another depending upon condition, sometimes Pagerank gives higher value, sometimes Pagerank (VOL). We can also observe from table that for small value of d that means if users often visit web pages randomly (d=0.25 means probability of user to follow link structure is 0.25) page rank of web page converge quickly as compare to large value of d.

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