

## Analysis of precoded orthogonal space-time block codes over fading MIMO channels

Shital R. Shegokar<sup>1</sup>

<sup>1</sup>(Asst. Professor, Department Of Electronics And Telecommunication Engineering, Mauli college of Engineering/SGBAU, India)

**ABSTRACT:** Recent information theory research has shown that the rich-scattering wireless channel is capable of enormous theoretical capacities if the multipath is properly exploited. This is possible with one of the most promising technology for future generation wireless communication called as Multiple-Input Multiple-Output (MIMO). MIMO architecture allows getting the diversity benefit or increased data rate. The work in the report will be aimed at evaluating performance of MIMO system targeted for improving data rate or capacity. Orthogonal space time coding (OSTBC) have been developed in order to exploit the advantages of MIMO system such as Diversity gain. Pre-coding technique can be combined rate with OSTBC to adapt to the current channel condition without changing the fixed structure of the transmitters and receiver. MIMO Pre-coding techniques aim at improving the system performance. Finally we present simulation results that validate our analysis.

**Keywords:** Diversity gain, MIMO, OSTBC, PRE-CODER

### I. INTRODUCTION

Impressive improvements in capacity and bit error rates (BERs) have motivated multiple-antenna radio systems, also known as multiple-input multiple-output (MIMO) systems. Along with the gains, however, comes a price in hardware complexity. The radio front end has a complexity, size, and cost of the hardware associated with each antenna (radio frequency power amplifiers, AD/DA converters etc) that scales with the number of antennas. Antenna sub-set selection where transmission and reception is performed through a selection of the total available antennas is a powerful solution that reduces the need for multiple RF chains yet retains many of the advantages of MIMO systems. This article is dedicated to a tutorial overview of MIMO antenna selection methods. MIMO signaling can improve wireless communication in two different ways: diversity methods (space time coding method) and spatial multiplexing. Diversity methods improve the robustness of the communication system in terms of BER by exploiting the multiple paths between transmit and receive antennas. Defining *diversity order* as the slope of the BER–signal-to-noise ratio (SNR) curve, space-time codes are capable of delivering diversity order of  $n_T n_R$  where  $n_R$  and  $n_T$  are the number of receive and transmit antennas, respectively.

The idea of space-time block coding (STBC) has been pro-posed by Alamouti [1] for transmitting signals in a complex modulation alphabet over two independently fading channels. This code achieves diversity two and has rate one, as it transmits two symbols in two time intervals. The theory of space-time block codes has been further developed by Tarokh, Jafarkhani and Calderbank [2]. They defined orthogonal space-time block codes (O-STBC) that lead to full diversity and low decoding complexity at the receiver, at the price of some loss in data rate. [3-4] Full data rate is achievable in connection with full diversity only in the case of two transmit antennas. *Orthogonal space time coding (OSTBC) has been developed in order to exploit the advantages of MIMO system such as Diversity gain. Pre-coding technique can be combined rate with OSTBC to adapt to the current channel condition without changing the fixed structure of the transmitters and receiver. MIMO Pre-coding techniques aim at improving the system performance and to maintain the full diversity.*

### 1.1 MIMO System model

With the integration of Internet and multimedia applications in next generation wireless communications, the demand for wide-band high data rate communication services is growing. As the available radio spectrum is limited, higher data rates can be achieved only by designing more efficient signaling techniques. Such large gains in capacity of communication over wireless channels are feasible in Multiple-input multiple-output (MIMO) systems.

The MIMO channel is constructed with multiple element array antennas at both ends of the wireless link. Let us consider a single point-to-point MIMO system with arrays of  $n_T$  transmit and  $n_R$  receive antennas. We focus on a complex baseband linear system model described in discrete time. The system block diagram is shown in Fig. 1. The transmitted signals in each symbol period are represented by an  $n_T \times 1$  column matrix  $\mathbf{x}$ , where the  $i$ th component  $x_i$ , refers to the transmitted signal from antenna  $i$ .

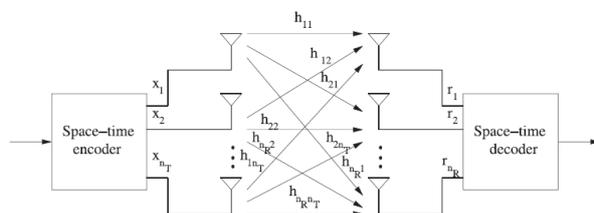


Fig 1. Block diagram of a MIMO System [8]

### 1.2 MIMO Channel Capacity

In this section, we study the capacity of a MIMO channel with  $n_R$  outputs. We assume that the receiver knows the realization of the channel that is it knows both  $\mathbf{r}$  and  $\mathbf{H}$ . For the transmitter, we study the case when the transmitter does not know the realization of the channel; however, it knows the distribution of  $\mathbf{H}$ . This corresponds to an open-loop system. The channel path gains follow a Rayleigh fading channel model.

At each symbol period, transmitted signals,  $x_i, i = 1, 2, \dots, n_T$  are transmitted simultaneously from  $n_T$  transmit antennas. The received signal  $r$ , which is received at antenna  $n_R$ , is given by [5]

- The capacity of a MIMO channel is a function of the channel matrix  $\mathbf{H}$ . Considering the random nature of the channel matrix  $\mathbf{H}$ , the capacity of a MIMO channel subject to the input covariance matrix being an iid Rayleigh fading model can be considered as the following random variable [8]:

$$C = W \log_2 \det \left( \mathbf{I}_{n_T} + \frac{P}{n_T \sigma^2} \mathbf{Q} \right) \dots\dots\dots(1)$$

$$\mathbf{Q} = \begin{cases} \mathbf{H}\mathbf{H}^H, & n_R < n_T \\ \mathbf{H}^H\mathbf{H}, & n_R \geq n_T \end{cases} \dots\dots\dots(2)$$

- If the number of transmit and receive antennas are the same, the capacity increases at least linearly as a function of number of antennas.
- At high SNRs, a 3 dB increase in SNR results in  $\min\{n_T, n_R\}$  extra bits of capacity.
- The ergodic capacity of a MIMO channel, when the channel is isotropic, given any side information that exists at the transmitter, is [8]:

$$C = E \left\{ W \log_2 \det \left[ \left( \mathbf{I}_r + \frac{P}{\sigma^2 n_T} \mathbf{Q} \right) \right] \right\} \dots\dots\dots(3)$$

$$\mathbf{Q} = \begin{cases} \mathbf{H}\mathbf{H}^H, & n_R < n_T \\ \mathbf{H}^H\mathbf{H}, & n_R \geq n_T \end{cases} \dots\dots\dots(4)$$

- If the number of transmit and receive antennas are the same, the capacity increases at least linearly as a function of number of antennas.
- At high SNRs, a 3 dB increase in SNR results in  $\min\{n_T, n_R\}$  extra bits of capacity.
- The ergodic capacity of a MIMO channel, when the channel is isotropic, given any side information that exists at the transmitter, is [8]:

$$C = E \left\{ W \log_2 \det \left[ \left( \mathbf{I}_r + \frac{P}{\sigma^2 n_T} \mathbf{Q} \right) \right] \right\} \dots\dots\dots(5)$$

$$\mathbf{r} = \mathbf{H} \mathbf{x} + \mathbf{n} \dots\dots\dots(6)$$

The resulting capacity of the channel is a random variable because the capacity is a function of the channel matrix  $\mathbf{H}$ . The distribution of the capacity is determined by the distribution of the channel matrix  $\mathbf{H}$ . The total transmitted power is kept the same for different numbers of transmit antennas to make a fair comparison.

Fig2. Capacity vs SNR for SISO, SIMO(1X4) and MIMO(2X4)

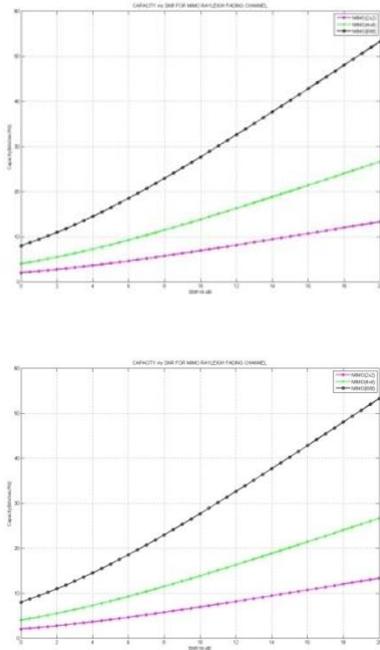


Fig3. Capacity vs SNR for MIMO (2X2), (4X4), (8X8) systems.

## II. CHANNEL CAPACITY OF OSTBC:

The SNR at receiver is given by :

$$\frac{\rho}{M_T} \|H\|_F^2 \dots\dots\dots (8)$$

and the capacity is given by [7]:

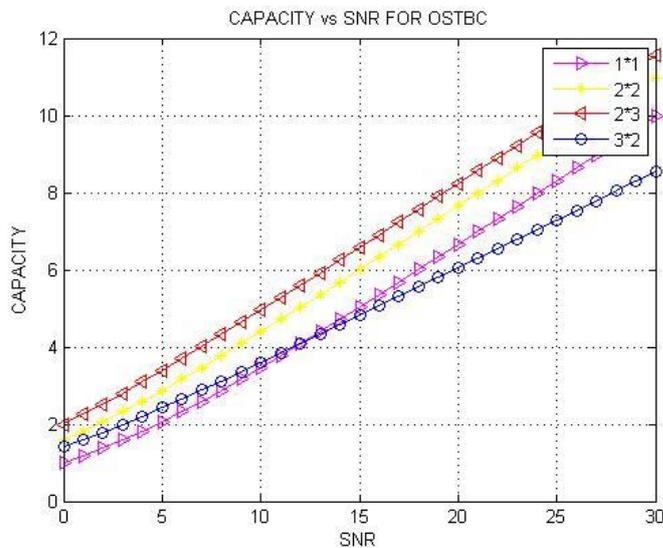
$$C_{OSTBC} = r_s \left( 1 + \frac{\rho}{M_T} \|H\|_F^2 \right) \dots\dots\dots (9)$$

where  $r_s$  is the code rate.

$$\begin{aligned}
 C &= \log_2 \left( \det \left( I_{M_T} + \frac{E_s}{M_T N_o} HH^H \right) \right) \dots\dots\dots (10) \\
 &= \log_2 \left( \prod_{i=1}^r \left( 1 + \frac{\rho}{M_T} \lambda_i \right) \right) \\
 &= \log_2 \left( 1 + \frac{\rho}{M_T} \|H\|_F^2 + 1 + \frac{\rho}{M_T} (\dots) + \dots \right) \\
 &\geq C_{OSTBC}
 \end{aligned}$$

where  $\lambda_k$  eigenvalues of  $HH^H$  and

$$\sum_{i=1}^r \lambda_i = \text{Tr}(HH^H) = \|H\|_F^2 \dots\dots\dots (11)$$



**Fig 4: Channel capacity of OSTBC**

### III. PRECODER FOR IMPROVING ERROR PROBABILITY & DIVERSITY ORDER

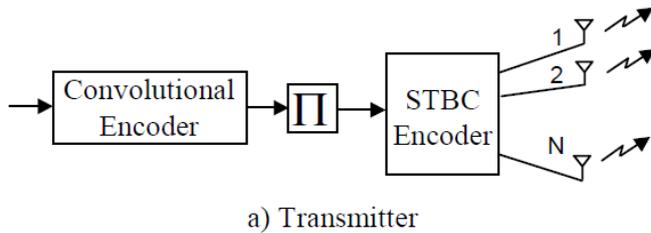
#### 3.1 System Model:

The system under consideration is shown Fig.5. As shown in figure, the incoming data is encoded by a convolution code of rate  $r$ , interleaved, and then encoded by STBC encoder. The output of the STBC encoder is then transmitted from the  $N$  transmit antennas.

At the receiver, after demodulation, matched-filtering, and sampling, the signal  $r_{jt}$  received by antenna  $j$  at time  $t$  is given by:

$$r_t^j = \sum_{i=1}^N \alpha_{i,j}(t) c_t^i + w_t^j \dots\dots\dots(12)$$

where  $c_t^i$  is the signal transmitted from antenna  $i$  at time  $t$  the noise  $w_t^j$  at time  $t$  is modeled as independent samples of a zero-mean complex Gaussian random variable (RV) with variance  $N_0/2$  per dimension. The coefficients  $\alpha_{i,j}(t)$  model fading between the  $i^{\text{th}}$  transmit and  $j^{\text{th}}$  receive antennas at time instant  $t$  and are assumed to be complex Gaussian random variables with variance 0.5 per dimension. In addition, the fading coefficients are assumed to be constant over a block of  $N$  consecutive symbols within a frame and vary independently from one block to another. [9]:



**Fig 5.** MIMO system model without antenna selection

**3.2 Performance Analysis of the Full-Complexity System:**

When a CC code is concatenated with a STBC code over fading channels, both time and space diversity gains can be achieved. The maximum diversity order that can be achieved in this case is  $NM d_{\min}$  where  $d_{\min}$  is the minimum Hamming distance of the CC code employed.

Assuming maximum likelihood decoding, the conditional pairwise error probability is then given by:

$$\begin{aligned}
 P_2(d | \alpha) &= Q \left( \sqrt{2 \frac{E_b}{N_0} \frac{r}{N} \sum_{l=1}^d \sum_{i=1}^N \sum_{j=1}^M |\alpha_{l,i,j}|^2} \right) \dots\dots\dots(13) \\
 &= Q \left( \sqrt{2 \frac{E_b}{N_0} \frac{r}{N} \sum_{k=1}^{NMd} |\alpha_k|^2} \right),
 \end{aligned}$$

where  $d$  is the Hamming distance between the transmitted codeword and the erroneous codeword that the receiver decides on, and  $\alpha_{l,i,j}$  for  $l = 1, 2, \dots, d$ ,  $i = 1, 2, \dots, N$ ,  $j = 1, 2, \dots, M$  is the fading coefficient of channel between  $i^{\text{th}}$  transmitter antenna and  $j^{\text{th}}$  receiver antenna at time index  $l$  at which the two code words differ. To compute the average bit error rate, we need to average the expression in (2) with respect to the random variables (RVs)  $|\alpha_k|^2$  for  $k = 1, \dots, NMd$ . To simplify the analysis, we first introduce an auxiliary RV that we denote by  $Y$ , defined as:

$$Y = \sum_{k=1}^{NMd} |\alpha_k|^2 \dots\dots\dots(14)$$

Note that Y is a Chi-square RV with 2NMd degrees of freedom with probability density function

$$f(y) = \frac{1}{(NMd - 1)!} y^{NMd-1} e^{-y}, \quad y \geq 0. \quad \dots\dots\dots (15)$$

(pdf) given by [9]:

Consequently, the average pair wise error probability can be shown to be [9]:

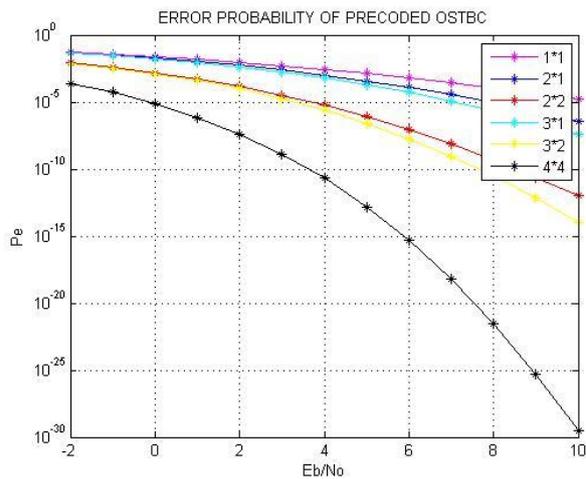
$$P_2(d) \approx \binom{2NMd - 1}{NMd} (4\gamma_s)^{-NMd} \quad \dots\dots\dots (16)$$

This clearly shows that the diversity order is NMdmin Accordingly; the BER of this concatenation scheme is upper bounded by:

$$P_b(e) \leq \sum_{d=d_{min}}^{\infty} \beta_d P_2(d) \quad \dots\dots\dots (17)$$

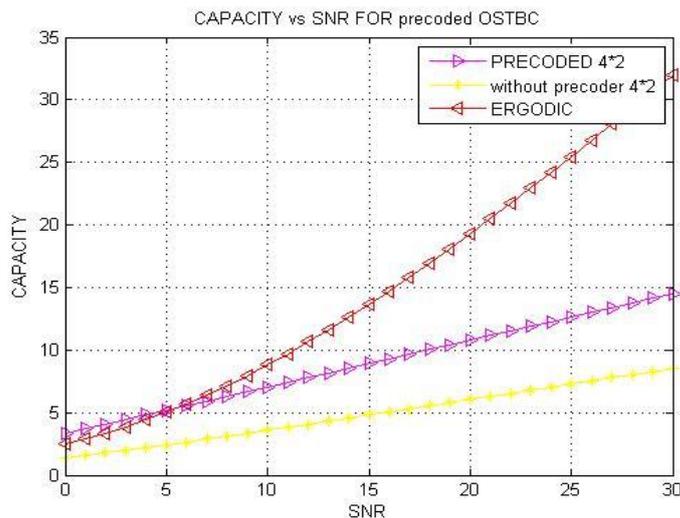
Where  $\beta_d$  is the multiplicity corresponding to distance d.

**3.3 Error Probability of Pre-coded OSTBC:**



**Fig 6. Error Probability of Pre-coded OSTBC**

**3.4 Capacity Comparison of with Pre-coder, without Pre-coder and Ergodic Capacity:**



**Fig 7: Capacity Comparison**

#### IV. CONCLUSION

1. Using Pre-coder, we achieved optimal capacity and better error performance.
2. By using linear pre-coding techniques, we improved the error probability and capacity.
3. By using a Convolution encoder and an Interleaver as a pre-coder, we achieved better error performance and diversity order.

It is clear that one of the advantages of concatenating a STBC code with an outer CC code is the significant increase in the diversity. On the other hand, it is also clear that the consequence of this concatenation is a reduction in the bandwidth efficiency, which may become crucial in high data rate applications.

Thus MIMO system is an important key for enabling the wireless industry to deliver on the vast potential and promise of wireless broadband.

#### REFERENCES

- [1] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE JSAC*, vol. 16, no. 8, pp. 1451-1458, 1998.
- [2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456-1467, 1999.
- [3] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge University Press, 2003.
- [4] E. G. Larsson and P. Stoica, *Space-time block coding for wireless communications*. Cambridge, UK: Cambridge University Press, 2003.
- [5] B. Vucetic and J. Yuan, *Space-Time Coding*, John Wiley, 2003.
- [6] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, July 1999.
- [7] S. Zhou and G. B. Giannakis, "MIMO Communications with Partial Channel State Information," chapter in the book *Space-Time Processing for MIMO Communications*, A.B. Gershman and N. Sidiropoulos, Editors, John Wiley & Sons, 2005.
- [8] B. Vucetic, J. Yuan, "Space Time Coding", John Wiley and Sons.
- [9] Performance Analysis of Combined Convolutional Coding and Space-Time Block Coding with Antenna Selection† Xiang Nian Zeng and Ali Ghayeb.- 2 0-7803-8794-5/04/ 004 IEEE