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Multi-Resolution Image Processing Techniques

Harshula Tulapurkar [1], Rajesh Bansode [2]
[1](Department of Information technology, Thakur College of Engineering and Technology/ Mumbai University, India)
[2](Head of Department of Information technology, Thakur College of Engineering and Technology/ Mumbai University,)

Abstract: *A digital image is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels. This pixel value represents various aspects of the image like color, brightness, etc. In recent years Multi-Resolution Analysis (MRA) techniques are applied for image analysis. MRA offers a framework for extracting information from image at various resolutions and can be applied to variety of problems in signal and image processing. It is believed that the Human Visual System (HVS) offers a hierarchical approach of extracting details from the view. For e.g. when we visit an unfamiliar area first thing that catches our sight is large buildings, trees etc. and once we are familiar with the view the subtle and minor features like foliage, color hues etc. are noticed. Large amount of data in digital images poses problem in storage, display and processing. These and other requirements like feature detection and extraction can be addressed by MRA. In this paper we take a look at the various MRA techniques like Wavelet, Ridgelet, Curvelet and Contourlet*
Index Keywords: *Multi Resolution, Frequency Domain, Discrete Wavelet Transform, Ridgelet Transform, Contourlet Transform, Curvelet Transform*

I. Introduction

A digital image is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels. An image is a two-dimensional function f(x,y), where x and y are the spatial (plane) coordinates, and the amplitude of values, called picture elements or p. Pixel values typically represent gray levels, colors, heights, opacities etc. In most natural images, the energy is concentrated on the lower frequency range. If we see images we see that connected regions of similar texture or gray level combine to form objects. If objects are small in size or low in contrast we examine at high resolutions. MRA, as implied by its name, analyzes the signal at different frequencies with different resolutions. Multi-resolution offers a natural, hierarchical view of information.

II. Why Transform?

Transform theory plays a fundamental role in image processing, as working with the transform of an image instead of the image itself may give us more insight into the properties of the image. Transform offers

- Better image Processing.
- Conceptual insights into spatial frequency information. What it means to be smooth, moderate change , fast change etc.
- Fast computation.
- Alternative representation and sensing.
- Efficient storage and transmission

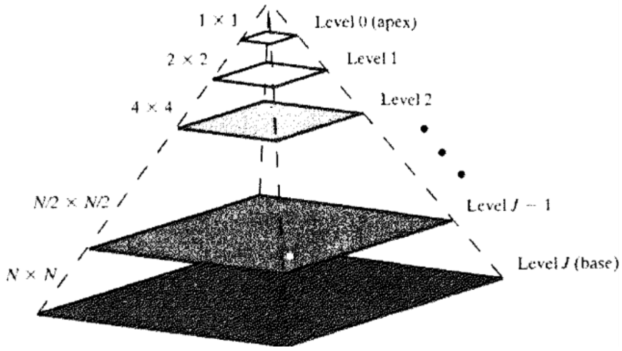
There are two domains of transformation-Spatial and Frequency. Here in this paper we focus on Frequency domain transformations in which the image is converted into its frequency distribution. Image has two-frequency components- High frequency components correspond to edges in an image whereas Low frequency components in an image correspond to smooth regions.

Discrete Fourier Transform (DFT) is the most common and powerful procedure to analyze, manipulate and synthesize digital signals. It is used to determine the harmonic or frequency content of a signal. Fourier theorem states that a "Periodic function f(x) may be expressed as the sum of a series of sine or cosine terms" (called the Fourier series), each of which has specific amplitude and phase coefficients known as Fourier coefficients. DFT decomposes an image in sine and cosine form.

The big disadvantage of a Fourier expansion however is that it has only frequency resolution and no time resolution. This means that although we might be able to determine all the frequencies present in a signal, we do not know when they are present. The wavelet transform or wavelet analysis overcomes this shortcoming of the Fourier transform by giving a time-frequency joint representation.

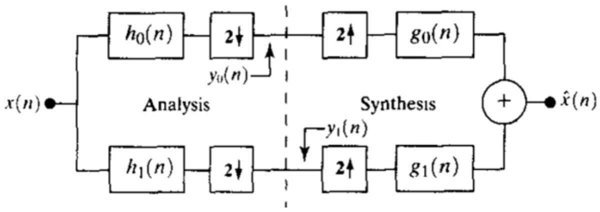
The idea behind these time-frequency joint representations is to cut the signal of interest into several parts and then analyze the parts separately. It is clear that analyzing a signal this way will give more information about the when and where of different frequency components. In wavelet analysis a fully scalable modulated window is used to cut the signal. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of this collection of representations we can speak of a multiresolution analysis (MRA).

III. Image Pyramids And Sub band Coding



Simple structure of representing images at different resolutions is Image Pyramid. An Image Pyramid is collection of decreasing resolution images arranged in shape of a pyramid.[3]

Figure 1 Image Pyramid The Base is high-resolution representation of image of size N x N being processed whereas the apex contains low-resolution approximation. As one moves up the pyramid, both size and resolution decrease. Fully populated pyramids are composed of j+1 levels where J=LOG2N Most pyramids are truncated to P+1 levels where 1<P<J



Subband Coding: In Subband coding an image is decomposed into a set of band limited components called subbands, which can be reassembled to reconstruct the original image without errors. .[3]

Figure 2 two-band filter bank for 1D subband coding.

IV. Continuos And Discrete Wavelet Transform

Unlike Fourier transform, whose basis function is sinusoids, wavelet transforms are based on small waves called wavelets of varying frequency and limited duration. The continuous wavelet transform uses a single function ψ(t) and all its dilated and shifted versions to analyze functions.

$$CWT_x^{\psi}(\tau, s) = \Psi_x^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t)\psi^*\left(\frac{t-\tau}{s}\right) dt$$ **.........**Eq. 1.1

Eq 1.1 [1] states the transformed signal is a function of two variables tau and s, the translation and scale parameters, respectively. psi(t) is the transforming function, and it is called the mother wavelet . The term tau (translation) is related to the location of the window, as the window is shifted through the signal. Scale parameter, which is defined, as 1/frequency is similar to the scale used in maps. As in the case of maps, high scales correspond to a non-detailed global view (of the signal), and low scales correspond to a detailed view. Similarly, in terms of frequency, low frequencies (high scales) correspond to global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to detailed information of a hidden pattern in the signal.

COMPUTATION OF CWT

- Let x (t) is the signal to be analyzed. The mother wavelet (common ones are Haar, Morlet, Meyer) All the windows that are used are the dilated (or compressed) and shifted versions of the mother wavelet.
- Initial scale of s=1 is chosen. The CWT is computed for different values of s. Wavelet computation is done by multiplying the signal with wavelet function and then integrated over all times However, depending on the signal, a complete transform is usually not necessary. For all practical purposes, the signals are band limited, and therefore, computation of the transform for a limited interval of scales is usually adequate. For convenience, the procedure will be started from scale s=1 and will continue for the increasing values of s, i.e., the analysis will start from high frequencies and proceed towards low frequencies. This first value of s will correspond to the most compressed wavelet. As the value of s is increased, the wavelet will dilate.

– The wavelet is placed at the beginning of the signal at the point which corresponds to time=0. The wavelet coefficients for s=1,t-0 are computed. The result of the integration is then multiplied by the constant number 1/sqrt{s}. This multiplication is for energy normalization purposes so that the transformed signal will have the same energy at every scale. The final result is the value of the transformation, i.e., the value of the continuous wavelet transform at time =0and scale s=1. In other words, it is the value that corresponds to the point tau =0 , s=1 in the time-scale plane.

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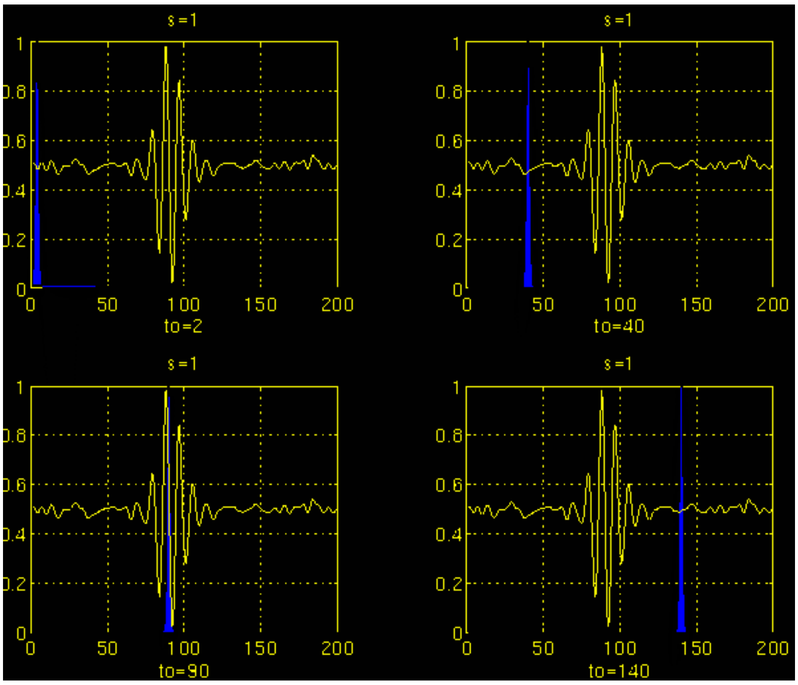


Figure 3 CWT process step by step. Scale=1 and varying version of Tau. Scale=1 captures highest frequency. Notice width of blue band indicates scale. .[2] Fundamental concepts & an overview of the wavelet theory Robi Polikar

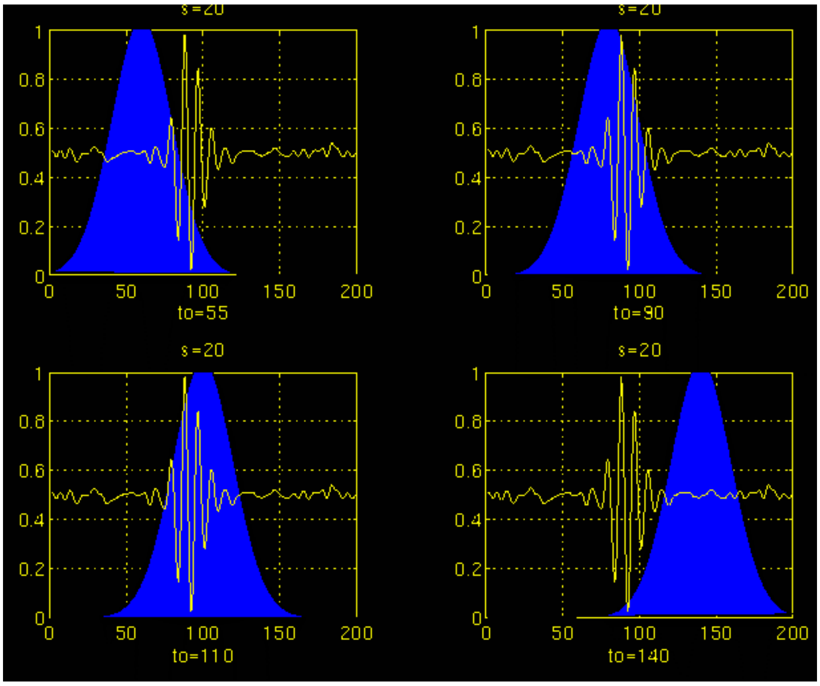


Figure 4 - Notice width of blue band is more as scale has increased indicating low frequency.[2] Fundamental concepts & an overview of the wavelet theory Robi Polikar

– The wavelet at scale s=1 is then shifted towards the right by tau amount to the location t=tau , and the wavelet coefficients are computed to get the transform value at t=tau , s=1 in the time- frequency plane. This procedure is repeated until the wavelet reaches the end of the signal. One row of points on the time-scale plane for the scale s=1 is now completed.
– The above procedure is repeated for every value of s. Every computation for a given value of s fills the corresponding single row of the time-scale plane. When the process is completed for all desired values of s, the CWT of the signal has been calculated.

The main idea of Discrete Wavelet Transform (DCT) is the same as it is in the CWT. A time-scale representation of a digital signal is obtained using digital filtering techniques. Changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times computed CWT. In the discrete case, filters of different cutoff frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters to analyze the high frequencies, and it is passed through a series of low pass filters to analyze the low frequencies. The resolution of the signal, which is a measure of the amount of detail information in the signal, is changed by the filtering operations, and the scale is changed by up-sampling and down-sampling (subsampling) operations. Subsampling a signal means removing some of the samples of the signal. For example, subsampling by two refers to dropping every other sample of the signal.

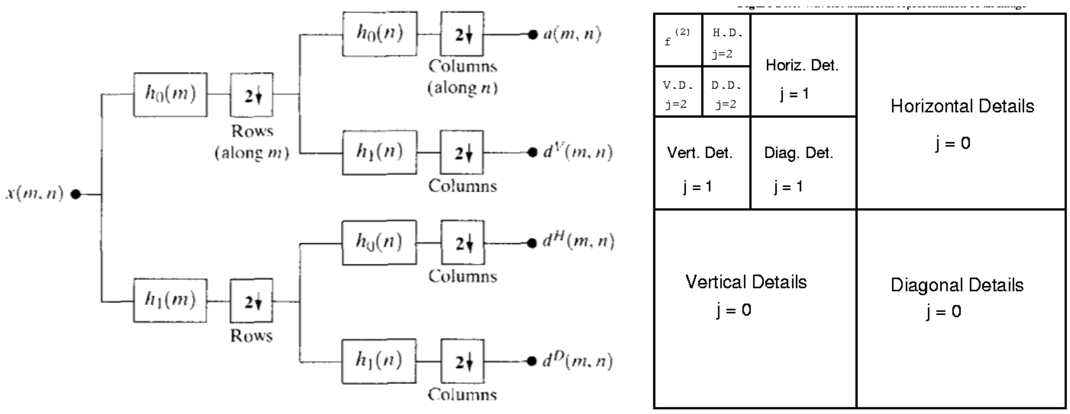


Figure 5- Wavelet Transform representation of an image[1] Sparse Image and Ridgelet Processing

"Approximation" coefficients

$$W_{\Phi}(j_0,k) = \frac{1}{\sqrt{M}} \sum_x f(x)\,\varphi_{j_0,k}(x)$$

"Detail" coefficients

$$W_{\Psi}(j,k) = \frac{1}{\sqrt{M}} \sum_x f(x)\,\psi_{j,k}(x)$$

The wavelet coefficients measure how closely correlated the wavelet is with each section of the signal. For compact representation, choose a wavelet that matches the shape of the image components – Example: Haar wavelet for black and white drawings

Merits of DWT over DCT:
– DWT gives better visual image quality as it understands the working of HVS more clearly.
– DWT defines the multi resolution description of the image. So, the image can be shown in different levels of resolution and proceed from low resolution to high resolution.

Demerits of DWT over DCT:
– DWT is more complex than the DCT. It takes 54 multiplications for computing block of 8x8; distinct wavelet calculation depends upon the length of the filter used.
– Computation cost is higher and its computation time is longer.

V. Ridgelet And Curvelet Transform

Ridgelets and curvelets are special members of the family of multiscale orientation-selective transforms and were developed as an answer to the weakness of the separable wavelet transform in sparsely representing what appears to be simple building-block atoms in an image, that is, lines, curves, and edges.[2] Ridgelet transform involves taking a wavelet transform (1-D WT) along the radial variable in the Radon domain. The curvelet transform, like the wavelet transform, is a multiscale transform, with frame elements indexed by scale and location parameters. It preserves the same time frequency localization property as for wavelets and at the same time, with their elongated support in the Fourier domain, curvelet become directional. It acts like a bandpass filter. In addition, anisotropic scaling principle, which is quite different from the isotropic scaling of wavelets, helps in sparse representation. The elements obey a special scaling law, where the length of the support of a frame elements and the width of the support are linked by the relation width ≈ $length^{2.}$

Figure 6- Curvelet with fixed orientation and location and varying scale[7]

Figure 7- Curvelet with fixed orientation and scale and varying location [7]

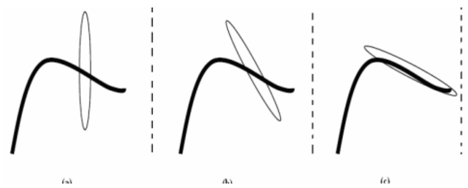


Figure 8- Curvelet with fixed scale and location and varying orientation [7]

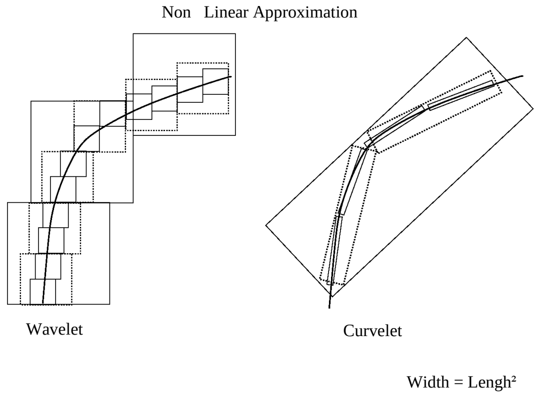


Figure 9 Parabolic scaling, Non-linear approximation [7]

Motivated by the need of image analysis, Candes and Donoho developed Curvelet transform in 2000.

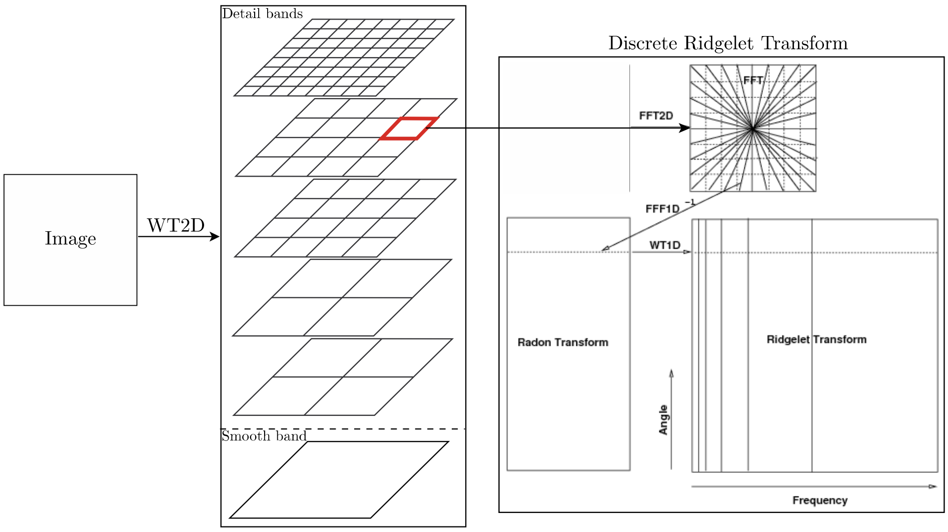
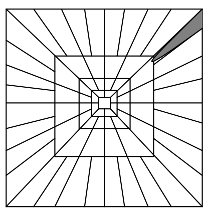


Figure 9 First generation Curvelet transform DGCT1[1]

Computation cost of DCTG1 is very high and so Second Generation Curvelet transform was designed (DCTG2). In DCTG2, first 2D Fast Fourier Transform (FFT) of the image is taken. The 2D Fourier frequency plane is then divided into wedges. The parabolic shape of wedges is the result of partitioning the Fourier plane into dyadic squares and angular divisions. Each square represents a scale and acts like a bandpass filter and the angular divisions partition the band passed image into different angles or orientations. Thus if we want to deal with a particular wedge we'll need to define it's scale j and angle l . Each of the wedges



corresponds to a particular Curvelet (shown as ellipses) at a given scale and angle. This indicates that the inverse FFT of a particular wedge if taken, will determine the Curvelet coefficients for that scale and angle. Outer scales represent the higher frequency components whereas the innermost square represents the low frequency components. Intermediate ones represent medium frequencies. It is these scales that will be used for embedding the watermarks. There are two implementation of DCTG2- USFFT and wrapping. The unequispaced FFT (USFFT) implementation uses a nonstandard interpolation and has a drawback of a higher computational burden compared to the wrapping-based implementation.

The Contourlet's tight frame of Do and Vetterli (2003b) implements the CurveletG2 idea directly on a discrete grid using a perfect reconstruction filter bank procedure. In Lu and Do (2003), the authors proposed a modification of the Contourlet with a directional filter bank that provides a frequency partitioning which is close to the curvelets, but with no redundancy.

Algorithm 1 DCTG1.

Require: Input $n \times n$ image $f[i_1, i_2]$, type of DRT (see above).

Apply the à trous isotropic WT2D with J scales,
Set $B_1 = B_{\min}$,
for $j = 1, \ldots, J$ **do**
  Partition the sub-band w_j with a block size B_j and apply the DRT to each block,
  if j modulo $2 = 1$ **then**
    $B_{j+1} = 2B_j$,
  else
    $B_{j+1} = B_j$.
  end if
end for

Task: Compute the DCTG2 of an $N \times N$ image f.
Parameters: Coarsest decomposition scale, curvelets, or wavelets at the finest scale.
Apply the 2-D FFT and obtain Fourier coefficients $\hat{f}$.
for each scale j and orientation ℓ **do**

1. Form the product $\hat{f}\hat{u}_{j,\ell}$.
2. Wrap this product around the origin.
3. Apply the inverse 2-D FFT to the wrapped data to get discrete DCTG2 coefficients $\alpha_{j,\ell,\mathbf{k}}$.

Output: $\mathcal{C} = (\alpha_{j,\ell,\mathbf{k}})_{j,\ell,\mathbf{k}}$, the DCTG2 of f.

Algorithm 2 DCTG2-wrapping[1]

VI. Conclusion

Multi-resolution Analysis is an effective paradigm that offers a hierarchical view of information. Features that go undetected in one resolution may be easy to spot in another. As a computational tool it can be offered to a variety of problems in image processing. For e.g. feature detection and extraction can be performed effectively and quickly using multiresolution techniques to analyze it. the surface of the huge body of material recently produced about the discrete wavelet transform.

This report gave an overview of some multiscale transforms; namely wavelets, ridgelets and curvelets and their potential applicability on a wide range of image processing problems. Although these transforms are not adaptive, they are strikingly effective both theoretically and practically on piecewise images away from smooth contours.

However, in image processing, the geometry of the image and its regularity is generally not known in advance. Therefore, to reach higher sparsity levels, it is necessary to find representations that can adapt themselves to the geometrical content of the image. Geometric transforms such as wedgelets or bandlets allow defining an adapted multiscale geometry. These transforms perform a non-linear search for an optimal representation. They offer geometrical adaptivity together with stable algorithms.

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